Control of multi-mobile agent interconnected dynamic of motion

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This work treats the dynamic control of multi-mobile robot formation taking robot dynamic interconnections. The dynamic of each agent is modeled by a nonlinear second order differential equation, and its behavioral control will depends on attractive or repulsive interconnection function. The interconnection dynamic function is built around certain estimated parameters, and taking the dynamic of agents in neighbor. Once the target/objectif is fixed, the formation convergence in presence of known obstacles is obtained through a stabilizing nonlinear sliding mode controller, and under the bound of the interconnection parameters. Some bio-inspired examples can be concerned by our modeling and control approaches, one thinks to the autonomy of a herd of sheep in displacement, a flock of birds or a school of fish, and in generally the problem of swarms.

Keywords: multi-mobile robot; formation stabilization, dynamic interconnection.

1. Introduction

The automatic control algorithm resulting from the stability analyzes of a multi-agent system in formation is an attractive area in control theory. Some results issue from the linear or nonlinear stability theory cannot be applied directly to systems in formation, because, one thinks not only to the stability of each *agent* but also to the formation shape stability. Such an investigation will led to us as centralized or decentralized agents behavior. A common approaches for formation feedback control are based on multi-agent kinematics.[1–3] While the kinematic behaviors of swarms, especially for micro/nano robots, the models are inspired from continuous physical behavior of particles like the diffusion equations, reaction-diffusion phenomena[4] or wave equation.[5] In order to ensure information exchange, a unified, distributed formation control architecture that accommodates an arbitrary number of group leaders and arbitrary information flow among vehicles is proposed by Ren.[2] The architecture requires only local neighbor-to-neighbor information exchange. The formation coalition and its synchronization was studied by Olfati-Saber[6] where a particle that evolves on a sphere (swarms on sphere) exhibits self-organization. The self-organization process around a target was studied also by EL kamel[1] using decentralizing control, and general rules are given to overcome obstacles as perturbation while the formation reach the target. In multi-agent dynamic behavior, the acceleration of each agent is introduced and the problem deals with input and interconnection in forces. Thinking of multi-agent control with dynamics, one cites the work of Chang,[7] Kowalczyk,[8] Essaghaier[9] and Gazi.[10] In Chang,[7] techniques using gyroscopic forces and scalar potentials are used to create swarming behaviors for multiple agent systems. The methods result in collision avoidance between the agents as well as with obstacles. While in Kowalczyk,[8] the paper expands existing framework proposing attraction area potential function which allows to build formation and repulsion potential function which allows to avoid collisions with obstacles. The flexible virtual structure which is an attractive area in formation control was analytically solved by Essaghaier.[9] In this paper the dynamic control of multi-agent system is treated including an unknown interconnection function between the agents and their neighbors. Considering that this function is not well known, the estimated interconnection parameters are combined with a sliding mode control law ensuring the robust stabilization of the formation with obstacles avoidance.

2. Problem description

Consider n agents moving in the plane where the dynamic behavior of the i^{th} agent is described by

$$\begin{pmatrix} \ddot{x}_i \\ \ddot{y}_i \end{pmatrix} = \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix} \tag{1}$$

where for $i \in \mathbb{N}$, (x_i, y_i) denote the cartesian coordinates (positions) of robot i. u_{x_i} and u_{y_i} are the inputs that should be defined with respect to the formation stabilizing problem and the regulation control including obstacles avoidance for targets capturing. More generally, one substitutes the behavior of the i^{th} agent by this writing

$$\ddot{q}_i = u_i \tag{2}$$

with $q_i = (x_i, y_i) \in \mathbb{R}^2$ and $u_i = (u_{x_i}, u_{y_i}) \in \mathbb{R}^2$. In formation regulating/tracking control, intercommunication between agents is necessary to success the mission. Such an investigation leads us to take into account interactions between agents. Hence, one modifies the dynamic behavior as follows

$$\ddot{q}_i = u_i + \sum_{\substack{j=1,\mathcal{N}^i \\ j \neq i}} Q_{ij}(t, q_i, q_j) \tag{3}$$

where $\mathcal{N}^i$ denotes the neighbor of agent i and $Q_{ij} \in \mathbb{R}^2$ defines the interconnection between agent i and agents j which is function of q_i and q_j positions. Note that Q_{ij} gives the interconnection into the two directions of motion. We suppose that $\hat{Q}_{ij}$ is the estimate of Q_{ij} such that the sum of the estimation error component is bounded by a known function $K_i(t, q_i, q_j) \in \mathbb{R}^2$:

$$\sum_{\substack{j=1,\mathcal{N}^i \\ j \neq i}} |\hat{Q}_{ij}(.) - Q_{ij}(.)| \leq K_i(t, q_i, q_j) \triangleq (K_{ix}, K_{iy})^T \tag{4}$$

We have the following result.

Theorem 2.1. *Let u_a a control law that stabilizes asymptotically the system (3) while it approaches the equilibrium point and let V the Lyapunov function associated to $\ddot{q} = u_a + \sum_{\substack{j=1,\mathcal{N}^i \\ j \neq i}} Q_{ij}(t, q_i, q_j)$. Given a scalar function ν from $\mathbb{R}_n$ to $\mathbb{R}$, the control law*

$$u = u_a + \nu \left(\frac{\partial V}{\partial \dot{q}} \right)^{\perp} \tag{5}$$

stabilizes, as well, asymptotically the solution of system (3) while approaching the equilibrium.

Proof. Let $V(q, \dot{q})$ be the Lyapunov function associated to $\ddot{q} = u_a + \sum_{\substack{j=1,\mathcal{N}^i \\ j \neq i}} Q_{ij}(t, q_i, q_j)$, then

$$\dot{V} = \frac{\partial V}{\partial q} \dot{q} + \frac{\partial V}{\partial \dot{q}} \ddot{q} \leq 0$$

or

$$\frac{\partial V}{\partial q}\dot{q} + \frac{\partial V}{\partial \dot{q}}(u_a + \sum_{\substack{j=1,\mathcal{N}^i \\ j\neq i}} Q_{ij}(.)) \leq 0$$

Now replace u_a by u from (5) in the inequality above, it leads to

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial q}\dot{q} + \frac{\partial V}{\partial \dot{q}}(u_a + \sum_{\substack{j=1,\mathcal{N}^i \\ j\neq i}} Q_{ij}(.)) + \nu\frac{\partial V}{\partial \dot{q}}\left(\frac{\partial V}{\partial \dot{q}}\right)^{\perp} \qquad (6) \\ &= \frac{\partial V}{\partial q}\dot{q} + \frac{\partial V}{\partial \dot{q}}\ddot{q} \leq 0 \end{aligned}$$

Hence, the solution of (3) subject to the control law u converges to the equilibrium. □

2.1. *Stabilizing results*

In the following one treats the motion of the formation and the sliding mode techniques will be considered in the stabilizing control investigation. It is well known that this approach is robust, and more details about this technique are in Slotine.[11] The sliding mode technique was also proposed by Gazi[10] in the case of swarms aggregation. An inter-agent connection function is added in our work and a non free environment is considered. Our main result is summarized in the following. Let us introduce this stabilizing control theorem.

Theorem 2.2. *Given an objective q_i^r and consider the error in position $e_{q_i} = q_i - q_i^r$, the slide surface $s_i = \dot{q}_i + \lambda_i e_{q_i}$, the following input,*

$$\begin{aligned} u_{ix} &= -\sum_{\substack{j=1,\mathcal{N}^i \\ j\neq i}} \hat{Q}_{ij}(.)_x - \lambda_i \dot{q}_{ix} - (K_{ix} + \eta_i)sgn(s_{ix}) \qquad (7) \\ u_{iy} &= -\sum_{\substack{j=1,\mathcal{N}^i \\ j\neq i}} \hat{Q}_{ij}(.)_y - \lambda_i \dot{q}_{iy} - (K_{iy} + \eta_i)sgn(s_{iy}) \end{aligned}$$

ensures the exponential stabilization of the agent i with $(q_i, \dot{q}_i) \to (q_i^r, 0)$ ($\eta_i > 0$, $\lambda_i > 0$).

Proof. As we consider only the stabilizing problem of agent i, then $\dot{q}_i^r = \ddot{q}_i^r = 0$. From the Lyapunov function $V_i = \frac{1}{2}s_i^2$, the time derivative is a:

$\dot{V}_i = \dot{s}_{ix}s_{ix} + \dot{s}_{iy}s_{iy}$ from the dynamic of s_i, we obtain:

$$\dot{s}_{ix}s_{ix} = \sum_{\substack{j=1,\mathcal{N}^i \\ j\neq i}} (Q_{ij}(.)_x - \hat{Q}_{ij}(.)_x)s_{ix} - (K_{ix} + \eta_{ix})s_{ix}sgn(s_{ix})$$

The same computation can be deduced to $\dot{s}_{iy}s_{iy}$, consequently it is straightforward to prove from (4)

$$\dot{V} \leq -\eta_{ix}|s_{ix}| - \eta_{iy}|s_{iy}| \leq 0$$

which means that for some constants $\eta_i > 0$, $s_i = 0$ will be achieved in finite time, and the exponential stabilization of each agent toward the objective is asserted. □

3. Dynamic interconnection function

In this section, we give more details about the interconnection function and its estimation which permits to guarantee the inter-agent collision. Also we have to define agents belonging to the neighbor $\mathcal{N}^i$, and to construct the function that leads to a free motion in regard to obstacles. The information exchanged between the i^{th} and the j^{th} agent can be modeled by[4]

$$Q_{ij}(t, q_i, q_j) = -(q_i - q_j)(a_i - b_i e^{-\frac{\|q_i - q_j\|^2}{c_i}}) \tag{8}$$

where a_i, b_i and c_i are positive scalar parameters determine the degree of rigidity of the connection. The Q_{ij} function can be associated to a repulsion and attraction potentials. These potential functions are such that

$$-\nabla_{q_i} V_a(\|q_i - q_j\|) - \nabla_{q_i} V_r(\|q_i - q_j\|) = Q_{ij}(t, q_i, q_j) \tag{9}$$

hence, from (8) we can deduce,

$$\begin{aligned} \nabla_{q_i} V_a(\|q_i - q_j\|) &= (q_i - q_j)a_i \qquad (10) \\ \nabla_{q_i} V_r(\|q_i - q_j\|) &= -(q_i - q_j)b_i e^{-\frac{\|q_i - q_j\|^2}{c_i}} \end{aligned}$$

With the given potential functions we assume that the estimate of Q_{ij} is subject to (a_i, b_i) parameter variation with some boundary conditions and $\hat{a}_i \in [(a_i)_{min}, (a_i)_{max}]$ and $\hat{b}_i \in [(b_i)_{min}, (b_i)_{max}]$. Consequently,

$$\hat{Q}_{ij}(t, q_i, q_j) = -(q_i - q_j)\left(\hat{a}_i - \hat{b}_i e^{-\frac{\|q_i - q_j\|^2}{c_i}}\right) \tag{11}$$

Hence,

$$| \hat{Q}_{ij}(.) - Q_{ij}(.) | \leq | q_i - q_j | \left(| a_i - \hat{a}_i | + | b_i - \hat{b}_i | e^{-\frac{\|q_i - q_j\|^2}{c_i}} \right) \quad (12)$$

$$\leq | q_i - q_j | \left((a_i)_{max} + (b_i)_{max} e^{-\frac{\|q_i - q_j\|^2}{c_i}} \right) \quad (13)$$

Then, we can take

$$K_i(t, q_i, q_j) \triangleq \sum_{\substack{j=1,\mathcal{N}^i \\ j \neq i}} | q_i - q_j | \left((a_i)_{max} + (b_i)_{max} e^{-\frac{\|q_i - q_j\|^2}{c_i}} \right) \quad (14)$$

The minimum value of $\hat{a}_i$ leads to a released connection while the maximum value means a rigid connection between the i^{th} agent and its neighbor. It remains to ensure the formation obstacle avoidance while it behaves to targets.

4. Obstacle avoidance

Assume that an agent governed by equation (3) is moving in a space containing one obstacle and let q_0 its initial position on $t = t_0$ and its final one corresponds to the frame origin. To move from an initial position q_0 to a final one q_f there exist an infinity of possible trajectories, having different behaviors during their movement. That is why it is very hard to handle a agent governed by a dynamical system in order to join initial and final desired positions while forcing it to obey to some behavioral criteria between t_0 and t_f. Now, we will design the function ν that ensures the stabilization toward the target while avoiding fixed obstacles. The result is summarized in the following theorem.

Theorem 4.1. *Consider the interconnected multi-agent dynamic (3). Let* $q_{i0} = (x_{i0}, y_{i0})$ *the initial condition of agent* i*, and* $L_i(x)$ *the equation of the* i^{th} *line joining the desired position* $q_i^r = (q_{ix}^r, q_{iy}^r)$ *and the obstacles which are assumed to be centered in a circle* $\mathcal{C}$*, with* $O = (O_x, O_y)$ *its center and* r *the radius. Let* $O_{q_i} = O + r\dfrac{q_i - O}{\|q_i - O\|} \in \mathcal{C}$*. The following control law,*

$$u_i = (u_{ix}, u_{iy})^T + \nu_i(\dot{q}_i + \lambda_i e_{q_i})^{\perp} \quad (15)$$

where

$$\nu_i = -\frac{sign([y_{i0} - L_i(x_{i0})][q_{ix}^r - O_x])}{\|q_i - O_{q_i}\|} \quad (16)$$

ensures the following,

(1) $\|q_i - O\| - r \neq 0$, $\forall t \geq 0$ *(obstacles avoidance).*
(2) q_i *converges asymptotically to* q_i^r *and no collision occurs among the* q_i.

Proof. One notes that $K = q \in \Omega / y_i \geq L(x_i)$, $H = q \in \Omega / y_i \leq L(x_i)$, where $L(x)$ is defined above, are invariant sets with respect to the interconnected second order dynamic system (3) with the control law (15). The reader can see El Kamel[1] where the polar coordinates have been used. Further, we can easily prove that ν never goes to infinity since $\|q_i - O\| - r \neq 0$ which imply that the proposed control law is bounded. The inter-collision avoidance and the stabilization are resulting from theorem 2.1. □

5. Simulation results

In order to test the given theoretical results, we limit our multi-agent system to four interconnected dynamic subsystems. For $i = 1, ..., 4$ the gain parameters are $\lambda_i = 0.4$ and $\eta_i = 0.2$, while the estimated parameters are such that $\hat{a}_i = 2.10^{-3}$, $\hat{b}_i = 0.02$, $(a_i)_{max} = 0.03$, $(b_i)_{max} = 0.05$, and the function interconnection parameters are $c = 1$, $a = 2.10^{-4}$ and $b = 0.01$. Figure 1 shows the behavior of agents in a free environment and the character of the sliding mode control once the sliding surfaces are reached. In figures 2, we sketch the target capturing and the stabilization/regulation results while the obstacles are avoided with resect to the given initial conditions. This confirm the proposed invariant set in reaching the objective.

6. conclusion

For a given fixed target and known non free environment, with obstacles circumscribed in a circle, the objective is reached asymptotically by the multi-agent system in formation. The formation control was presented as a stabilizing control problem including dynamic interconnection with bounded parameters. The proposed control law can take another interconnection function related to other behaviors of the formation. Our next investigation will concern a separate multi-agent navigation for multiple objectives.

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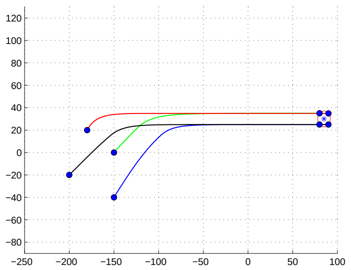

Fig. 1: Multi-agent stabilization in a free environment $\nu_i = 0$.

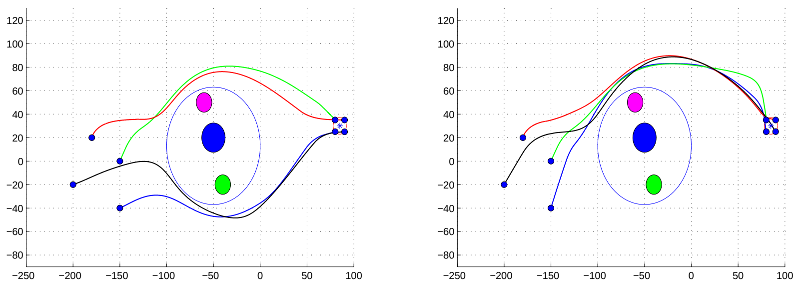

Fig. 2: Multi-agent in presence of obstacles.

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