On the stability of nonholonomic multi-vehicle formation

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Abstract

The focus of the paper is to solve the stability problem of a multi-agent system composed of nonholonomic vehicles. The multi-vehicle formation is achieved following a first step where targets are identified and attractive sets circumscribing this are constructed. In a second step, the initial position and orientation of each agent is specified. In a last stage, a decentralized controller integrating a regulation control law, ensures obstacles avoidance, is elaborated. The formation's stability is achieved if a group of agents reaches their common target with the right direction and without any motion planning. The LaSalle's invariance principle is the mathematical tool used to prove the invariance of the elaborated set with respect to the nonholonomic multi-vehicle transient behaviors. The simulations illustrate the effectiveness of our strategy and it can be exploited in swarm navigation case.

Key words

Multi-vehicle; nonholonomic constraints, stabilization; formation; LaSalle's principle

1 Introduction

The simultaneous stabilization of the position and orientation involves the control of the evolution of the complete state of an agent. The control of a multi-agent system, especially the system's of nonholonomic vehicles, is an active area of research because the vehicles on wheels are nowadays the main mode of personal transportation, industrial trucks in factories [8], convoys of vehicles on highway, intelligent agent optimization of urban transportation systems [7]. The applications requires the coordination of several vehicles giving rise to new problems in control [12] [13]. Another more technical reason is that the equations governing moving nonholonomic vehicles are highly nonlinear and are of particular theoretical interest in the field of nonlinear control theory. In the other hand and in order to solve the stability problem, the study of robot formation control are inspired from swarm evolution in nature. Thus, from the growth of the industry and military objectives the idea of using multiple small vehicles instead of one big one has been manifested. Because teams of inexpensive robots, performing cooperative tasks, may prove to be more cost and energy-effective than a single one. They are, in addition, capable of achieving a mission more efficiently. Starting from the seminal works of [5] and [6], several theoretical frameworks have been proposed to analyze the collective motion of multi-agent systems. A typical aim of these approaches is to formulate decentralized control laws driving the team of agents to pre-specified equilibrium configurations. Although a rigorous stability analysis of nonholonomic multi-agent systems is generally a very difficult task, nice theoretical results have been obtained in the case of linear motion models. Stability analysis becomes more challenging when kinematic constraints are taken into account, as in the case of wheeled nonholonomic vehicles. One cites, the stability analysis of collective circular motion for nonholonomic multi-vehicle systems in [3], the study of flocking in teams as nonholonomic agents in [9], the virtual structure in formation control of nonholonomic multi-vehicle systems in [10], The formation vector control of nonholonomic mobile robot groups proposed by [11] and the formation control strategy for unicycles which did not include obstacles avoidance in [4].

In this paper we propose a new methodology in formation control of non-holonomic vehicle system. The formation convergence with respect to targets uses the LaSalle's invariance principle, and this will characterize the stability's domain. Further, a regulation control input is proposed such that each agent's trajectory in closed loop can be adjusted for obstacles avoidance and without motion planning.

The paper is organized as following: a circumvention of a target by the group is proposed in section 2. Section 3 deals with the stability that integrates obstacles avoidance. Our simulation results are presented and commented in section 4. Some conclusions are given in section 5.

2 Circumvention of a target by the group

Via the nonlinear controller which will be designed, the objective is to ensure the convergence of the multi-vehicle in group toward an attractive set. This set must integrate the target and the control problem is different from the classical stabilization theory where the set is reduced to a point. Hence, a circle contains the target will be constructed and where each vehicle occupies a specified position with the adequate orientation. To do, the LaSalle's invariance theorem will be useful to solve the target's circumvention problem by a group of nonholonomic vehicles. Recall that . The LaSalle's theorem enures the following: if the vehicle is initialized in the set Ω, where Ω is considered invariant with respect to the system's kinematics, and if there exists a function V continuously differentiable such that its derivative with respect to time t is negative for each solution in Ω, then the solution which has the initial condition q_0 in Ω converges toward the largest invariant set defined by :

$$E = \{q \in \Omega / \dot{V}(q) = 0\}$$

Let us denote the position of a target by $A(a, b)$ in the two dimensions space. On defines a circle centered in A with a radius l which is considered large enough such that all vehicles can be positioned above and may include the target. In the following we give a mathematical description of the vehicle model. The system is a set of n two-wheeled vehicles with a parametrization as shown by Figure 1.

Let $q_i = (x_i, y_i)$ denotes the i^{th} vehicle's position and θ_i its orientation. The nonholonomic kinematic behavior of the i^{th} vehicle is described by:

$$\begin{aligned} \dot{x}_i = & \quad u_i \cos\theta_i \\ \dot{y}_i = & \quad u_i \sin\theta_i \\ \dot{\theta}_i = & \quad w_i \end{aligned} \tag{1}$$

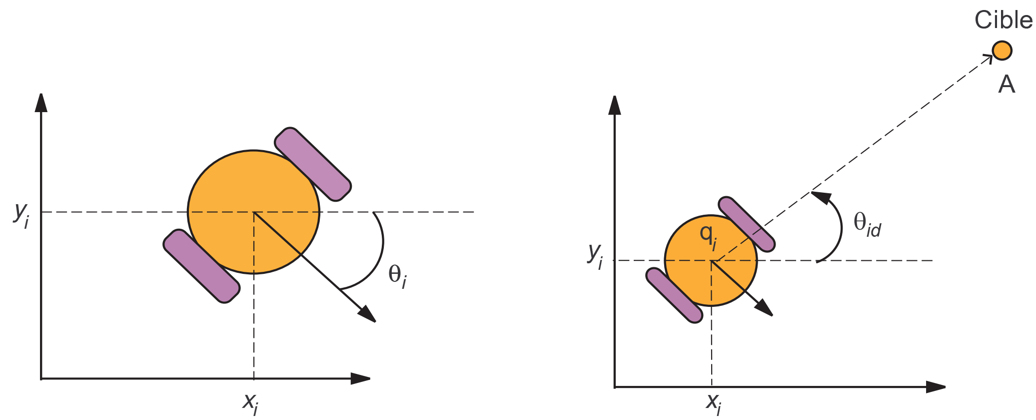


Figure 1: Parametrization of the i^{th} vehicle position and orientation

In order to avoid a spiral motion which may occur if no control is addressed to the vehicle's orientation, we propose not only to control the position such each vehicle reach the circle surrounding the target but also to ensure the adequate angle of direction. However due the nonholonomic behavior of kinematics the convergence set of the system must include the trajectory errors due to orientations. Consequently, one introduces the error as following:

$$e_i = \theta_i - \theta_{id} \tag{2}$$

with the desired angle $\theta_{id} = \arctan(\dfrac{y_i - b}{x_i - a})$ is the angle defined by the horizontal axis and the vector $\overrightarrow{q_i A}$ (see figure 1).

We will refer to a vehicle model in the following form:

$$\begin{aligned} \dot{x}_i = &\quad u_i \cos\theta_i \\ \dot{y}_i = &\quad u_i \sin\theta_i \\ \dot{e}_i = &\quad w_i - \dot{\theta}_{id} \end{aligned} \tag{3}$$

The stability objective consists to converge the rotation error e_i toward π and the vehicle's trajectory toward a circle circumscribing the target with the appropriate distance. Let $q_i \in \mathbb{R}^2$ denotes the position of the i^{th} vehicle and $e_i \in \mathbb{R}$ due to the directional error. The multi-vehicle system positions and orientations are regrouped in the vetors $q = (q_1, q_2, ..., q_N)$ and $e = (e_1, e_2, ..., e_N)$. The main result proposed in the following theorem shows the system's invariance.

Theorem 2.1 *Let Ω a compact set:*

$$\Omega = \{(q, e) \in \mathbb{R}^{2N} \times \mathbb{R}^N / l \leq \|q_i - A\| \leq k, |e_i| \leq c\}$$

We consider N nonholonomic vehicle with the kinematics (3) and the two control inputs (u_i, w_i). Then,

1. *Ω is invariant with respect to (3)*

2. The control laws

$$\begin{aligned} u_i &= (\|q_i - A\|^2 - l^2)\cos e_i \\ w_i &= -e_i + \dot{\theta}_{id} \end{aligned} \tag{4}$$

ensure that each solution with initial condition in Ω converges to the largest invariant set

$$M = \{(q, e) \in \Omega / \|q_i - A\| = l, e = 0_{\mathbb{R}^N}\}$$

with $0_{\mathbb{R}^N} = (0, 0, .., 0) \in \mathbb{R}^N$.

□

The result of the theorem consists to prove, firstly, that $\|q_i - A\| \to l$ as $t \to \infty$, meaning that q_i converge to a circle centered in A with the radius l. Secondly, $e_i \to 0$ as $t \to \infty$ leading to $\|\theta_i - \theta_{id}\| \to 0$. Further, we search to prove that the solutions of (3) remains into the set Ω. The three following Lemmas are useful to the analysis.

Lemma 2.2 *Let*

$$\Omega = \{(q, e) \in \mathbb{R}^{2N} \times \mathbb{R}^N / l \leq \|q_i - A\| \leq k, |e_i| \leq c\}$$

All solutions of (3) initialized in Ω remain in time in Ω. We must show that Ω is invariant with respect to nonholomic kinematics (3).

□

Proof. Let us start with $(q_0, e_0) \in \Omega$ and $S(q_i) = \|q_i - A\|$. The time derivative of $S(q_i)$ leads to:

$$\begin{aligned} \dot{S}(q_i) &= -\frac{< \dot{q}_i, A - q_i >}{S} \\ &= -\frac{u_i((a - x_i)\cos(\theta_i) + (b - y_i)\sin(\theta_i))}{S} \\ &= -\frac{u_i < A - q_i, \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix} >}{S} \end{aligned} \tag{5}$$

$<, >$ denotes the scalar product.

As $e_i = \theta_i - \theta_{id}$ which is the angle defined by $\left(A - q_i; \begin{pmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{pmatrix}\right)$, it is obvious that

$$\dot{S}(q_i) = -\frac{u_i S \cos e_i}{S} = -u_i \cos e_i \tag{6}$$

Taking into account the control law (4), we obtain:

$$\dot{S}(q_i) = -(\| q_i - A \|^2 - l^2) \cos^2 e_i \tag{7}$$

which is equivalent to

$$\frac{\dot{S}(q_i)}{S(q_i)^2 - l^2} = -\cos^2 e_i \tag{8}$$

We integrate this relation,

$$\begin{aligned}
\int_{S_0}^{S} \frac{dh}{h^2 - l^2} &= \int_{t_0}^{t} -\cos^2 e_i ds \\
\Leftrightarrow \ \ln(\frac{S-l}{S+1}) &= \ln(\frac{S_0 - l}{S_0+1}) + \int_{t_0}^{t} -2l\cos^2 e_i ds \\
\Leftrightarrow \ \frac{S-l}{S+1} &= \frac{S_0 - l}{S_0+1} exp(\int_{t_0}^{t} -2l\cos^2 e_i ds)
\end{aligned}$$

with $S = S(q_i)$ and $S_0 = S(q_{i0})$. As $(q_0, e_0) \in \Omega$ then $S_0 - l \geq 0$ which implies that $S(q_i) \geq l$. Consequently,

$$\| q_i - A \| \geq l$$

In the other hand, $S(q_i) \geq l$ implies that $\dot{S}(q_i) \leq 0$. S is a decreasing function with respect to time, then

$$\|q_i - A\| \leq \|q_{i0} - A\| \leq k$$

Finally, it is obvious that

$$l \leq \|q_i - A\| \leq k$$

Now we take into account the control law related to w_i. The dynamic of the i^{th} orientation error leads to,

$$\begin{aligned}
\dot{e}_i &= -e_i \\
\Leftrightarrow \ e_i &= e_{i0}\exp(t_0 - t) \quad \forall t \geq t_0
\end{aligned}$$

consequently, $|e_i| \leq |e_{i0}| \leq c$. As a result $(q, e) \in \Omega$ and Ω is invariant with respect to (3).

■

Lemma 2.3 *Assume that (q, e) is solution of (3) with $(q_0, e_0) \in \Omega$ it is initial condition, and the Lyapunov function candidate V*

$$V(q,e) = \sum_{i=1}^{n} \| q_i - A \| + \frac{1}{2} e_i^2$$

The time derivative of $V(q, e)$ is negative on Ω.

□

Proof. Taking the time derivative of $V(q, e)$,

$$\begin{aligned}
\dot{V}(q,e) &= \sum_{i=1}^{n} \frac{< \dot{q}_i, q_i - A >}{\| q_i - A \|} + (w_i + \dot{\theta}_{id}) e_i \\
&= \sum_{i=1}^{n} u_i \cos e + (w_i + \dot{\theta}_{id}) e_i
\end{aligned}$$

Inject the expressions of u_i and w_i into (2):

$$\dot{V}(q,e) = -\sum_{i=1}^{n} (\| q_i - A \|^2 - l^2) \cos^2 e_i + e_i^2 \leq 0$$

This achieves the result.

■

Lemma 2.4 *Assume that (q, e) is solution of (3). The set M given by*

$$M = \{(q, e) \in \Omega / \|q_i - A\| = l, e = 0_{\mathbb{R}^N}\}$$

is invariant with respect to (3)

□

Proof. Let $(q_0, e_0) \in M$ and $H(q_i) = S(q_i) - l$ with $S(q_i)$ is the function defined in the proof of Lemma 2.2. The time derivative of $H(q_i)$ is given by:

$$\dot{H}(q_i) = \dot{S}(q_i) = -(\| q_i - A \|^2 - l^2) \cos^2 e_i \leq 0$$

Hence, H is a decreasing function which implies that

$$0 \leq \| q_i - A \| - l \leq \| q_{i0} - A \| - l = 0$$

Consequently,

$$\| q_i - A \| - l = 0$$

In the other hand,

$$e_i \quad = e_{i0} exp(t_0 - t) \quad \forall t \geq t_0$$

and as $(q_0, e_0) \in M$ then $e_{i0} = 0$. So,

$$|e_i| = 0$$

Finally, one concludes that if $(q_0, e_0) \in M$ then $(q, e) \in M$. M is an invariant set with respect to the nonholonomic multi-vehicle system (3).

■

The statement of these three lemmas and their proofs will be used for proof of Theorem 2.1.

Proof.(Theorem 2.1). Following to Lemmas 2.2-2.3, The LaSalle's invariance principle confirms that the solutions of (3) initialized in Ω converge to the largest invariant set $E = \{(q, e) \in \Omega / \dot{V} = 0\}$ where E is no other than:

$$\begin{aligned} E \quad &= \left\{ \ (\| q_i - A \| - l = 0) \vee (\cos e_i = 0) \ \right\} \wedge (e_i = 0) \\ &= (\| q_i - A \| - l = 0) \wedge (e_i = 0) = M \end{aligned}$$

From Lemma 2.4, we may conclude that M is the largest invariant set of E.

■

In the following, the stability of the nonholonomic multi-vehicle formation integrates obstacles avoidance. This is an extension of Theorem 2.1.

3 Stability integrates obstacles avoidance

In the literature, little work has been invested on the nonholonomic multi-vehicle stability and obstacle avoidance. For only one vehicle, we cite [1] for the existence of a regular repulsive feedback stabilization. Tacking into account the nonholonomic constraint, our objective is to determine a continuous repulsive control law that permits to avoid obstacles. Theses obstacles are considered known on the multi-vehicle navigation environment. Recall that each agent of the multi-vehicle system is described by 3. A first result to the extension of the attractive control law is given by the following theorem.

Theorem 3.1 *Let Ω be a compact set*

$$\Omega = \{(q,e) \in \mathbb{R}^{2N} \times \mathbb{R}^N / V_i = \frac{1}{2}[\|q_i - A\|^2 + e_i^2] \leq \frac{1}{2}k^2,\ \forall\, i \in \{1,2,..,N\}\}$$

The N nonholonomic multi-vehicle formation is given by (3). Then

1. $\Gamma = \{(q,e) \in \Omega \times \mathbb{R}^N /\ l \leq \|q_i - A\| \leq k,\ \forall\, i \in \{1,2,..,N\}\}$ *is invariant with respect to (3).*

2. *The following control law,*

$$\begin{aligned} u_i &= (1-\nu_i(q_i)e_i^2)\frac{(\|q_i - A\|^2 - l^2)}{\|q_i - A\|}\cos e_i \\ w_i &= -e_i + \dot{\theta}_{id} - \nu_i(q_i)e_i(\|q_i - A\|^2 - l^2)\cos^2 e_i \end{aligned} \tag{9}$$

   for all scalar function ν_i independent of $S = \|q_i - A\|^2 - l^2$, and such that $\int_{t_0}^{t} -(1 - \nu_i e_i)\cos^2 e_i ds$ is convergent, then all solutions of (3) with the initial conditions in Ω converge toward the set

$$M = \{(q,e) \in \Omega / \|q_i - A\| = l, e = 0_{\mathbb{R}^N}\}$$

   with $0_{\mathbb{R}^N} = (0,0,..,0) \in \mathbb{R}^N$.

$\square$

Proof. It is clear that Ω is a compact set because $\Omega = V_i^{-1}([0, \frac{1}{2}k^2])$ and V_i is continuous on $\mathbb{R}^{2N} \times \mathbb{R}^N$. Firstly, we show that Ω is invariant. Let $(q_0, e_0) \in \Omega$ and the Lyapunov function candidate V is given by

$$V = \frac{1}{2}\sum_{i=1}^{N}(\|q_i - A\|^2 + e_i^2) = \sum_{i=1}^{N} V_i(q_i, e_i) \tag{10}$$

Its time derivative is as,

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N} -u_i\|q_i - A\| \cos e_i + (w - \dot{\theta}_d)e_i \\ &= \sum_{i=1}^{N} -(1-\nu_i(q_i)e_i^2)(\|q_i - A\|^2 - l^2)\cos^2 e_i - e_i^2 - \nu_i(q_i)e_i^2(\|q_i - A\|^2 - l^2)\cos^2 e_i \\ &= \sum_{i=1}^{N} -(\|q_i - A\|^2 - l^2)\cos e_i^2 - e_i^2\end{aligned} \tag{11}$$

it remains to prove that $(\|q_i - A\|^2 - l^2) \geq 0$. Let $S = \|q_i - A\|^2 - l^2$, its time derivative is given by,

$$\begin{aligned}\dot{S} &= -u_i\|q_i - A\| \cos e \\ &= -(1-\nu_i e_i^2)S\cos^2 e_i\end{aligned} \tag{12}$$

Par hypothesis ν_i can be constructed such that it is independent of S, hence we can write

$$\begin{aligned}\frac{\dot{S}}{S} &= -(1-\nu_i e_i^2)\cos^2 e_i \\ S &= S_0 \exp(\int_{t_0}^{t} -(1-\nu_i e_i^2)\cos^2 e_i ds)\end{aligned} \tag{13}$$

Following to hypothesis given in the theorem 3.1, the $\displaystyle\int_{t_0}^{t} -(1-\nu_i e_i^2)\cos^2 e_i ds$ is finite, the we may conclude that the derivative of V_i with respect to t is negative semidefinite. As $(q_0, e_0) \in \Omega$ then,

$$V_i(q) \leq V_i(q_0) \leq \frac{1}{2}k^2$$

$\forall\ (q_0, e_0) \in \Omega$. By now, for the system (3) let us show that $\Gamma \subset \Omega$. Taking $(q_0, e_0) \in \Gamma$, and as we have proved that $V_i(q) = \frac{1}{2}[\|q_i - A\|^2 + e_i^2] \leq \frac{1}{2}k^2$, $\forall\ i \in \{1, 2, .., N\}$ and $\forall\ (q_0, e_0) \in \Gamma \subset \Omega$, consequently $\frac{1}{2}\|q_i - A\|^2 \leq \frac{1}{2}k^2$ and

$$\|q_i - A\| \leq k \tag{14}$$

As it was proved that $S = \|q_i - A\|^2 - l^2$ is positive semidefinite for all $(q_0, e_0) \in \Gamma \subset \Omega$, meaning that,

$$\|q_i - A\| \geq l \tag{15}$$

Then Γ is an invariant set. It remains to prove that all solutions of (3) converge toward

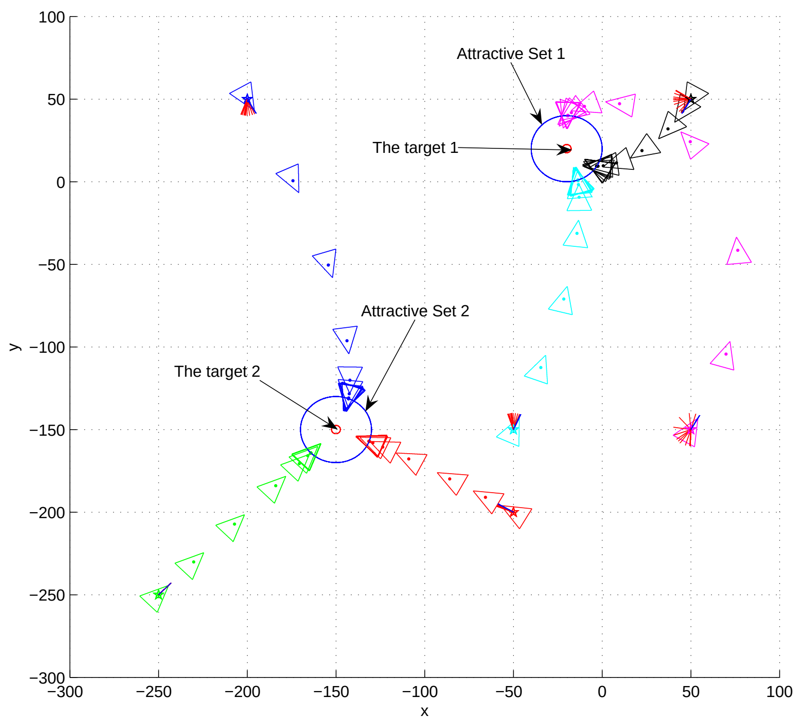
$$M = \{(q, e) \in \Omega / \|q_i - A\| = l, e = 0_{\mathbb{R}^N}\} \tag{16}$$

Applying the LaSalle's invariance principle [2], as Ω is compact and invariant with respect to (3), and the time derivative of V is negative semidefinite on Ω, then all solutions starting in Ω approach the largest invariant set of $E = \{(q, e) \in \Omega / \dot{V} = 0\} = M$.

■

4 Simulation results

In order to check our theoretical results, one carried out several numerical tests with a Matlab program. One chose a finite number of nonholonomic mobile robots going to six. Figure 4 presents the six mobile robots, distributed in a random manner within the $2D$ space. The effectiveness of the proposed controller is subject of two targets surrounding with a predefined distribution to each target. The six mobile robots are divided on two sub-group where each sub-group is composed of three sub-entities. In this case, each target is surrounded and the steering angle of each entity converges to the desired one. One notes that the proposed controllers in velocities, are limited in magnitude (bounded). However, the limitation depends strongly on the distant mobile robot. In the future of this work, many questions should be treated. One quotes the convergence of the robots to the same attractive set requires the same fixed time of realization. The largest invariance set Ω, which is parameterized by k and should contains the target and all the formation, depends on the initial position of the distant robot. Hence, this initial position cannot be as large as possible as this introduces a limitation in velocity magnitudes.



5 Conclusion

In this paper, the stability of the formation, is subdivided in three steps. In the first, one was to identify one or more target(s) and to create an attractive set around each one. In the second, one had identified the initial position and orientation of each entity of the formation.

In the last stage, one applies to the formation, the decentralized controller found above. As a result, the formation was to reach each target with the right direction, this being without any motion planning. One notes that the convergence to the right steering angle is asserted at the same time as the circle that surround the target is reached. Moreover, we extended the term of the controller to include a new scalar regulation control input ν. We addressed more flexibility in designing ν such that a new trajectory's behavior could be obtained while the convergence to the desired set is kept. As a perspective, and more than the existence of this function, our goal is to find an explicit form to ν that permits to avoid all obstacles in the navigation area, and guarantees that no collision occur for the multi-vehicle formation.

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