Tracking Control of an Underactuated Underwater Vehicle ROV-Observer

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***Abstract*— This paper addresses the problem of trajectory tracking control of an underactuated autonomous underwater vehicle (ROV-Observer) in the horizontal and vertical plane. The underwater vehicle is not actuated in the sway and yaw directions, and the mass matrix is not assumed to be diagonal. Using a reference values the dynamics of the vehicle is transformed to the error one. Using backstepping technique and the tracking error dynamics, the system states are stabilized and the tracking errors converge to an arbitrarily small neighborhood of zero.**

I. INTRODUCTION

The problem of stabilizing an underactuated underwater at a desired trajectory is an important issue in many offshore applications such as, for example, solving trajectory tracking, path-following, path-tracking and stabilization problems. Modern developments in the fields of control, sensing, and communications have contributed to make very complex and dedicated underwater robot systems a reality. This kind of vehicle could be used in highly hazardous and unknown environments. the autonomy and control of the robot is the key factor to its mission success. In spite of highly coupled and non-linear the dynamics of underwater vehicle system in nature, decoupled linear control system strategy is widely used for practical applications. As autonomous underwater vehicle needs intelligent control system, it is necessary to develop control system that really takes into account the coupled and non-linear characteristics of the system. In addition, most of the AUVs are underactuated, i.e., they have fewer actuated inputs than the degrees of freedom (DOF), imposing non-integrable acceleration constraints. The underwater robot is the subject of numerous papers and thesis; ([2], [3]) and references therein. As cited in [9], continuous time-invariant controller was developed to achieve global exponential position tracking for underactuated ships. However, the orientation of the ship was not controlled. By applying a cascaded approach, a global tracking result was obtained in [13]. Based on Lyapunov's direct method and passivity approach, two constructive tracking solutions were proposed in [10] for an underwater robot. In [5], a single controller was proposed to solve both stabilization and tracking simultaneously. Some related independent work includes ([11], [12]) local H_∞ tracking control and differential flatness approach.

In [7], a trajectory planning and a tracking control algorithm for an underactuated AUV moving on the horizontal plane was studied but the drag forces model used in this work was taken linear with respect to velocities; this confining assumption is rectified in [8]. A new type of control law is developed in [15] to steer an autonomous underwater vehicle (AUV) along a desired path, it overcomes stringent initial condition constraints that are present in a number of path-following control strategies described in the literature. In this paper, we study an ultraportable submarine vehicle, belong to the family of ROVs called ROV, and is expected for observation and exploration insubsea historical sites. The ROV is equipped with two cameras wich will permits to Tele-exploration in mixed reality sites (*). The ROV has a close frame structure (see Fig.1). This vehicle is actuated with two reversible horizontal thrusters F_{1x} and F_{2x} for surge and yaw motion, and a reversible vertical thruster F_{3z} for heave motion. A 150 meters cable provides electric power to the thrusters and enables communication between the vehicle sensors and the surface equipment (Fig.1).

In this paper, we propose a solution to overcome these problems for the tracking control of underactuated ROV moving in horizontal and vertical plane. Controller design builds on Lyapunov theory and backstepping techniques. The paper is organized as the following: In section II the dynamics and kinematics of ROV is described in a vertical plane and a methodology is proposed to design a controller that forces position and orientation of the ROV to track a reference trajectory. In section III, a control design is proposed in the horizontal plane to track a given trajectory. The theoretical results are supported by simulations in section IV.

Assumptions : Vehicle has an XZ and YZ-plane of symmetry; surge is decoupled from pitch and sway is decoupled from roll.

II. TRACKING CONTROL IN THE VERTICAL PLANE (XZ)

A. ROV kinematics and dynamics

In this section, the kinematic and dynamic equations of the motion of a ROV moving on the vertical plane are described. Further details on the ROV's dynamical modeling are given in [1] . Using an inertial reference frame R and a body-fixed frame R_v Fig.1. the kinematic equations of motion of the center of mass (G) for a ROV on the vertical xz plane can

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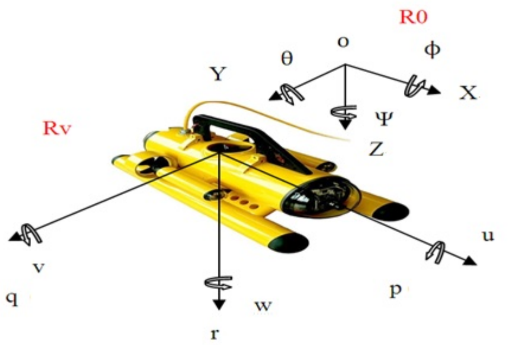


Fig. 1. Body-fixed frame and earth-fixed reference frame for the SUBSEA-TECH ROV.

be written as:

$$\dot{x} = c\theta u + s\theta w,\ \dot{z} = -s\theta u + c\theta w,\ \dot{\theta} = q \tag{1}$$

where x and z represent the inertial coordinates of the center of the mass G of the vehicle and u , w are the surge and heave velocities respectively defined in the body-fixed frame. The orientation of the vehicle is described by the angle θ and q is its pitch(angular) velocity. We denote by: $c\theta = \cos\theta$; $s\theta = \sin\theta$. The dynamic of the ROV is expressed by the following differential equations (2):

$$\begin{aligned}
\dot{u} =& (1/\delta)\{-J_y(X_u + X_{uu}|u|)u - \alpha_{uq}(M_q + M_{qq}|q|)q \\
&+(\alpha_{uq} z_g F_B + J_y g(F_W - F_B))s\theta - (J_y m_z - \alpha_{uq}^2)wq \\
&-\alpha_{uq}(Z_{\dot{w}} - X_{\dot{u}})wu + \tau_1\} \\
\dot{w} =& (1/m_z)\{-(Z_w + Z_{ww}|w|)w + (F_W - F_B)c\theta + m_z uq \\
&+\alpha_{uq} q^2 + \tau_3\} \\
\dot{q} =& (1/\delta)\{-\alpha_{uq}(X_u + X_{uu}|u|)u - m_x(M_q + M_{qq}|q|)q \\
&+(m_x z_g F_B + \alpha_{uq}(F_W - F_B))s\theta + (m_x\alpha_{uq} - \alpha_{uq} m_z)wq \\
&-m_x(Z_{\dot{w}} - X_{\dot{u}})wu + (\alpha_{uq}/J_y)\tau_1\}
\end{aligned} \tag{2}$$

where $\tau_1 = F_{1x} + F_{2x}$, $\tau_3 = F_{3z}$ and $(F_B,\ F_W)$ are the buoyancy and gravity magnitudes.

B. Coordinate transformations

We introduce the following coordinate transformation to the vehicle for getting the vehicle system matrix in to a diagonal form:
$x_1 = x + \alpha s\theta$, $z_1 = z + \alpha c\theta$, $u_1 = u - \alpha q$, where $\alpha = J_y/\alpha_{uq}$. We obtain the model given by:

$$\begin{aligned}
&\dot{x}_1 = c\theta u_1 + s\theta w,\ \dot{z}_1 = -s\theta u_1 + c\theta w,\ \dot{\theta} = q \\
&\dot{u}_1 = a_1 u_1 + a_2|u_1 + \alpha q|(u_1 + \alpha q) + a_3 q + a_4 q|q| \\
&+a_5 s\theta + a_6 wq + a_7 wu_1 \\
&\dot{w} = b_1 w + b_2|w|w + b_3\cos\theta + b_4 uq + b_5 q^2 + (1/m_z)\tau_3 \\
&\dot{q} = d_1 u + d_2 u_1|u_1| + d_3 q + a_4 q|q| + d_5\sin\theta + d_6 wq \\
&+d_7 wu_1 + (1/\delta\alpha)\tau_1
\end{aligned} \tag{3}$$

where a_i, d_i and b_i depend on the ROV fixed parameters.

C. Error dynamics formulation

The aim here is to track the following reference variables: $x_{1r}, z_{1r}, \theta_r, u_{1r}, w_r, q_r$. To this end, we define the following tracking errors: $x_{1e} = x_1 - x_{1r}$, $z_{1e} = z_1 - z_{1r}$, $\theta_e = \theta - \theta_r$, $u_{1e} = u_1 - u_{1r}$, $w_e = w - w_r$, $q_e = q - q_r$. According to (3) and the definition of the tracking errors we obtain the error dynamics as the kinematic ones:

$$\begin{aligned}
\dot{X}_e =& \underbrace{\begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}}_{R_\theta} \underbrace{\begin{pmatrix} u_{1e} \\ w_e \end{pmatrix}}_{U_e} \\
&+ \underbrace{\begin{pmatrix} c\theta_c\theta_r & s\theta - s\theta_r \\ -s\theta + s\theta_r & c\theta - c\theta_r \end{pmatrix}}_{R_{\theta_r}} \underbrace{\begin{pmatrix} u_{1r} \\ w_r \end{pmatrix}}_{U_r},\ \dot{\theta}_e = q_e
\end{aligned} \tag{4}$$

and the dynamic ones:

$$\begin{aligned}
\dot{u}_{1e} =& a_2(u_{1e} + u_{1r} + \alpha(q_e + q_r))|u_{1e} + u_{1r} + \alpha(q_e + q_r)| \\
&+a_1(u_{1e} + u_{1r}) + a_3(q_e + q_r) + a_4(q_e + q_r)|q_e + q_r| \\
&+a_5\sin(\theta_e + \theta_r) + a_6(w_e q_e + w_r q_e + q_r w_e + w_r q_r) \\
&+a_7(w_e u_{1e} + w_r u_{1e} + u_{1r} w_e + u_{1r} w_r) - \dot{u}_{1r} \\
\dot{w}_e =& b_1(w_e + w_r) + b_2|w_e + w_r|(w_e + w_r) - \dot{w}_r \\
&+b_4(u_{1e} q_e + q_r u_{1e} + u_{1r} q_e + u_{1r} q_r) \\
&+b_5(q_e^2 + 2q_r q_e + q_r^2) + b_3\cos(\theta_e + \theta_r) + (1/m_z)\tau_3 \\
\dot{q}_e =& d_1(u_{1e} + u_{1r}) + d_2(u_{1e} + u_{1r})|u_{1e} + u_{1r}| \\
&+d_3(q_e + q_r) + a_4(q_e + q_r)|q_e + q_r| - \dot{q}_r \\
&+d_5\sin(\theta_e + \theta_r) + d_6(w_e q_e + w_r q_e + q_r w_e + w_r q_r) \\
&+d_7(w_e u_{1e} + w_r u_{1e} + u_{1r} w_e + u_{1r} w_r) + (1/\delta\alpha)\tau_1
\end{aligned} \tag{5}$$

The surge motion is not directly actuated.

D. Control design

The tracking control objective has been transformed to a stabilizing problem given by the system (4)-(5). Thus, we consider the following first feedback law:

$$\begin{aligned}
\tau_1 =& \alpha\delta\{\tau_q - d_1(u_{1e} + u_{1r}) - d_2(u_{1e} + u_{1r})|u_{1e} + u_{1r}| \\
&-d_3(q_e + q_r) - d_4(q_e + q_r)|q_e + q_r| \\
&-d_5\sin(\theta_e + \theta_r) - d_6(w_e q_e + w_r q_e + q_r w_e + w_r q_r) \\
&-d_7(w_e u_{1e} + w_r u_{1e} + u_{1r} w_e + u_{1r} w_r) + \dot{q}_r\} \\
\tau_3 =& m_z\{\tau_w - b_1(w_e + w_r) - b_2|w_e + w_r|(w_e + w_r) \\
&-b_3\cos(\theta_e + \theta_r) - b_4(u_{1e} q_e + q_r u_{1e} + u_{1r} q_e + u_{1r} q_r) \\
&-b_5(q_e^2 + 2q_r q_e + q_r^2) + \dot{w}_r\}
\end{aligned} \tag{6}$$

with τ_q and τ_w being considered as new controls to be designed later. The corresponding system of errors can be easily written as:

$$\begin{aligned}
&\dot{X}_e = R_\theta U_e + R_{\theta_r} U_r;\ \dot{\theta}_e = q_e \\
&\dot{u}_{1e} = a_1 u_{1e} + a_3 q_e + a_6(w_e q_e + w_r q_e + q_r w_e) \\
&+a_7(w_e u_{1e} + w_r u_{1e} + u_{1r} w_e) + \xi \\
&\dot{w}_e = \tau_w;\ \dot{q}_e = \tau_q
\end{aligned} \tag{7}$$

where

$$\begin{aligned}\xi =&a_2(u_{1e}+u_{1r}+\alpha(q_e+q_r))|u_{1e}+u_{1r}+\alpha(q_e+q_r)|\\&+a_1u_{1r}+a_3q_r+a_4(q_e+q_r)|q_e+q_r|\\&+a_5\sin(\theta_e+\theta_r)+a_6w_rq_r+a_7u_{1r}w_r-\dot{u}_{1r}\end{aligned} \quad (8)$$

The examination of equation (7) shows that there is direct control capability on the forward (heave) and on the rotational motion of the vehicle but not on the side (surge) motion, i.e., we can control the linear velocity w and the corresponding error w_e as well as the angular velocity q and the corresponding error q_e . We also observe that we have indirect control of the side velocity error u_e through the coupling of the controlled variables in the term $a_6(w_eq_e+w_rq_e+q_rw_e+w_rq_r)$. Consistent with backstepping design techniques for the side velocity error u_e, we can choose as an auxiliary control variable one of the controlled velocities and, then, stabilize the latter using the corresponding actual control variable. The same observations hold for the linear and angular position errors: we first use the velocities as control variables for the position, then we stabilize the velocities w_e and q_e with τ_w and τ_q and u_e using the coupling term.

step 1) In order to stabilize the vector position X_e, we assume u_{1e} and w_e as virtual controls. We start by defining the following Lyapunov function candidate:

$$V_1=\frac{1}{2}X_e^TX_e\Rightarrow\dot{V}_1=X_e^T(R_\theta U_e+R_{\theta_r}U_r)$$

We suggest as desired expressions for the virtual controls $(\alpha_{u_1},\alpha_w)^T=-R_\theta^T(KX_e+R_{\theta_r}U_r)$ then,

$$\begin{aligned}\alpha_{u_1}=&-k(x_{1e}c\theta-z_{1e}s\theta)+c\theta\gamma_1-s\theta\gamma_2\\\alpha_w=&-k(x_{1e}s\theta+z_{1e}c\theta)+s\theta\gamma_1+c\theta\gamma_2\end{aligned} \quad (9)$$

where$[\gamma_1,\gamma_2]^T=R_{\theta_r}U_r$, $K=diag(k,k)$. The time derivative of V_1 becomes

$$\dot{V}_1=-X_e^TKX_e.$$

step 2) Since the components of the vector U_e are not true controls, we need to introduce new error variables ν_{u_1} and ν_w defined as: $\nu=[\nu_{u_1},\nu_w]^T=[u_{1e}-\alpha_{u_1},w_e-\alpha_w]^T$. Then, the controlled position equations are rewritten as

$$\dot{X}_e=-KX_e+R_\theta\nu.$$

In the following we would force ν_w to zero, so we consider the following Lyapunv function:

$$V_2=V_1+\frac{1}{2}\nu_w^2 \quad (10)$$

The time derivative of V_2 can be expressed as:

$$\dot{V}_2=-X_e^T[KX_e-R_\theta\nu]+\nu_w(\tau_w+k(w_e+s\theta\gamma_1+c\theta\gamma_2)) \quad (11)$$

Setting

$$\begin{aligned}\tau_w=&-c_{1w}\nu_w-c_{2w}\nu_w^3-k(w_e+s\theta\gamma_1+c\theta\gamma_2)\\&-(x_{1e}s\theta+z_{1e}c\theta)+f_w\end{aligned} \quad (12)$$

where f_w is a design variable for subsequent use, c_{1w} and c_{2w} are positive constants. The time derivative of V_2 becomes,

$$\dot{V}_2=-X_e^TKX_e-c_{1w}\nu_w-c_{2w}\nu_w^3+f_w\nu_w+\nu_{u_1}(x_{1e}c\theta-z_{1e}s\theta) \quad (13)$$

So far, the controlled subsystem of error dynamics equations has been transformed to,

$$\begin{aligned}&\dot{X}_e=-KX_e+R_\theta\nu\\&\dot{\nu}_w=-c_{1w}\nu_w-c_{2w}\nu_w^3-(x_{1e}s\theta+z_{1e}c\theta)+f_w\end{aligned} \quad (14)$$

Before starting the next step, we perform some manipulations on the sway error dynamic equations. Then $\dot{\nu}_{u_1}$ can be expanded to:

$$\begin{aligned}\dot{\nu}_{u_1}=&a_1u_{1e}+a_3q_e+a_6(w_eq_e+w_rq_e+q_rw_e)\\&+a_7(w_eu_{1e}+w_ru_{1e}+u_{1r}w_e)+\xi\\&-k(u_{1e}+c\theta\gamma_1-s\theta\gamma_2)\\=&(a_3+a_6w)q_e+(a_6q_r+a_7u_{1r})\nu_w+\varrho_\nu\end{aligned} \quad (15)$$

where $\varrho_\nu=(a_6q_r+a_7u_{1r})\alpha_w+(a_1-k)u_{1e}$
$+\xi+k(s\theta\gamma_2-c\theta\gamma_1)$

step 3) In order to stabilize ν_{u_1} and θ_e we consider the velocity in yaw as an auxiliary control, assuming $(a_3+a_6w)\neq 0$. We note α_q as a virtual control of q. Choosing the following Lyapunov function

$$V_3=V_2+(\nu_{u_1}^2+\theta_e^2)/2 \quad (16)$$

and taking into account Eq.(11), its time derivative is:

$$\begin{aligned}\dot{V}_3=&-X_e^TKX_e-c_{1w}\nu_w^2-c_{2w}\nu_w^4+\nu_w[f_w+a_6q_r+a_7u_{1r}]\\&+\varrho_\nu\nu_{u_1}+[(a_3+a_6w)\nu_{u_1}+\theta_e]\alpha_q\\&+\nu_{u_1}(x_{1e}c\theta-z_{1e}s\theta)+\nu_{u_1}((a_3+a_6w)\alpha_q\end{aligned} \quad (17)$$

Setting $\alpha_q=-c_q((a_3+a_6w)\nu_{u_1}+\theta_e)$, $c_q>0$. Then

$$\begin{aligned}\dot{V}_3=&-X_e^TKX_e-c_{1w}\nu_w^2-c_{2w}\nu_w^4\\&-c_q((a_3+a_6w)\nu_{u_1}+\theta_e)^2+\nu_{u_1}(x_{1e}c\theta-z_{1e}s\theta)\\&+\nu_w[f_w+\nu_{u_1}(a_6q_r+a_7u_{1r})]+\varrho_\nu\nu_{u_1}\end{aligned} \quad (18)$$

step 4) The variable $q_e=\alpha_q$ is not a true control. Thus, we have to introduce an error $\nu_q=q_e-\alpha_q$ in place of α_q and we use τ_q to stabilize the subsystem:

$$\begin{aligned}&\dot{\nu}_{u_1}=(a_3+a_6w)q_e+(a_6q_r+a_7u_{1r})\nu_w+\varrho_\nu\\&\dot{\nu}_q=\tau_q-\dot{\alpha}_q\\&\dot{\theta}_e=\nu_q+\alpha_q\end{aligned} \quad (19)$$

Finally, for the complete system, we choose the lyapunov function:

$$V_4 = V_3 + \nu_q^2/2 \tag{20}$$

and its time derivative is:

$$\begin{aligned}\dot{V}_4 =& -X_e^T K X_e - c_{1w}\nu_w^2 - c_{2w}\nu_w^4 - c_q(a_6 w \nu_{u_1} + \theta_e)^2\\ &+\nu_w[f_w + \nu_{u_1}(a_6 q_r + a_7 u_{1r})] + \nu_{u_1}(x_{1e}c\theta - z_{1e}s\theta)\\ &+\nu_q(\tau_q + c_q((a_3+a_6w)^2+1)q_e + c_q(a_3+a_6w)\zeta_\nu\\ &+c_q(a_3+a_6w)(a_6q_r + a_7u_{1r})\nu_w) + \varrho_\nu \nu_{u_1}\end{aligned} \tag{21}$$

We consider the following control law:

$$\begin{aligned}\tau_q =& -c_{1q}\nu_q - c_{2q}\nu_q^3 - c_q((a_3+a_6w)^2+1)q_e\\ &-c_q(a_3+a_6w)\varrho_\nu - c_q(a_3+a_6w)(a_6q_r + a_7u_{1r})\nu_w\end{aligned} \tag{22}$$

with c_{1q} and c_{2q} are positives scalar. Then, equation (21) becomes:

$$\begin{aligned}\dot{V}_4 =& -X_e^T K X_e - c_{1w}\nu_w^2 - c_{2w}\nu_w^4 - c_q((a_3+a_6w)\nu_{u_1})^2\\ &+\nu_w[f_w + \nu_{u_1}(a_6q_r + a_7u_{1r})] - 2c_q(a_3+a_6w)\nu_{u_1}\theta_e\\ &+\nu_{u_1}(x_{1e}c\theta - z_{1e}s\theta) + \varrho_\nu\nu_{u_1} - c_{1q}\nu_q^2 - c_{2q}\nu_q^4 - c_q(\theta_e)^2\end{aligned} \tag{23}$$

In order to deal with the quantities with uncertain sign we conduct some algebraic manipulations. Firstly, we set

$$f_w = -\nu_{u_1}(a_6q_r + a_7u_{1r}) - c_1\nu_{u_1}^2\nu_w - c_2\nu_{u_1}^4\nu_w \tag{24}$$

Then, Eq (23) becomes:

$$\begin{aligned}\dot{V}_4 =& -kx_{1e}^2 - kz_{1e}^2 - c_{1w}\nu_w^2 - c_{2w}\nu_w^4 - c_1\nu_{u_1}^2\nu_w^2\\ &-c_2\nu_{u_1}^4\nu_w^2 - c_q((a_3+a_6w)\nu_{u_1})^2 - c_q(\theta_e)^2\\ &-2c_q((a_3+a_6w)\nu_{u_1}\theta_e) + \nu_{u_1}(x_{1e}c\theta - z_{1e}s\theta)\\ &-c_{1q}\nu_q^2 - c_{2q}\nu_q^4 + \varrho_\nu\nu_{u_1}\end{aligned} \tag{25}$$

In the above expression, we remark that the last three terms have uncertain signs. For the analysis we will use the Young's inequality, with the quantities $\epsilon_i,\ i = 1....7$ as positive constants, we obtain:

$$\begin{aligned}(x_{1e}c\theta - z_{1e}s\theta)\nu_{u_1} \le& \frac{1}{4\epsilon_1}|x_{1e}|^2 + \frac{1}{4\epsilon_1}|z_{1e}|^2 + 2\epsilon_1|\nu_{u_1}|^2\\ 2c_q(a_3+a_6w)\nu_{u_1}\theta_e \le& \frac{1}{\epsilon_2}a_6^2|w|^2|\theta_e|^2 + \frac{1}{\epsilon_2}a_3^2|\theta_e|^2\\ &+2\epsilon_2c_q^2|\nu_{u_1}|^2\end{aligned} \tag{26}$$

Now, we will expand the expression of $\varrho_\nu\nu_{u_1}$, we obtain the following expression,

$$\begin{aligned}(a_6q_r + a_7u_{1r})\alpha_w\nu_{u_1} \le& \frac{(a_6)^2}{4\epsilon_3}(k^2|x_{1e}|^2 + k^2|z_{1e}|^2 + |\gamma_1|^2\\ &+|\gamma_2|^2)|q_r|^2 + 2\epsilon_3|\nu_{u_1}|^2\\ &+\frac{(a_7)^2}{4\epsilon_4}(k^2|x_{1e}|^2 + k^2|z_{1e}|^2 + |\gamma_1|^2\\ &+|\gamma_2|^2)|u_{1r}|^2 + 2\epsilon_4|\nu_{u_1}|^2\\ (a_1-k)u_{1e}\nu_{u_1} \le& \frac{(a_1-k)^2}{4\epsilon_5}|u_{1e}|^2 + \epsilon_5|\nu_{u_1}|^2\\ k(s\theta\gamma_2 - c\theta\gamma_1)\nu_{u_1} \le& \frac{k^2}{4\epsilon_6}(|\gamma_1|^2 + |\gamma_2|^2) + 2\epsilon_6|\nu_{u_1}|^2\end{aligned} \tag{27}$$

$$\xi\nu_{u_1} \le \frac{1}{4\epsilon_7}|\xi|^2 + \epsilon_7|\nu_{u_1}|^2 \tag{28}$$

Angular rates and velocities are considered to have maximum values and they verify:
$|u_1|^2 \le u_{1,max}^2$, $|w|^2 \le w_{max}^2$, $|u_{1e}|^2 \le u_{1e,max}^2$
$|q_r|^2 \le q_{r,max}^2$, $|w_r|^2 \le w_{r,max}^2$, $|u_{1r}|^2 \le u_{1r,max}^2$
Taking into account the results from (25) to (28), the time derivative of V_4 in (19), becomes:

$$\begin{aligned}\dot{V}_4 \le& -[k - \frac{1}{4\epsilon_1} - \frac{k^2(a_6)^2}{4\epsilon_3}q_{r,max}^2 - \frac{k^2(a_7)^2}{4\epsilon_4}u_{1r,max}^2]x_{1e}^2\\ &-[k - \frac{1}{4\epsilon_1} - \frac{k^2(a_6)^2}{4\epsilon_3}q_{r,max}^2 - \frac{k^2(a_7)^2}{4\epsilon_4}u_{1r,max}^2]z_{1e}^2\\ &-(c_q - \frac{1}{\epsilon_2}\beta_3^2 - \frac{1}{\epsilon_2}\beta_6^2w_{max}^2)\theta_e^2 - c_{1q}\nu_q^2 - c_{1w}\nu_w^2\\ &-(c_1\nu_w^2 - 2\epsilon_1 - 2\epsilon_2c_3^2 - 2\epsilon_3 - 2\epsilon_4 - \epsilon_5 - 2\epsilon_6 - \epsilon_7)\nu_{u_1}^2\\ &-c_q\beta_6^2w_{max}^2\nu_{u_1}^2 + \mu_1\end{aligned} \tag{29}$$

where

$$\begin{aligned}\mu_1 =& (\gamma_{1,max}^2 + \gamma_{2,max}^2)(\frac{(a_6)^2}{4\epsilon_3}q_{r,max}^2 + \frac{(a_7)^2}{4\epsilon_4}u_{1r,max}^2 + \frac{k^2}{4\epsilon_6})\\ &+\frac{1}{4\epsilon_7}|\xi|^2\end{aligned} \tag{30}$$

The coefficient $c_1\nu_w^2 - \epsilon$ must be positive, where $\epsilon = 2\epsilon_1 + 2\epsilon_2c_3^2 + 2\epsilon_3 + 2\epsilon_4 + \epsilon_5 + 2\epsilon_6$. Consequently $|\nu_w|$ must be above $\sqrt{\frac{\epsilon}{c_1}}$, which holds for large c_1 and small ϵ. So, from the inequality, we obtain:

$$\begin{aligned}\dot{V}_4 \le& -k_1x_{1e}^2 - k_1z_{1e}^2 - k_2\theta_e^2 - c_{1w}\nu_w^2 - k_2\nu_{u_1}^2 - c_{1q}\nu_q^2\\ &-(c_1\nu_w^2 - \epsilon)\nu_{u_1}^2 + \mu_1\end{aligned} \tag{31}$$

$$\begin{aligned}k_1 =& k - \frac{1}{4\epsilon_1} - \frac{k^2(a_6)^2}{4\epsilon_3}q_{r,max}^2 - \frac{k^2(a_7)^2}{4\epsilon_4}u_{1r,max}^2 > 0\\ k_2 =& c_q - \frac{1}{\epsilon_2}a_3^2 - \frac{1}{\epsilon_2}{}_6^2 w_{max}^2 > 0\\ k_3 =& c_q\beta_6^2w_{max}^2\end{aligned} \tag{32}$$

From Eq. (31) it is:

$$\dot{V}_4 \le -k_1x_{1e}^2 - k_1z_{1e}^2 - k_2\theta_e^2 - c_{1w}\nu_w^2 - k_3\nu_{u_1}^2 - c_{1q}\nu_q^2 + \mu_1$$

By using the comparison lemma [14], the previous equation leads to:

$$\dot{V}_4 \leq -2\varpi_1 V_4 + \mu \Rightarrow V_4(t) \leq V_4(0)e^{-2\varpi_1 t} + (\mu_1/2\varpi_1)$$

where $\varpi_1 = min\{k_1, k_2, k_3, c_{1w}, c_{1q}\}$. If we define $\xi = [x_e, z_e, \theta_e, \nu_u, \nu_w, \nu_q]^T$, then, considering equation (20) it is $2V_4 = \|\xi\|^2$ we conclude

$$\|\xi(t)\| \leq \|\xi(0)\| e^{-\varpi_1 t} + \sqrt{\frac{\mu_1}{\varpi_1}} \tag{33}$$

Eq. (33) means that the states of the error dynamics remain in a small, bounded set around zero, which can be reduced using an appropriate combination of the controller gains. At this result we arrived using (6) along with (12) and (22).

III. TRACKING CONTROL IN THE LATERAL PLANE (XY)

A. ROV kinematics and dynamics

The marine vehicle has two back thrusters for moving along the surge and the yaw degree of freedom, but no side (lateral) thruster for moving along the sway [1]. The kinematic and dynamic equations of motion are analytically written as:

$$\begin{aligned}
\dot{x} &= c\psi u - s\psi v,\ \dot{y} = s\psi u + c\psi v,\ \dot{\psi} = r \\
\dot{u} &= (1/\delta)\{-X_u u - X_{uu}|u|u + (J_y m_y + \alpha_{uq}\alpha_{vp})vr + \tau_1\} \\
\dot{v} &= (1/\delta)\{-Y_v v - Y_{vv}|v|v - (J_x m_x + \alpha_{vp}\alpha_{uq})ur\} \\
\dot{r} &= (1/J_z)\{-N_r r - N_{rr}|r|r - (X_{\dot{u}} - Y_{\dot{v}})uv + \tau_2\}
\end{aligned} \tag{34}$$

where x and y represent the inertial coordinates of the center mass of the vehicle and u , v are respectively the surge and sway velocities in the body-fixed frame. The orientation of the vehicle is described by the angle ψ and r is its yaw(angular) velocity.

B. Error dynamics formulation

The aim here is to track the following reference variables: $x_r, y_r, \psi_r, u_r, v_r, r_r$. To this end, we define the following tracking errors:
$X_e = \begin{pmatrix} x_e = x - x_r \\ y_e = y - y_r \end{pmatrix}, U_e = \begin{pmatrix} u_e = u - u_r \\ v_e = v - v_r \end{pmatrix}$
$\psi_e = \psi - \psi_r,\ r_e = r - r_r$.
According to (3) and the definition of the tracking errors we obtain the error dynamics as the kinematic ones:

$$\begin{aligned}
\dot{X}_e = &\underbrace{\begin{pmatrix} c\psi & -s\psi \\ s\psi & c\psi \end{pmatrix}}_{R_\psi}\underbrace{\begin{pmatrix} u_e \\ v_e \end{pmatrix}}_{U_e} \\
&+ \underbrace{\begin{pmatrix} c\psi_- c\psi_r & -s\psi + s\psi_r \\ s\psi - s\psi_r & c\psi - c\psi_r \end{pmatrix}}_{R_{\psi_r}}\underbrace{\begin{pmatrix} u_r \\ v_r \end{pmatrix}}_{U_r};\ \dot{\psi}_e = r_e
\end{aligned} \tag{35}$$

and the dynamic ones:

$$\begin{aligned}
\dot{u}_e = &\alpha_1(u_e + u_r) - \alpha_2|u_e + u_r|(u_e + u_r) \\
&-\alpha_3(v_e r_e + r_r v_e + v_r r_e + v_r r_r) - \dot{u_r} + \tau_1
\end{aligned}$$

$$\begin{aligned}
\dot{v}_e = &-\beta_1(v_e + v_r) - \beta_2(v_e + v_r)|v_e + v_r| \\
&-\beta_3(u_e r_e + u_r r_e + r_r u_e + u_r r_r) - \dot{v}_r
\end{aligned}$$

$$\begin{aligned}
\dot{r}_e = &-\gamma_1(r_e + r_r) - \gamma_2(r_e + r_r)|r_e + r_r| \\
&-\gamma_3(u_e v_e + u_r v_e + v_r u_e + u_r v_r)u - \dot{r}_r + \tau_2
\end{aligned} \tag{36}$$

where α_i, β_i and γ_i depend on the ROV fixed parameters. When moving in the horizontal plane, the ROV is not actuated in the sway direction.

C. Control design

The tracking control objective has been transformed to a stabilizing problem given by the system (4)-(5). Thus, we consider the following first feedback law:

$$\begin{aligned}
\tau_u = &\alpha_1(u_e + u_r) - \alpha_2|u_e + u_r|(u_e + u_r) \\
&-\alpha_3(v_e r_e + r_r v_e + v_r r_e + v_r r_r) - \dot{u_r} + \tau_1
\end{aligned}$$

$$\begin{aligned}
\tau_r = &-\gamma_1(r_e + r_r) - \gamma_2(r_e + r_r)|r_e + r_r| \\
&-\gamma_3(u_e v_e + u_r v_e + v_r u_e + u_r v_r)u - \dot{r}_r + \tau_2
\end{aligned} \tag{37}$$

with τ_u and τ_r being considered as new controls to be designed later. The corresponding system of errors can be easily written as:

$$\begin{aligned}
&\dot{X}_e = R_\psi U_e + R_{\psi_r} U_r;\ \dot{\psi}_e = r_e \\
&\dot{u}_e = \tau_u \\
&\dot{v}_e = -\beta_1 v - \beta_2 v|v| - \beta_3(u_e r_e + u_r r_e + r_r u_e + u_r r_r) - \dot{v}_r \\
&\dot{r}_e = \tau_r
\end{aligned} \tag{38}$$

The examination of equation (38) shows that there is direct control capability on the forward (surge) and on the rotational motion of the vehicle but not on the side (sway) motion, i.e., we can control the linear velocity u and the corresponding error u_e as well as the angular velocity r and the corresponding error r_e . We also observe that we have indirect control of the side velocity error v_e through the coupling of the controlled variables in the term $\beta_3(u_e r_e + u_r r_e + r_r u_e + u_r r_r)$. Consistent with backstepping design techniques for the side velocity error v_e, we can choose as an auxiliary control variable one of the controlled velocities and, then, stabilize the latter using the corresponding actual control variable. The same observations hold for the linear and angular position errors: we first use the velocities as control variables for the position, then we stabilize the velocities u_e and r_e with τ_u and τ_r and v_e using the coupling term.

step 1) In order to stabilize the vector position X_e, we assume u_e and v_e as virtual controls. We start by defining the following Lyapunov function candidate:

$$V_1 = \frac{1}{2}X_e^T X_e \Rightarrow \dot{V}_1 = X_e^T(R_\psi U_e + R_{\psi_r} U_r)$$

We suggest as desired expressions for the virtual controls $U_e = (\alpha_u, \alpha_v)^T = -R_\psi^T[KX_e + R_{\psi_r} U_r]$ then,

$$\begin{aligned}\alpha_u =& - k(x_e c\psi + y_e s\psi) - \cos\psi\delta_1 - \sin\psi\delta_2\\ \alpha_v =& - k(-x_e s\psi + y_e c\psi) + \sin\psi\delta_1 - \cos\psi\delta_2\\ \dot{\alpha}_u =& - k(u_e + \cos\psi\delta_1 + \sin\psi\delta_2)\\ \dot{\alpha}_v =& - k(v_e - \sin\psi\delta_1 + \cos\psi\delta_2)\end{aligned} \tag{39}$$

where$[\delta_1, \delta_2]^T = R_{\psi_r} U_r$, $K = diag(k, k)$. The time derivative of V_1 becomes

$$\dot{V}_1 = -X_e^T K X_e.$$

step 2) Since the components of the vector U_e are not true controls, we need to introduce new error variables ϖ_u and ϖ_v defined as: $\varpi = [\varpi_u, \varpi_v]^T = [u_e - \alpha_u, v_e - \alpha_v]^T$. Then, the controlled position equations are rewritten as

$$\dot{X}_e = - KX_e + R_\psi \varpi \tag{40}$$

In the following we would force ϖ_u to zero, so we consider the following Lyapunv function:

$$V_2 = V_1 + (\varpi_u^2)/2 \tag{41}$$

The time derivative of V_2 can be expressed as:

$$\dot{V}_2 = - X_e^T K X_e + R_\psi \varpi + \varpi_u(\tau_u - \dot{\alpha}_u) \tag{42}$$

Setting

$$\tau_u = - c_{1u}\varpi_u - c_{2u}\varpi_u^3 + \dot{\alpha}_u - x_e c\psi - y_e s\psi + f_u \tag{43}$$

where f_w is a design variable for subsequent use, the time derivative of V_2 becomes,

$$\begin{aligned}\dot{V}_2 =& - X_e^T K X_e - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4\\ &+ f_u\varpi_u + \varpi_v(y_e\cos\psi - x_e\sin\psi)\end{aligned} \tag{44}$$

wherec_{1w} and c_{2w} are positive constants. So far, the controlled subsystem of error dynamics equations has been transformed to,

$$\begin{aligned}\dot{X}_e &= -KX_e + R_\psi\varpi\\ \dot{\varpi}_u &= -c_{1u}\varpi_u - c_{2u}\varpi_u^3 + \dot{\alpha}_u - x_e c\psi - y_e s\psi + f_u\end{aligned} \tag{45}$$

Before starting the next step, we perform some manipulations on the sway error dynamic equations. Then $\dot{\varpi}_v$ can be expanded to:

$$\dot{\varpi}_v = - \beta_3 u r_e - \beta_3 r_r \varpi_u + \varrho_v \tag{46}$$

where

$$\begin{aligned}\varrho_v =& k(c\psi\delta_2 - s\psi\delta_1) + (k - \beta_1)v_e - \beta_2|v|v - \beta_3 u_r r_r - \dot{v}_r\\ &- \beta_1 v_r + \beta_3 r_r(kx_e c\psi + ky_e s\psi + \cos\psi\delta_1 + \sin\psi\delta_2)\end{aligned} \tag{47}$$

step 3) In order to stabilize ϖ_v and ψ_e we consider the velocity in yaw as an auxiliary control. Choosing the following Lyapunov function

$$V_3 = V_2 + (\varpi_v^2 + \psi_e^2)/2 \tag{48}$$

and taking into account Eq.(48), its time derivative is:

$$\begin{aligned}\dot{V}_3 =& - X_e^T K X_e - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 + [f_u - \beta_3 r_r \varpi_v]\varpi_u\\ &+ \varpi_v(y_e\cos\psi - x_e\sin\psi) + (\psi_e - \beta_3 u\varpi_v)\alpha_r + \varrho_v\varpi_v\end{aligned} \tag{49}$$

Setting $\alpha_r = -c_r(\psi_e - \beta_3 u \varpi_v)$, $c_r > 0$. Then

$$\begin{aligned}\dot{V}_3 =& - X_e^T K X_e - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 + [f_u - \beta_3 r_r \varpi_v]\varpi_u\\ &+ \varpi_v(y_e\cos\psi - x_e\sin\psi) - c_r(-\beta_3 u\varpi_v + \psi_e)^2 + \varrho_v\varpi_v\end{aligned} \tag{50}$$

step 4) The variable $r_e = \alpha_r$ is not a true control. Thus, we have to introduce an error $\varpi_r = r_e - \alpha_r$ in place of α_r and we use τ_r to stabilize the subsystem:

$$\begin{aligned}\dot{\varpi}_v &= -\beta_3 u r_e - \beta_3 r_r \varpi_u + \varrho_v\\ \dot{\varpi}_r &= \tau_r - \dot{\alpha}_r\\ \dot{\psi}_e &= \varpi_r + \alpha_r\end{aligned} \tag{51}$$

Finally, for the complete system, we choose the lyapunov function:

$$V_4 = V_3 + \varpi_r^2/2 \tag{52}$$

and its time derivative is:

$$\begin{aligned}\dot{V}_4 =& - X_e^T K X_e - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 + [f_u - \beta_3 r_r \varpi_v]\varpi_u\\ &+ \varpi_v(y_e\cos\psi - x_e\sin\psi) - c_r(\psi_e - \beta_3 u\varpi_v)^2 + \varrho_v\varpi_v\\ &+ \varpi_r(\tau_r + c_r((\beta_3 u)^2 + 1)r_e - c_r\beta_3 u\varrho_v + c_r\beta_3^2 u r_r \varpi_u)\end{aligned} \tag{53}$$

We consider the following control law:

$$\begin{aligned}\tau_r =& - c_{1r}\varpi_r - c_{2r}\varpi_r^3 - c_r((\beta_3 u)^2 + 1)r_e + c_r\beta_3 u\varrho_v\\ &- c_r\beta_3^2 u r_r \varpi_u\end{aligned} \tag{54}$$

with c_{1r} and c_{2r} are positives scalar. Then, equation (53) becomes:

$$\begin{aligned}\dot{V}_4 =& - X_e^T K X_e - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 + [f_u - \beta_3 r_r \varpi_v]\varpi_u\\ &+ \varpi_v(y_e\cos\psi - x_e\sin\psi) - c_r(\psi_e - \beta_3 u\varpi_v)^2 + \varrho_v\varpi_v\\ &- c_{1r}\varpi_r^2 - c_{2r}\varpi_r^4\end{aligned} \tag{55}$$

In order to deal with the quantities with uncertain sign we conduct some algebraic manipulations. Firstly, we set

$$f_u = \beta_3 r_r \varpi_v - c_3\varpi_v^2\varpi_u - c_4\varpi_v^4\varpi_u \tag{56}$$

Then, Eq (55) becomes:

$$\begin{aligned}\dot{V}_4 =& - X_e^T K X_e - c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 - c_3\varpi_v^2\varpi_u^2 - c_4\varpi_v^4\varpi_u^2\\ &- c_{1r}\varpi_r^2 - c_{2r}\varpi_r^4 + \varpi_v(y_e\cos\psi - x_e\sin\psi) - c_r\beta_3^2 u^2\varpi_v^2\\ &- c_r\psi_e^2 + 2c_r\beta_3 u\psi_e\varpi_v + \varrho_v\varpi_v\end{aligned} \tag{57}$$

In the above expression, we remark that the last three terms have uncertain signs. For the analysis we will use the Young's inequality, with the quantities $\epsilon_i,\ i = 1....7$ as positive constants, we obtain:

$$\begin{aligned}
\varpi_v((y_e\cos\psi - x_e\sin\psi)) \leq& \frac{|\varpi_v|^2}{4\epsilon_1} + \epsilon_1|x_e|^2 \\
&+\epsilon_1|y_e|^2 \\
\varpi_v(kc\psi\delta_2 - ks\psi\delta_1) \leq& \frac{|\varpi_v|^2}{4\epsilon_2} + \epsilon_2 k^2|\delta_1|^2 \\
&+\epsilon_2 k^2|\delta_2|^2 \\
2c_r\beta_3 u\psi_e\varpi_v \leq& \frac{|\varpi_v|^2}{\epsilon_4} \\
&+\epsilon_4 c_r^2\beta_3^2|\psi_e|^2|u|^2 \\
-\beta_2|v|v\varpi_v \leq& \beta_2 v^2|\varpi_v| \\
\leq& \frac{|\varpi_v|^2}{4\epsilon_5} + \epsilon_5\beta_2^2 v^4 \\
-(\beta_3 u_r r_r + \dot{v}_r + \beta_1 v_r)\varpi_v \leq& \frac{|\varpi_v|^2}{4\epsilon_6} + \epsilon_6|\xi|^2 \\
(k-\beta_1)v_e\varpi_v \leq& \frac{|\varpi_v|^2}{\epsilon_7} \\
&+\epsilon_7(k-\beta_1)^2|v_e|^2
\end{aligned} \tag{58}$$

Angular rates and velocities are considered to have maximum values and they verify:

- $|r_r|^2 \leq r_{r,max}^2,\ |u|^2 \leq u_{max}^2$
- $|v_e|^2 \leq v_{e,max}^2,\ |v|^4 \leq v_{max}^4$
- $|\delta_1|^2 \leq \delta_{1,max}^2,\ |\delta_2|^2 \leq \delta_{2,max}^2$

Taking into account the results from (58) , the time derivative of V_4 in (57), becomes:

$$\begin{aligned}
\dot{V}_4 \leq& -[k-\epsilon_1-\epsilon_3 k^2 r_{r,max}^2]x_e^2 - [k-\epsilon_1-\epsilon_3 k^2 r_{r,max}^2]y_e^2 \\
&-\varpi_v^2[c_3\varpi_u^2 - \frac{1}{2\epsilon_1} - \frac{1}{2\epsilon_2} - \frac{1}{2\epsilon_3} - \frac{1}{\epsilon_4} - \frac{1}{4\epsilon_5} - \frac{1}{4\epsilon_6} - \frac{1}{4\epsilon_7}] \\
&-c_{1u}\varpi_u^2 - c_{2u}\varpi_u^4 - c_4\varpi_v^4\varpi_u^2 - c_{1r}\varpi_r^2 - c_{2r}\varpi_r^4 \\
&-c_r\beta_3^2 u_{max}^2\varpi_v^2 - [c_r - \epsilon_4 c_r^2\beta_3^2 u_{max}^2]\psi_e^2 + \mu_2
\end{aligned} \tag{59}$$

where

$$\begin{aligned}
\mu_2 =& [\epsilon_2 k^2 + \epsilon_3 k^2 r_{r,max}^2]\delta_{1,max}^2 + [\epsilon_2 k^2 + \epsilon_3 k^2 r_{r,max}^2]\delta_{2,max}^2 \\
&+\epsilon_7(k-\beta_1)^2 v_{e,max}^2 + \epsilon_6|\xi|^2 + \epsilon_5\beta_2^2 v_{max}^4
\end{aligned} \tag{60}$$

The coefficient $c_3\varpi_u^2 - \frac{1}{2\epsilon_1} - \frac{1}{2\epsilon_2} - \frac{1}{2\epsilon_3} - \frac{1}{\epsilon_4} - \frac{1}{4\epsilon_5} - \frac{1}{4\epsilon_6} - \frac{1}{4\epsilon_7}$ must be positive, where $\epsilon = \frac{1}{2\epsilon_1} + \frac{1}{2\epsilon_2} + \frac{1}{2\epsilon_3} + \frac{1}{\epsilon_4} + \frac{1}{4\epsilon_5} + \frac{1}{4\epsilon_6} + \frac{1}{4\epsilon_7}$. Consequently $|\varpi_u|$ must be above $\sqrt{\frac{\epsilon}{c_3}}$, which holds for large c_3 and small ϵ. So, from the inequality, we obtain:

$$\dot{V}_4 = -k_1x_e^2 - k_1y_e^2 - k_2\psi_e^2 - k_3\varpi u^2 - k_4\varpi_v^2 - k_5\varpi_r^2 + \mu_2 \tag{61}$$

where $k_1 = k - \epsilon_1 - \epsilon_3 k^2 r_{r,max}^2 > 0,\ k_2 = c_r - \epsilon_4 c_r^2\beta_3^2 u_{max}^2 > 0$, $k_3 = c_{1u} > 0,\ k_4 = c_r\beta_3^2 u_{max}^2 > 0,\ k_5 = c_{1r} > 0$. By using the comparison lemma [14], the previous equation leads to:

$$\dot{V}_4 \leq -2\varpi_2 V_4 + \mu_2 \Rightarrow V_4(t) \leq V_4(0)e^{-2\varpi_2 t} + (\mu_2/2\varpi_2)$$

where $\varpi_2 = min\{k_1, k_2, k_3, c_{1w}, c_{1q}\}$. If we define $\xi = [x_e, z_e, \theta_e, \nu_u, \nu_w, \nu_q]^T$, then, considering equation (52) it is $2V_4 = \|\xi\|^2$ we conclude

$$\|\xi(t)\| \leq \|\xi(0)\|e^{-\varpi_2 t} + \sqrt{\frac{\mu_2}{\varpi_2}} \tag{62}$$

Eq. (62) means that the states of the error dynamics remain in a small, bounded set around zero, which can be reduced using an appropriate combination of the controller gains. At this result we arrived using (37) along with (43) and (54).

IV. Simulation Results

In this section, we give a numerical simulation to illustrate our theoretical results. Before starting, we will present the system parameter values (IS units) used for simulations.

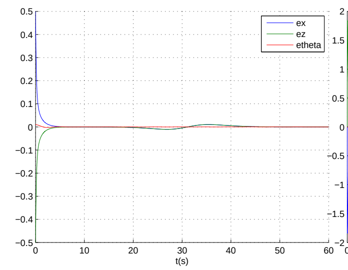
The reference trajectory is described by the following equations

TABLE I
RIGID BODY AND HYDRODYNAMIC PARAMETERS

Parameter	Symbol	Value
mass	m	10.84
Added mass in surge	$X_{\dot{u}}$	-1.0810
Added mass in sway	$Y_{\dot{v}}$	-0.3848
Added mass in heave	$Z_{\dot{w}}$	-0.3.848
Added inertia in roll	$K_{\dot{p}}$	0
Added inertia in yaw	$N_{\dot{r}}$	-0.0075
Added inertia in pitch	$M_{\dot{q}}$	-0.0075
Surge linear drag	X_u	0.9613
sway linear drag	Y_v	2.4674
heave linear drag	Z_w	2.4674
yaw linear drag	N_r	5.3014×10^{-6}
Surge linear drag	M_q	5.3014×10^{-6}
Surge quadratic drag	X_{uu}	4.4674
Sway quadratic drag	Y_{vv}	5.989
heave quadratic drag	Z_{ww}	5.989
Quadratic yaw drag	N_{rr}	0.1011
Quadratic pitch drag	M_{qq}	0.1011

$$x_r = y_r = z_r = h_r\frac{t^5}{t^5 + (t_f - t)^5}$$

where h_r is the desired altitude and t_f is the final time. The simulation results are obtained with these gains:
$k = 0.1,\ c_{1u} = 10,\ c_{2u} = 10,\ c_{1w} = 10,\ c_{2w} = 10,\ c_{1q} = 1,\ c_{2q} = 1,\ c_{1r} = 1,\ c_{2r} = 1,\ c_q = 10,\ c_r = 10,\ h_r = 10.$
In Fig (4,5,8,9), the reference and the actual trajectory of the ROV in the inertial space are displayed. We see the convergence of the center of the mass G trajectory to the desired one. The error in the linear and angular velocities which converge are depicted in Fig (3,7). In Fig (2,6), we can see that the inertial position errors and the Euler angles errors in a small neighborhood of zero.



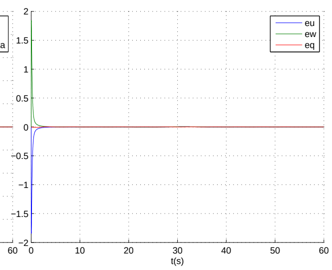
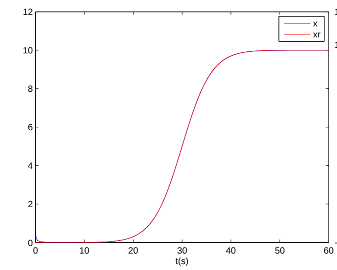


Fig. 2. the inertial position errors and the Euler angles errors in the XZ plane

Fig. 3. The error in the linear and angular velocities in the XZ plane



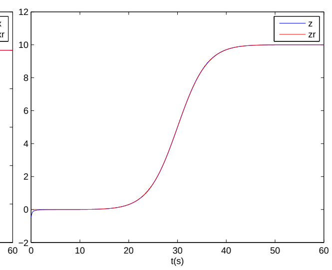
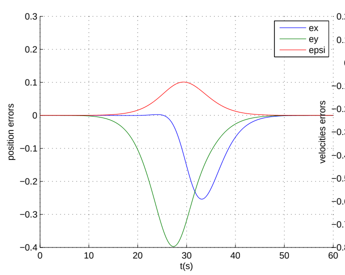


Fig. 4. The actual trajectory and the reference trajectory in the XY plane,

Fig. 5. The actual trajectory and the reference trajectory in the XY plane,



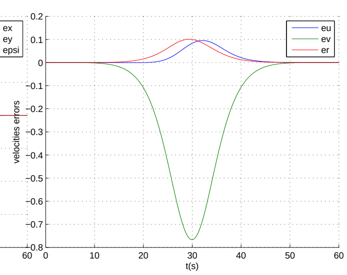
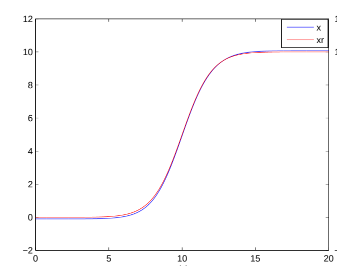


Fig. 6. the inertial position errors and the Euler angles errors in the XY planE

Fig. 7. The error in the linear and angular velocities in the XY plane



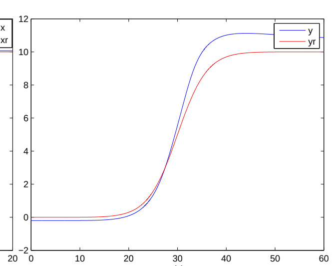


Fig. 8. The actual trajectory and the reference trajectory in the XY plane,

Fig. 9. The actual trajectory and the reference trajectory in the XY plane,

V. CONCLUSIONS

In this paper, the problem of trajectory tracking control for underactuated ROV on the vertical and horizontal plane was addressed. In the first section, the kinematic and dynamic on the vertical plane are described. Given a reference trajectory to be followed by the ROV, using these reference values, the dynamic of the ROV was transformed to the error one. Backstepping techniques were utilized to stabilize the above system and force the tracking error to a neighborhood about zero. In the second section The control problems of tracking on the horizontal plane for a ROV has be considered. A time-varying feedback control laws were derived using a combined integrator backstepping and averaging approach. The trajectories of the controlled ROV were proved to converge to the reference trajectory.

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