Streamlined Rotors Mini Rotorcraft : Trajectory Generation and Tracking

Lotfi Beji and Azgal Abichou

Abstract: We present in this paper the stabilization (tracking) with motion planning of the six independent configurations of a mini unmanned areal vehicle equipped with four streamlined rotors. Naturally, the yaw-dynamic can be stabilized without difficulties and independently of other motions. The remaining dynamics are linearly approximated around a small roll and pitch angles. It will be shown that the system presents a flat output that is likely to be useful in the motion generation problem. The tracking feedback controller is based on receding horizon point to point steering. The resulting controller involves the lift (collective) time derivative for what flatness and feedback linearization are used. Simulation tests are performed to progress in a region with approximatively ten-meter-buildings.

Keywords: Mini-UAV, tracking control, flatness, motion planning.

1. INTRODUCTION

Unmanned Areal Vehicles (UAV) terrain mission control is a matter of both interest and controversy for scientific research and engineering design. A large class of industrial and military control constraints consist in planning and following predefined trajectories. Examples range from unmanned and remotely piloted airplanes and submarines performing surveillance and inspection, mobile robots moving on factory floors and multi-fingered robot hands performing inspection and manipulation inside the human body under a surgery control. All these systems are highly nonlinear and require accurate performance.

Modeling and controlling aerial vehicles (blimps [2], mini rotorcraft) are the principal preoccupation of the *Lsc, Lim-* groups. In this topic, a mini-UAV is under construction by the Lsc-group taking into account industrial constraints. The areal flying engine couldn't exceed $2kg$ in mass, and $50cm$ in diameter with a $30mn$ flying-time. Within this optic, it can be held that our system belongs to a family of mini-UAV. A mini-flyer with streamlined rotors and blades was envisaged by the group. It is an autonomous hovering system, capable of vertical take-off, landing and quasi-stationary (hover or near hover) flight conditions. Compared to helicopters, named quad-rotor, [1,7,11] the four-rotor rotorcraft has some advantages [5,10]: given that two motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend, in trimmed flight, to cancel. An X4-flyer operates as an omnidirectional UAV. Vertical motion is controlled by collectively increasing or decreasing the power for all motors. Lateral motion, in x-direction or in y-direction, is achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations.

A model for the dynamic and configuration stabilization of quasi-stationary flight conditions of a four rotor vertical take-off and landing (VTOL) was studied by Hamel [5] where the dynamic motor effects are incorporating and a bound of perturbing errors was obtained for the coupled system. The stabilization problem of a four rotor rotorcraft is also studied and tested by Castillo [3] where the nested saturation algorithm is used. The idea is to guarantee a bound of the roll and pitch angles with a fixed bounded in control inputs. With the intent to stabilize an aircraft that is able to take-off vertically as helicopters, the control problem was solved for the planar vertical take-off and landing (PVTOL) with the input/output linearization procedure [6] and theory of flat systems [4,8,9].

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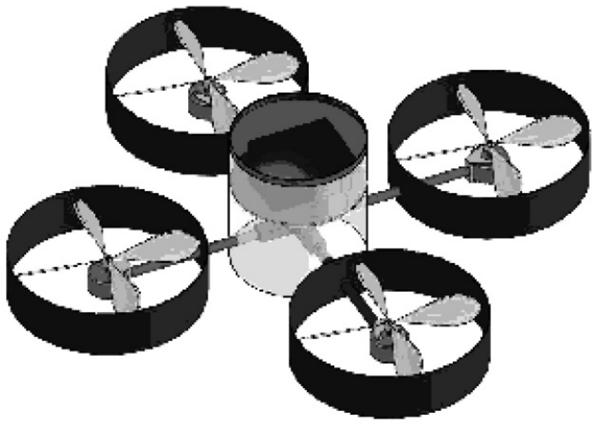

Fig. 1. General view of the four rotors rotorcraft.

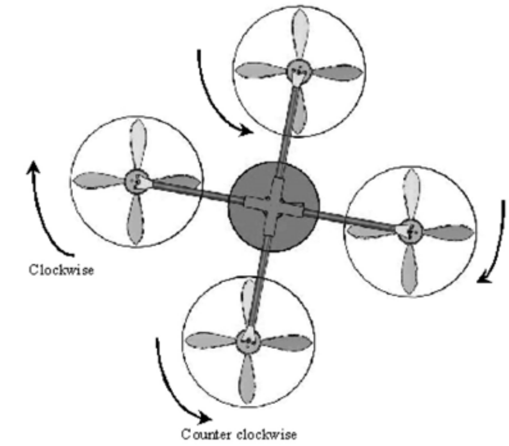

Fig. 2. Upper view to the four rotors rotorcraft.

In this paper, flatness and motion planning are combined to solve the point per point control of the X4-flyer. We show that the system is flat with $\xi=(x,y,z)$ as a flat-output. By virtue of the system being flat, we can write all state and input trajectories satisfying the differential equation in terms of the flat output and its time derivatives. The idea will be considered here for point to point control problem with a predefined path following.

The paper presents as follows: the translational and rotational motions, described by the Newton-Euler formalism, are detailed in Section 2. Section 3 deals with the flatness of the system and the way the reference motion is scheduled. The stabilization of the relative equilibrium is addressed in Section 4 where the stability of altitude/attitude motion is accomplished. A strategy to solve the tracking problem through point to point steering is shown in Section 5; incorporating the real time control and the trajectory realization. Finally, simulation tests, results and comments are put at work.

2. CONFIGURATION DESCRIPTION AND MODELLING

The X4-flyer is a system consisting of four individual electrical fans attached to a rigid cross frame. It is an omnidirectional VTOL vehicle ideally suited to stationary and quasi-stationary flight conditions. We consider a local reference airframe $\Re_G=\{G,E_1^g,E_2^g,E_3^g\}$ attached to the center of mass G of the vehicle. The center of mass is located at the intersection of the two rigid bars, each of which supports two motors. Equipment (controller cartes, sensors, etc.) onboard are placed not far from G. The inertial frame is denoted by $\Re_o=\{O,E_x,E_y,E_z\}$ such that the vertical direction E_z is upwards. Let the vector $\xi=(x,y,z)$ denote the position of the center of mass of the airframe in the frame $\Re_o$. While the rotation of the rigid body is determined by a rotation $R:\Re_G\to\Re_o$, where $R\in SO(3)$ is an orthogonal rotation matrix. This matrix is defined by the three Euler angles, θ (pitch), φ (roll) and ψ (yaw) which are regrouped in $\eta=(\varphi,\theta,\psi)$. A sketch of the X4-flyer is given in Figs. 1, 2, and 3. Here-after, we give details about the equation of motion, obtained with the Newton-Euler method.

2.1. Translation motion

We consider the translation motion of $\Re_G$ with respect to (wrt) $\Re_o$. The position of the center of mass wrt $\Re_o$ is defined by $\overrightarrow{OG}=(x\ y\ z)^t$, its time derivative gives the velocity wrt to $\Re_o$ such that $\frac{d\overrightarrow{OG}}{dt}=(\dot{x}\ \dot{y}\ \dot{z})^t$, while the second time derivative permits to get the acceleration: $\frac{d^2\overrightarrow{OG}}{dt^2}=(\ddot{x}\ \ddot{y}\ \ddot{z})^t$ denoted by

$$\frac{d^2\overrightarrow{OG}}{dt^2}=\overrightarrow{\gamma_G}\,|_{\Re_O}. \qquad (1)$$

Applying the first Newton equation of mechanics, we obtain the following compact expression of the translational motion

$$m\,\overrightarrow{\gamma_G}\,|_{\Re_O}=-mg\overrightarrow{e_z}+R^t_{\varphi\theta\psi}\vec{u}, \qquad (2)$$

where m is the total mass of the vehicle. The vector $\vec{u}$ combines the principal non conservative forces applied to the X4-flyer airframe including forces generated by the motors (Fig. 2) and drag terms. Drag forces and gyroscopic due to motors effects will be not considered in this work. $\overrightarrow{e_z}$ is the unit vector of E_z. The lift (collective) force $\vec{u}$ is the sum of the four forces, such that

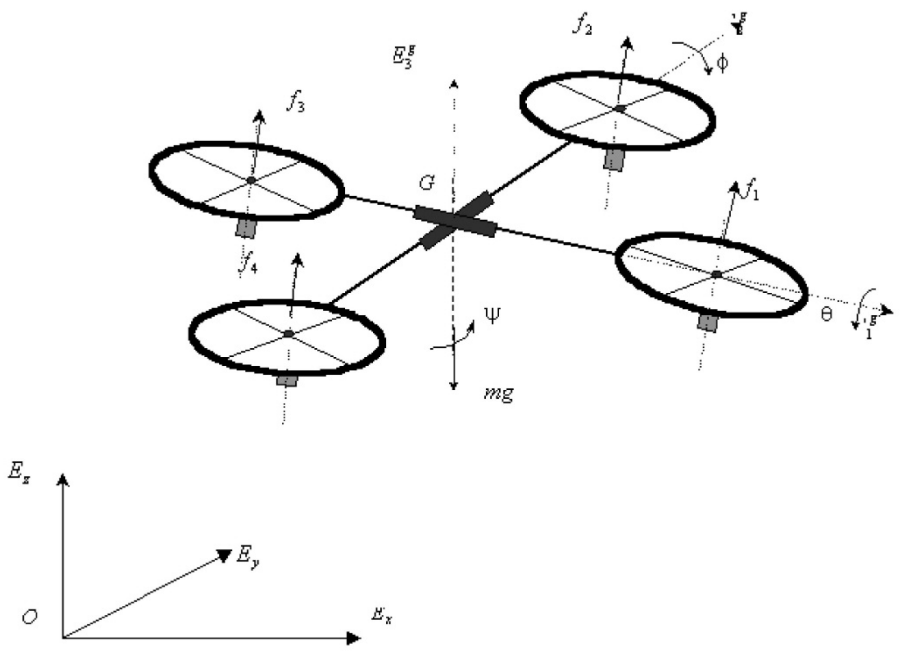

Fig. 3. Frames attached to the four rotor rotorcraft.

$$\vec{u} = \sum_{i=1}^{4} \overrightarrow{f_i} \tag{3}$$

with $\overrightarrow{f_i} = k_i \omega_i^2 \overrightarrow{e_3}$ and $\vec{e}_3$ is the unit vector along E_3^g. $k_i > 0$ is a given constant (we consider $k_i = k$) and ω_i is the angular speed resulting of motor i. The form of the rotation matrix used in (2) is as follow:

$$R_{\varphi\theta\psi} = \begin{pmatrix} c_\theta c_\psi & s_\psi C_\theta & -s_\theta \\ c_\psi s_\theta s_\varphi - s_\psi c_\varphi & s_\theta s_\psi s_\varphi + c_\psi c_\varphi & c_\theta s_\varphi \\ c_\psi s_\theta c_\varphi + s_\psi s_\varphi & s_\theta s_\psi c_\varphi - c_\psi s_\varphi & c_\theta c_\varphi \end{pmatrix}. \tag{4}$$

One substitute (3) into (2), we obtain

$$m\ \overrightarrow{\gamma_G}\, |_{\Re_O} = -mg\overrightarrow{e_z} + uR^t_{\varphi\theta\psi}\overrightarrow{e_3}\ , \tag{5}$$

where the scalar $u = \sum_{i=1}^{4} k_i \omega_i^2$ represents the system'input ($u > 0$).

2.2. Rotational motion

The rotational motion of the X4-flyer will be defined *wrt* to the local frame but expressed in the inertial frame. According to Classical Mechanics, and knowing the inertia matrix I_G of the X4-flyer at the center of mass and its local velocity of rotation Ω, the kinetic moment $\overrightarrow{\sigma_G}$ is defined by

$$\overrightarrow{\sigma_G} = I_G \vec{\Omega} \tag{6}$$

or the rotational velocity vector is related to $\eta = (\varphi\ \theta\ \psi)^t$ vector through

$$\Omega = J(\eta)\dot{\eta}\ , \tag{7}$$

where in the local frame $\vec{\Omega} = \Omega\vec{e}$, $\vec{e} = (\overrightarrow{e_1}\ \overrightarrow{e_2}\ \overrightarrow{e_3})^t$ and the jacobian

$$J(\eta) = \begin{pmatrix} c_\theta & 0 & -s_\theta \\ 0 & c_\varphi & s_\varphi c_\theta \\ 0 & -s_\varphi & c_\varphi c_\theta \end{pmatrix}.$$

Then, from (6) and (7) we get

$$\overrightarrow{\sigma_G} = I_G J(\eta)\dot{\eta}\vec{e}\ . \tag{8}$$

In the following let $\Pi_G(\eta) \triangleq I_G J(\eta)$. Using the derivative of (8), the dynamic moment

$$\overrightarrow{\delta_G} = \dot{\Pi}_G(\eta)\dot{\eta}\vec{e} + \Pi_G(\eta)\ddot{\eta}\vec{e} + \dot{\eta}^t\Pi_G(\eta)\dot{\eta}\vec{e}\ , \tag{9}$$

where the superscript t denotes the transpose. The rotational motion is subject of the following relation

$$\dot{\Pi}_G(\eta)\dot{\eta}\vec{e} + \Pi_G(\eta)\ddot{\eta}\vec{e} + \dot{\eta}^t\Pi_G(\eta)\dot{\eta}\vec{e} = \sum \overrightarrow{M}_{ext}\ , \tag{10}$$

where the external moments *wrt* G ($l_i = l, i = 1,...,4$ distance from G to motor i, which are considered identical)

$$\sum \overrightarrow{M}_{ext} = \tau_\theta \overrightarrow{e_1} + \tau_\varphi \overrightarrow{e_2} + \tau_\psi \overrightarrow{e_3}, \quad (11)$$

where

$$\begin{aligned} \tau_\theta &= lk(\omega_2^2 - \omega_4^2), \\ \tau_\varphi &= lk(\omega_3^2 - \omega_1^2), \\ \tau_\psi &= \kappa(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2). \end{aligned} \quad (12)$$

The equality from (10) is ensured, meaning that

$$\ddot{\eta} = \Pi_G(\eta)^{-1}(\tau - \dot{\Pi}_G(\eta)\dot{\eta}) \quad (13)$$

with $\tau = (\tau_\theta \ \tau_\varphi \ \tau_\psi)^t$.

Explicitly writing, we get

$$\begin{aligned} \ddot{\theta} &= \tfrac{1}{I_{xx}c_\varphi}(\tau_\theta + I_{xx}s_\varphi \dot{\phi}\dot{\theta}), \\ \ddot{\phi} &= \tfrac{1}{I_{yy}c_\varphi c_\theta}(\tau_\varphi + I_{yy}s_\varphi c_\theta \dot{\phi}^2 + I_{yy}s_\theta c_\varphi \dot{\theta}\dot{\phi}), \\ \ddot{\psi} &= I_{zz}\tau_\psi. \end{aligned} \quad (14)$$

As a first step, the model given above can be input/output linearized by the following decoupling feedback laws

$$\begin{aligned} \tau_\theta &= -I_{xx}s_\varphi \dot{\phi}\dot{\theta} + I_{xx}c_\varphi \tilde{\tau}_\theta, \\ \tau_\varphi &= -I_{yy}s_\varphi c_\theta \dot{\phi}^2 - I_{yy}s_\theta c_\varphi \dot{\theta}\dot{\phi} + I_{yy}c_\varphi c_\theta \tilde{\tau}_\varphi, \\ \tau_\psi &= I_{zz}\tilde{\tau}_\psi, \end{aligned} \quad (15)$$

and the decoupled dynamic model of rotation can be written as

$$\ddot{\eta} = \tilde{\tau} \quad (16)$$

with $\tilde{\tau} = (\tilde{\tau}_\theta \ \tilde{\tau}_\varphi \ \tilde{\tau}_\psi)^t$.

Using the translational and rotational motions (2) and (16), equations of the dynamic are detailed by

$$\begin{aligned} m\ddot{x} &= -us_\theta, \\ m\ddot{y} &= uc_\theta s_\varphi, \\ m\ddot{z} &= uc_\theta c_\varphi - mg, \\ \ddot{\theta} &= \tilde{\tau}_\theta, \\ \ddot{\phi} &= \tilde{\tau}_\varphi, \\ \ddot{\psi} &= \tilde{\tau}_\psi, \end{aligned} \quad (17)$$

which is a set of nonlinear differential equations with drift.

The four inputs u, $\tilde{\tau}_\theta$, $\tilde{\tau}_\varphi$ and $\tilde{\tau}_\psi$ will be calculated to ensure the point to point stabilization with motion planning. It is clear that our device belongs to families of underactuated systems. System (17) seems to be treatable compared to blimps [2].

Note that in (17) the appropriate choice of $\tilde{\tau}_\psi$ permits to stabilize ψ at any desired value ψ_d modulo 2π. As well as for its first and second time derivatives. While θ and φ variables are limited to an open set defined by $\pm\frac{\pi}{2}$.

3. MOTION PLANNING AND FLATNESS

A system is flat if we can find a set of outputs (equal in number to the number of inputs) such that all states and inputs can be determined from these outputs without integration [9]. Flatness was first defined by Fliess et al. [4]. It was considered by Martin et al. [9] with motion planing. More precisely, if the system has states $x \in \mathbb{R}^n$, and inputs $u \in \mathbb{R}^m$, with

$$\dot{x} = f(x,u), \quad (18)$$

where f is a smooth vector field, then the system is flat if we can find outputs $y \in \mathbb{R}^m$ of the form

$$y = h(x, u, \dot{u}, ..., u^{(r)})$$

such that

$$x = \varphi(y, \dot{y}, ..., y^{(q)}),$$
$$u = \alpha(y, \dot{y}, ..., y^{(q)}).$$

When a system is flat it is an indication that the nonlinear structure of the system is well characterized and one can exploit that structure in designing control algorithms for motion planing, trajectory generation, and stabilization.

Proposition 1: The X4-flyer described by the dynamic (17) is flat with $\xi = (x, y, z)$ is its flat output.

Proof: First, we define the state by $X = (x, y, z, \theta, \varphi, \dot{x}, \dot{y}, \dot{z}, \dot{\theta}, \dot{\phi})$, we denote $\dot{X}$ its time derivative, and the input vector is regrouped in $U = (u, \tilde{\tau}_\theta, \tilde{\tau}_\varphi)$, then the system can be written as

$$\dot{X} = f(X, U). \quad (19)$$

To prove that the state and the control vector are function of the flat output and their derivatives, for any given trajectory ($x(t), y(t), z(t)$) smooth enough, we get ($u > 0$)

$$\begin{aligned} &u = m\left(\ddot{x}^2 + \ddot{y}^2 + (\ddot{z}+g)^2\right)^{\frac{1}{2}}, \\ &\varphi = arctg\left(\frac{\ddot{y}}{\ddot{z}+g}\right), \\ &\theta = -arctg\left(\frac{c_\varphi \ddot{x}}{\ddot{z}+g}\right). \end{aligned} \tag{20}$$

Indeed, u, θ, φ, $\dot{u}$, $\dot{\theta}$ and $\dot{\phi}$ are function of $\ddot{\xi}, \xi^{(3)}$. So, it is straightforward to verify that $X = \varphi(\xi, \dot{\xi}, \xi^{(2)}, \xi^{(3)})$. Moreover, we can derive $\theta(t)$, $\varphi(t)$ and prove the ξ -dependence of $\tilde{\tau}_\theta = \alpha_{\tilde{\tau}_\theta}(\xi^{(2)}, \xi^{(3)}, \xi^{(4)})$ and $\tilde{\tau}_\varphi = \alpha_{\tilde{\tau}_\varphi}(\xi^{(2)}, \xi^{(3)}, \xi^{(4)})$. □

It follows from the fact that the system is flat that the feasible trajectories of the system are completely characterized by the motion of the center of mass of the X4-flyer. By converting the input constraints on the system to constraints on the curvature and higher derivatives of the motion of G, it is possible to compute efficient techniques for trajectory generation.

4. STABILIZATION OF THE RELATIVE EQUILIBRIUM

The relative equilibrium of the flying machine is subject of $\ddot{x} = \ddot{y} = \ddot{z} = 0$ and $\ddot{\theta} = \ddot{\phi} = \ddot{\psi} = 0$. It leads to $\theta = \varphi = 0$, $u = mg$ and $\tilde{\tau}_\theta = \tilde{\tau}_\varphi = 0$. In the following we stabilize an equilibrium of the form $(x_d, y_d, z_d, 0, 0, \psi_d)$ integrating motion planning. The flatness property of the system will serve for the trajectory planning between the given initial flat output (ξ_i, t_i) and the final one (ξ_f, t_f) where t_i and t_f are the initial and final time, respectively. As we have demonstrated in section III, we can write all trajectories ($X(t), U(t)$) satisfying the differential equation type (19) in terms of the flat output and its derivatives. In what following, we will see that time derivatives at fourth order of the flat output will be needed. In the simple stabilization control problem, i.e. without motion planning, time derivatives of the reference flat output are equal to zero. In our case, these derivations appear. Thus, our investigation can be viewed like case of tracking problem.

At first, we assume that $(\theta, \varphi) \in (0,0)$ such that (17) can be transformed to

$$\begin{aligned} &m\ddot{x} = -u\theta, \\ &m\ddot{y} = u\varphi, \\ &m\ddot{z} = u - mg, \\ &\ddot{\theta} = \tilde{\tau}_\theta, \\ &\ddot{\phi} = \tilde{\tau}_\varphi, \\ &\ddot{\psi} = \tilde{\tau}_\psi. \end{aligned} \tag{21}$$

4.1. Altitude z -stabilization and ψ -control

The control of the vertical position (altitude) can be obtained considering the following control input

$$u = mg + \ddot{z}_d - mk_1^z(\dot{z} - \dot{z}_d) - mk_2^z(z - z_d), \tag{22}$$

where k_1^z, k_2^z are the coefficients of stable polynomial and z_d is the desired altitude.

The yaw attitude can be stabilized to a desired value with the following tracking feedback control

$$\tilde{\tau}_\psi = \ddot{\psi}_d - k_1^\psi(\dot{\psi} - \dot{\psi}_d) - k_2^\psi(\psi - \psi_d), \tag{23}$$

where k_1^ψ, k_2^ψ are stable coefficients.

Indeed, introducing (22) into (21), we obtain

$$\begin{aligned} &\ddot{x} = -(g + f(z, z_d))\theta, \\ &\ddot{y} = (g + f(z, z_d))\varphi, \\ &\ddot{z} = f(z, z_d), \\ &\ddot{\theta} = \tilde{\tau}_\theta, \\ &\ddot{\phi} = \tilde{\tau}_\varphi, \\ &\ddot{\psi} = \ddot{\psi}_d - k_1^\psi(\dot{\psi} - \dot{\psi}_d) - k_2^\psi(\psi - \psi_d), \end{aligned} \tag{24}$$

where the function $f(z, z_d) = \ddot{z}_d - k_1^z(\dot{z} - \dot{z}_d)$ $-k_2^z(z - z_d)$ is assumed to be regular wrt to their arguments.

The following investigation concerns the system of the form

$$\begin{aligned} &\ddot{x} = -(g + f(z, z_d))\theta, \\ &\ddot{y} = (g + f(z, z_d))\varphi, \\ &\ddot{z} = f(z, z_d), \\ &\ddot{\theta} = \tilde{\tau}_\theta, \\ &\ddot{\phi} = \tilde{\tau}_\varphi, \end{aligned} \tag{25}$$

which can be subdivided on two independent cascade dynamics, the first one is given by

$$\begin{aligned} &\ddot{x} = -(g + f(z, z_d))\theta, \\ &\ddot{\theta} = \tilde{\tau}_\theta, \end{aligned} \tag{26}$$

and the second is

$$\begin{aligned} &\ddot{y} = (g + f(z, z_d))\varphi, \\ &\ddot{\phi} = \tilde{\tau}_\varphi. \end{aligned} \tag{27}$$

Designing the control $\tilde{\tau}_\theta$ in the dynamic (26) permits to stabilize (bound) the pitch angle which will be viewed as a control input for the x-motion. As soon as for (27), where $\tilde{\tau}_\varphi$ will be determined and φ is the input to stabilize the y-motion.

4.2. x-stabilization and θ-control

As the output x is flat, then its dynamic is transformed in order to make appear the control $\tilde{\tau}_\theta$. Recall that

$$\ddot{x} = -(g + f(z, z_d))\theta\,. \tag{28}$$

When one derive twice this expression, we get

$$x^{(4)} = -\ddot{f}(z, z_d)\theta - 2\dot{f}(z, z_d)\dot{\theta} - (g + f(z, z_d))\tilde{\tau}_\theta\,. \tag{29}$$

Proposition 2: By the fact that $g + f(z, z_d) = \frac{1}{m}u$, which is by hypothesis positif as $u > 0$ (see (3)), the asymptotic stability of x, consequently of θ is asserted by (property of the flat output)

$$\tilde{\tau}_\theta = -\frac{1}{g + f(z, z_d)}(\nu_x + \ddot{f}(z, z_d)\theta + 2\dot{f}(z, z_d)\dot{\theta}) \tag{30}$$

with

$$\begin{aligned}\nu_x = x_d^{(4)} - k_1^x(x^{(3)} - x_d^{(3)}) - k_2^x(\ddot{x} - \ddot{x}_d)\\ -k_3^x(\dot{x} - \dot{x}_d) - k_4^x(x - x_d),\end{aligned} \tag{31}$$

where $k_1^x, k_2^x, k_3^x, k_4^x$ are positives and stable coefficients.

Proof: Incorporating (30) into (29), it leads to the decoupled x-motion

$$x^{(4)} = \nu_x \tag{32}$$

further, taking ν_x as given in (31), x and their successive time derivatives are asymptotically stable. It means, by virtue of the original system (17), θ reaches its equilibrium ($\theta(t_f) = 0$). □

4.3. y-stabilization and φ-control

As detailed above, φ denotes the roll angle. This attitude has the same behavior like for θ. Roll allure is necessary to the X4-flyer to correct motion in the y-direction. These variables are related by the cascade system

$$\begin{aligned}\ddot{y} &= (g + f(z, z_d))\varphi,\\ \ddot{\phi} &= \tilde{\tau}_\varphi.\end{aligned} \tag{33}$$

As before, we will proceed by four derivatives of the flat output y with respect to time

$$y^{(4)} = \ddot{f}(z, z_d)\varphi + 2\dot{f}(z, z_d)\dot{\phi} + (g + f(z, z_d))\tilde{\tau}_\varphi. \tag{34}$$

Proposition 3: By the fact that $g + f(z, z_d) = \frac{1}{m}u$, which is by hypothesis positif definite as $u > 0$, the asymptotic stability of y, consequently of φ is such that

$$\tilde{\tau}_\varphi = -\frac{1}{g + f(z, z_d)}(\nu_y + \ddot{f}(z, z_d)\varphi + 2\dot{f}(z, z_d)\dot{\phi}) \tag{35}$$

with

$$\begin{aligned}\nu_y = y_d^{(4)} - k_1^y(y^{(3)} - y_d^{(3)}) - k_2^y(\ddot{y} - \ddot{y}_d)\\ -k_3^y(\dot{y} - \dot{y}_d) - k_4^y(y - y_d),\end{aligned} \tag{36}$$

where $k_1^y, k_2^y, k_3^y, k_4^y$ are positives and stable coefficients.

Proof: Incorporate (35) into (34), it leads to a decoupled y-motion

$$y^{(4)} = \nu_y \tag{37}$$

further, tacking ν_y as given in (36), y and their successive time derivatives are asymptotically stable. It means, by virtue of the original system (17), φ reaches its equilibrium ($\varphi(t_f) = 0$).

Remark 1: The proposed stabilizing controllers $\tilde{\tau}_\theta$ and $\tilde{\tau}_\varphi$ involve the first and second time derivatives of $f(z, z_d)$. We can easily calculate it from (22) and (17). Therefore, $\dot{f}(z, z_d) = z_d^{(3)} + ((k_1^z)^2 - k_2^z)\dot{e}_z + k_1^z k_2^z e_z$ and $\ddot{f}(z, z_d) = z_d^{(4)} -((k_1^z)^3 - 2k_1^z k_2^z)\dot{e}_z - ((k_1^z)^2 k_2^z - (k_2^z)^3)e_z$.

Remark 2: The new controllers shown in (31) and (36) involve the third time derivative of x and y, respectively. Easily, this dynamic can be obtained from the X4-flyer initial equations (17), as

$$\begin{aligned}x^{(3)} &= -\tfrac{1}{m}(\dot{u}s_\theta + uc_\theta\dot{\theta}),\\ y^{(3)} &= \tfrac{1}{m}(\dot{u}c_\theta s_\varphi - us_\theta s_\varphi\dot{\theta} + uc_\theta c_\varphi\dot{\phi}).\end{aligned}$$

Terms involving the time derivative $\dot{u} = \dot{f}(z, z_d)$, $\dot{\theta}$ and $\dot{\phi}$ should be measured or reconstructed from the flat properties of the system. However, the time derivative of $f(.)$ was detailed in Remark 1.

5. TRAJECTORY GENERATION AND POINT TO POINT STEERING

Due to the structure limit of the *X4-flyer*: motion can be asserted only in straight line along the *x*, *y* and *z* directions. In our case, that is sufficient to navigate in a region. Otherwise, an other version of the engine is under study by the group. The version flyer is to make easy maneuvers in corners with arc of circle. In the following, we solve the tracking problem as point to point steering one over a finite interval of time. Then we take each ending point with its final time as a new starting point. Fig. 4 illustrates the reference trajectory along the *x*, *y* and *z* directions. As we see, the X4-flyer will fly in the z-direction followed by the x -motion and the y -motion. The reference trajectory is parameterized as

$$z^r(t) = h_d \frac{t^5}{t^5 + (T_f^1 - t)^5}, \tag{38}$$

where h_d is the desired altitude and T_f^1 the final time. In order to solve the point to point steering control, the end point of the trajectory (38) can be adopted as initial point to move along x, then we have

$$x^r(t) = h_d \frac{(t - T_f^1)^5}{(t - T_f^1)^5 + (T_f^2 - (t - T_f^1))^5}. \tag{39}$$

Identically for $y^r(t)$

$$y^r(t) = h_d \frac{(t - T_f^2)^5}{(t - T_f^2)^5 + (T_f^3 - (t - T_f^2))^5}. \tag{40}$$

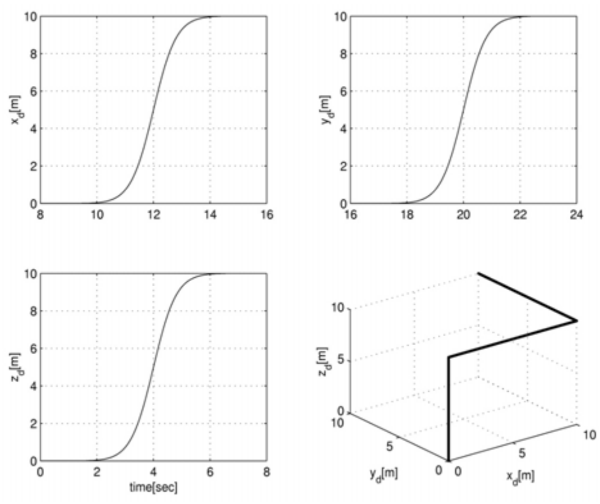

Fig. 4. Motion planning with $h_d = 10m$.

Constraints to perform these trajectories are such that

$$\begin{aligned}
&z^r(0) = x^r(T_f^1) = y^r(T_f^2) = 0, \\
&z^r(T_f^1) = x^r(T_f^2) = y^r(T_f^3) = h_d, \\
&\dot{z}^r(0) = \dot{x}^r(T_f^1) = \dot{y}^r(T_f^2) = 0, \\
&\dot{z}^r(T_f^1) = \dot{x}^r(T_f^2) = \dot{y}^r(T_f^3) = 0, \\
&\ddot{z}^r(T_f^1) = \ddot{x}^r(T_f^2) = \ddot{y}^r(T_f^3) = 0, \\
&\ddot{z}^r(0) = \ddot{x}^r(T_f^1) = \ddot{y}^r(T_f^2) = 0,
\end{aligned} \tag{41}$$

minimizing the time of displacement implies that the X4-flyer accelerates at the beginning and decelerates at the arrival.

5.1. Real time control and trajectory investigation

From a experimental point of view, a joystick with three degrees of freedom for the X4-flyer animation will be considered. This material is interpreted as a commanded position signal helping the user to progress in hostile environment. The user will be informed about the vehicle positions by a visual feedback. The study of visual feedback involves image based visual servo control. This investigation would be the subject of future work. In this paragraph, we incorporate relations between torques, motor velocities and the command referenced positioning. Recall that the X4-flyer equipped with four brushless dc-motors which are commanded in voltages (currents) and not directly in torques. Brushless motors deliver high rate, largely boarded on mini flying machines. The variation of current permits to adjust speeds and forces given by relations (3) and (12). Feasible trajectories are subject of tests on limits and constraints related by (20). In order to interpret the control in term of velocities, recall that (3) and (12) permit to write

$$\begin{pmatrix} u \\ \tau_\theta \\ \tau_\varphi \\ \tau_\psi \end{pmatrix} = k \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \bar{\omega}_1^2 \\ \bar{\omega}_2^2 \\ \bar{\omega}_3^2 \\ \bar{\omega}_4^2 \end{pmatrix} \triangleq k \Upsilon \bar{\omega}^2. \tag{42}$$

The proposed control law in $(u, \tau_\theta, \tau_\varphi, \tau_\psi)$ gives an $\bar{\omega}^2 = (\bar{\omega}_1^2, \bar{\omega}_2^2, \bar{\omega}_3^2, \bar{\omega}_4^2)$ different of ω_i^2 ($i = 1,4$) developed by the actuators. Differences are due to the presence of motor dynamics which should be integrated to the system equations. Such an idea allows to control the system in velocities. To do so, let it be born in mind that the motor shaft dynamic is connected to the rigid body dynamics via the velocity component ($i = 1,...,4$)

$$I_r \dot{\omega}_i = \tau_{mi} - k\overline{\omega}_i^2, \tag{43}$$

where $k\overline{\omega}_i^2 = \sum_{j=1}^{4} \Upsilon_{ij}^{-1}\Gamma_j$ and we assume that $\overline{\omega}_i = \dot{\omega}_i$. The constant I_r represents the shaft inertia and τ_{mi} is the torque transmitted by the shaft (assumed to be rigid). Perturbation due to frictions and/or backlash can be easily incorporated in the model. The following analysis shows that such an undesirable phenomenon influences accuracy in motion. Then it should be compensated by the control τ_m.

Given the reference flat output with their derivatives $(\xi_d, \dot{\xi}_d, \ddot{\xi}_d, \xi_d^3, \xi_d^4, \xi_d^5)$, the reference velocities obtained from (42) ($l, k > 0$) verify

$$\begin{pmatrix} \overline{\omega}_{1d}^2 \\ \overline{\omega}_{2d}^2 \\ \overline{\omega}_{3d}^2 \\ \overline{\omega}_{4d}^2 \end{pmatrix} = k^{-1}\Upsilon^{-1} \begin{pmatrix} u^d \\ \tau_\theta^d \\ \tau_\varphi^d \\ \tau_\psi^d \end{pmatrix} \triangleq k^{-1}\Upsilon^{-1}\Gamma_d, \tag{44}$$

where u_d is given by (20) with ξ is replaced by ξ_d. The other elements of Γ_d are in (15) where we substitute current states by the reference ones. Identically in (43); the shaft reference velocity should verify ($\overline{\omega}_{id} = \dot{\omega}_{id}$)

$$I_r \dot{\omega}_{id} = \tau_{mi}^d - k\overline{\omega}_{id}^2. \tag{45}$$

Therefore, we have

$$\begin{aligned} I_r(\dot{\omega} - \dot{\omega}_d) &= \tau_m - \tau_m^d - k(\overline{\omega}^2 - \overline{\omega}_d^2) \\ &= \tau_m - \tau_m^d - \Upsilon^{-1}(\Gamma - \Gamma_d) \end{aligned}$$

with the proposed input τ_m

$$\tau_m = \tau_m^d + \Upsilon^{-1}(\Gamma - \Gamma_d) - k_\omega(\omega - \omega_d) \tag{46}$$

such that $k_\omega > 0$, we can assert the convergence of ω to ω_d. Moreover $\tau_m \to \tau_m^d$ and $\Gamma \to \Gamma_d$.

6. SIMULATIONS

Recall that the objectives consist in testing the point to point stabilizing configuration. What we need to compare is the tracking problem with and without motion planning. Motion generation is described here by an important and limited acceleration in ascent following with an important deceleration which permits to reach at t_f the desired point. The generated motion could satisfy $\xi_d(t_i) = \dot{\xi}_d(t_i) = 0$ and $\xi_d(t_f) = \xi_d$, $\dot{\xi}_d(t_f) = 0$. The final time t_f shouldn't be reduced enough to limit an excessive reference acceleration. Without motion planning ξ_d can't be more then $1m$, otherwise the system diverges. Tests have been effectuated as follows: for $\xi_d = \xi_d(t_f) = 1m(t_f = 4s)$ (with and without motion planning) and $\xi_d(t_f) = 10m(t_f = 8s)$ (only with motion planning). All control parameters are $k_1^z = 8$, $k_2^z = 16$, $k_1^x = k_1^y = 20$, $k_2^x = k_2^y = 150$, $k_3^x = k_3^y = 500$ and $k_4^x = k_4^y = 625$. The masse is $m = 2kg$.

6.1. Results and comments

Any equilibrium of the X4-flyer is defined by ($x_d, y_d, z_d, 0, 0, \psi_d$). Let $\xi_d(x_d, y_d, z_d) = 10m$. To reach this configuration, an example of motion planning within the z-direction is given in Fig. 5. Motions along x, y-directions are similar to that of z. The last sub-figure shows that $f(z, z_d) > -g$, then the validity of the proposed controllers. The derivatives $\dot{z}_d$ and $\ddot{z}_d$ behavior are sketched in Fig. 5. The allure of inputs (Fig. 6) shows that $u > 0$ and $u = mg$ when the X4-flyer reaches the relative equilibrium. In addition we show that the behavior of errors, given by Fig. 7 is verified. At the equilibrium, attitudes of $\varphi(t_f)$ and $\theta(t_f)$ are equal to zero. Without motion planning, the system becomes instable and incapable to reach this configuration. To compare the flight with/without a predefined path, we will consider Figs. 8 and 10. Without motion planning, the amplitude of controllers is important (Fig. 10), chattering dominates the behavior of inputs when the system leaves its initial configuration. While with a predefined path a minimum of energy is asserted which is requested for flying vehicles. Motion in different directions *z, x* and y is also tested and shown by Fig. 12. With motion planning, we can assert a good behavior of the X4-flyer even in presence of drag forces. Drag forces ($0.5\dot{x}, 0.5\dot{y}, 0.5\dot{z}$) influence motion along the x and y directions (Fig. 13), but with a good allure of motion. In order to guarantee $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, robustness of τ_θ and τ_φ towards singularities should be deeply studied. Simulation tests are accomplished with Fig. 11, where we prove the well tracking of shafts velocities. Recall that these velocities are subject of motion planning integrating constraints of the system. The simulated parameters, and used in Fig. 11, are $I_r = k = 1$ and $k_\omega = 1000$.

7. CONCLUSIONS

We have presented a rotorcraft with streamlined four rotors. The dynamic model involves four control inputs used to stabilize the engine with predefined paths. The system presents a flat output which was efficiency exploited in motion planning, in point to point stabilization and in tracking control with respect to a region with ten-meter-buildings. It was shown that the algorithm is sensible to the necessary final-time of the reference trajectory. Due to limits in autonomy of batteries in fly, acceleration/deceleration of the vehicle in motion is justified. The proposed control law is extended to the actuator dynamics which permits to control the shaft and the blade velocities. With the proposed motion planning, actuator saturations can be overcomed, consequently economy in energy of batteries can be asserted during the fly. This work will be extended to systems with delay and flatness based-visual feedback control.

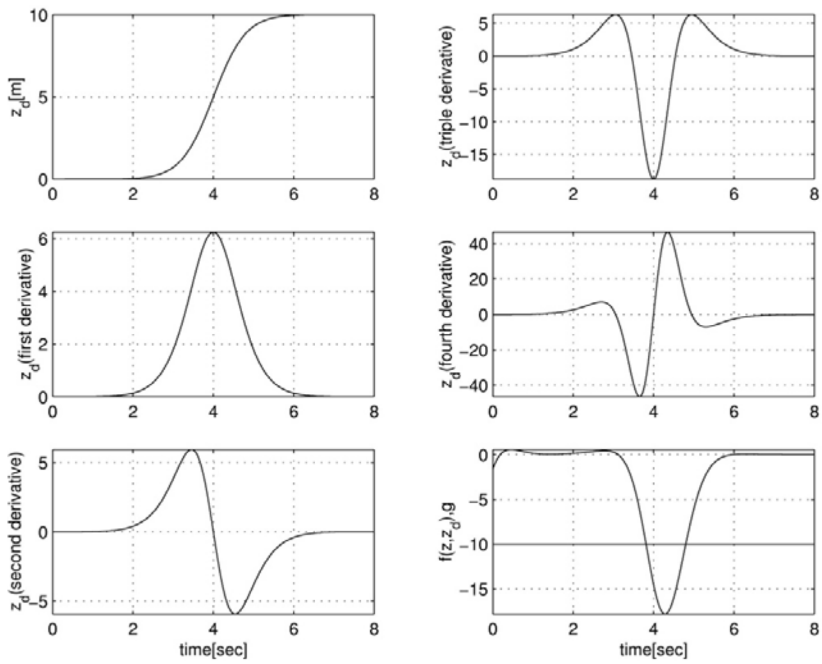

Fig. 5. Motion planning within the z-direction (h_d = 10m).

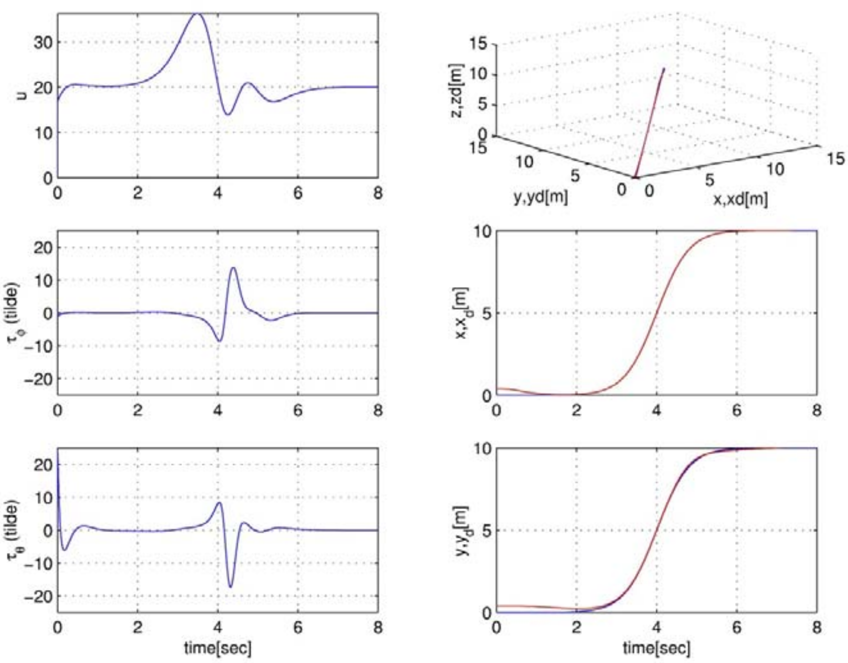

Fig. 6. Necessary inputs to stabilize (h_d = 10m).

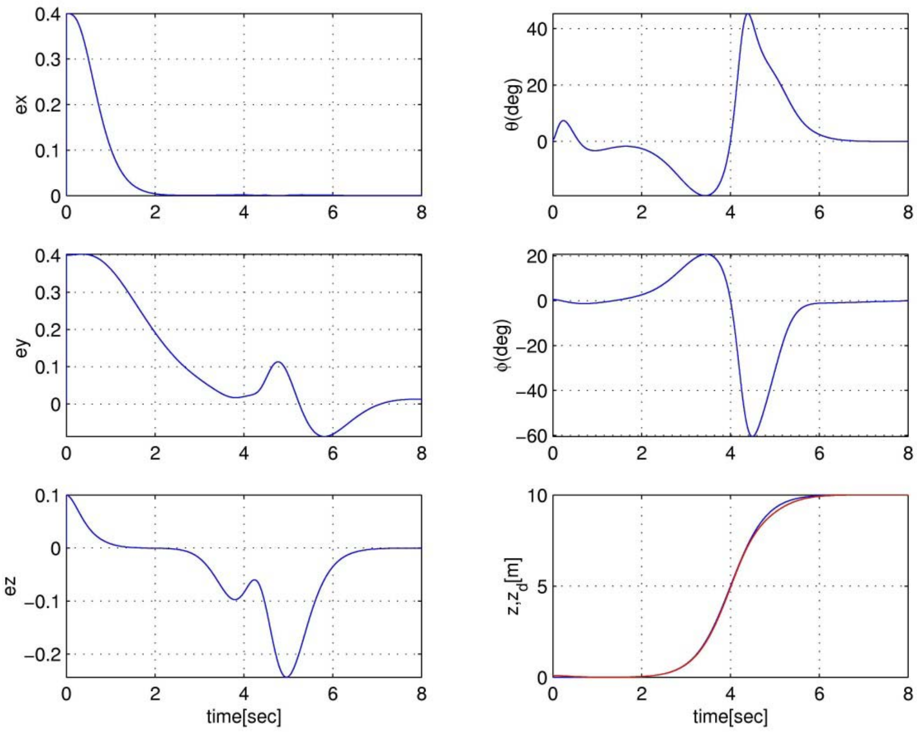

Fig. 7. Stabilization errors with motion planning ($h_d = 10m$).

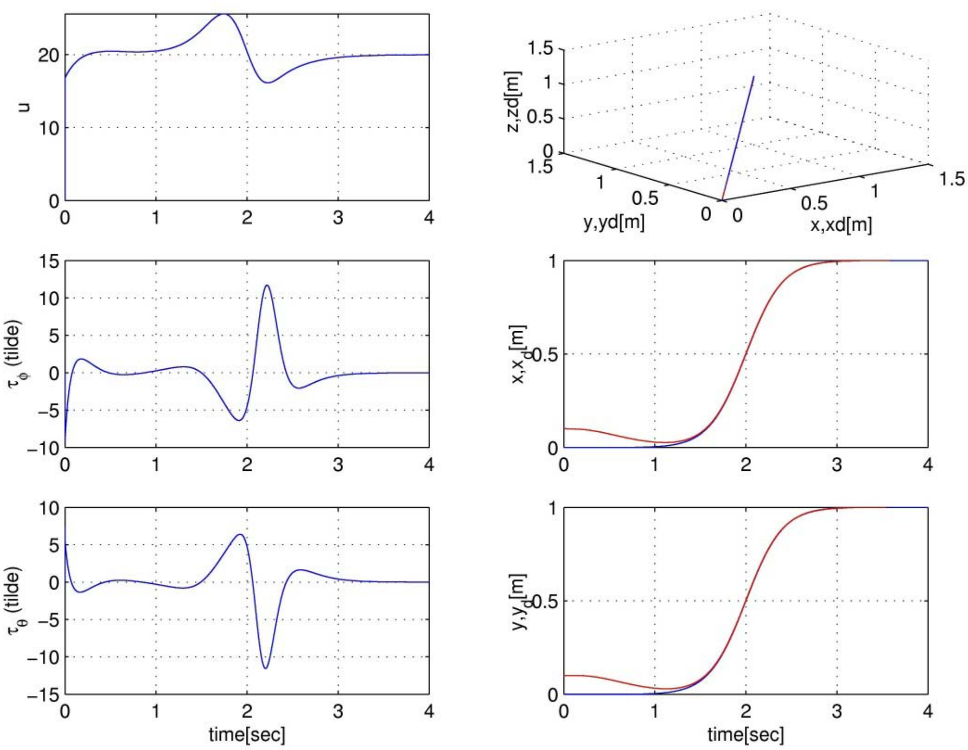

Fig. 8. Inputs and tracking behaviors ($h_d = 1m$).

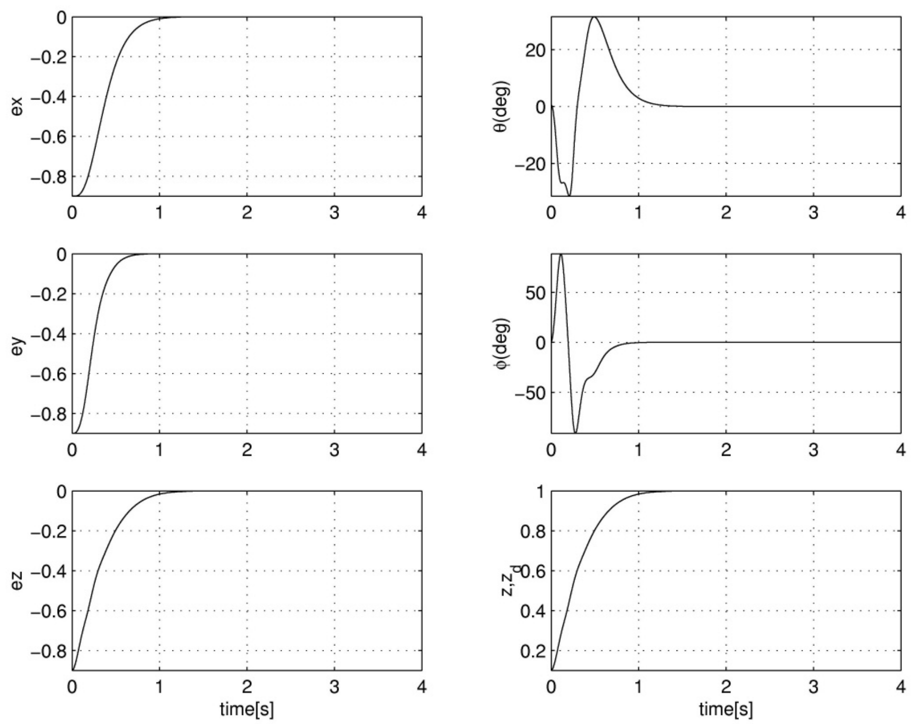

Fig. 9. Tracking errors for (h_d = 1m) without planning.

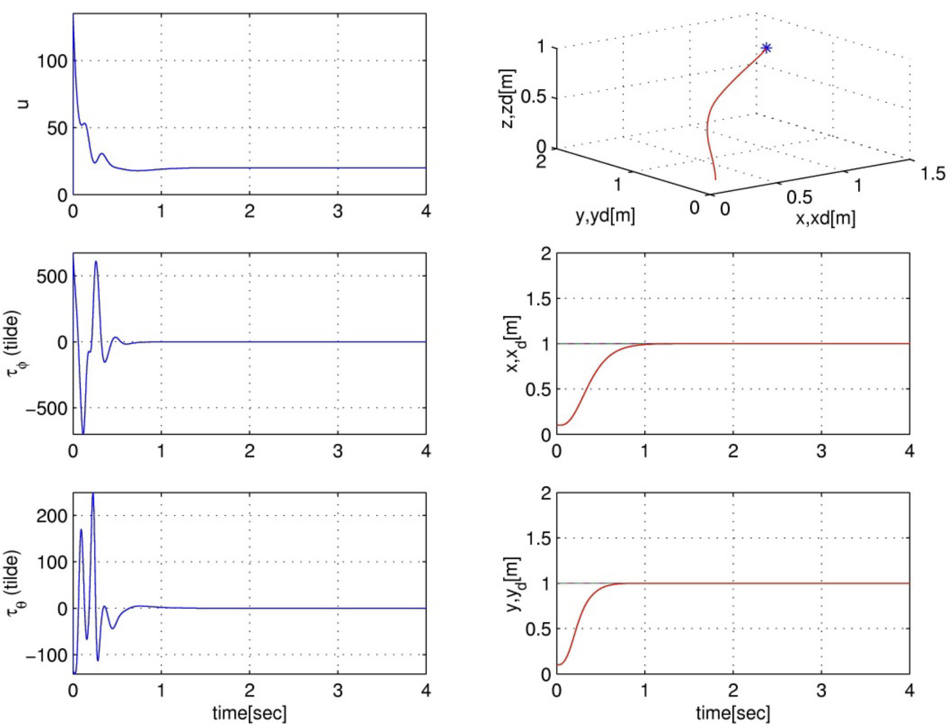

Fig. 10. Necessary inputs to stabilize (h_d = 1m) without motion planning.

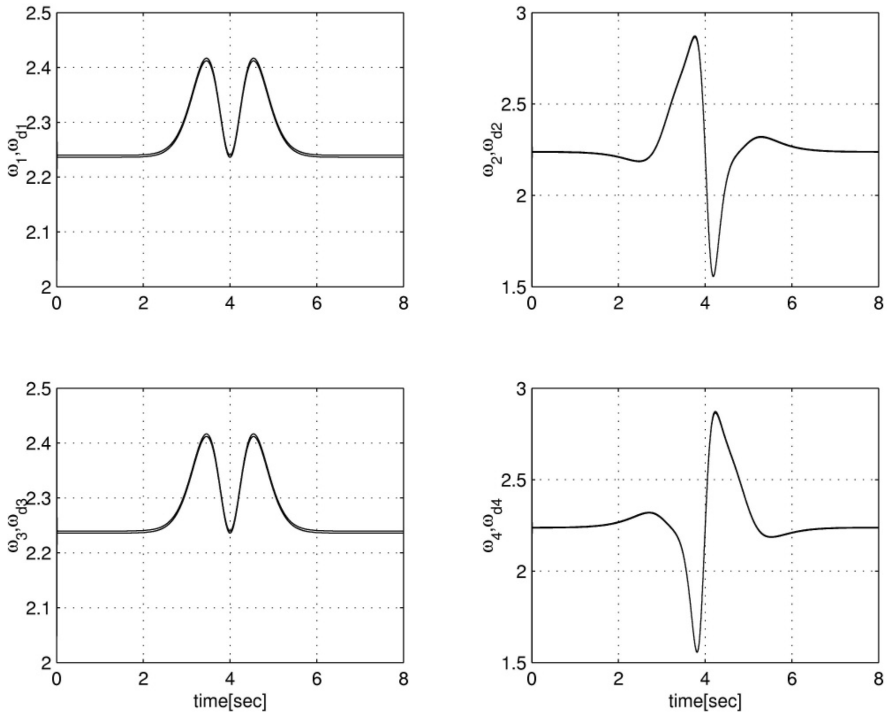

Fig. 11. Behaviour of rotors velocities compared to reference velocities (motion within the *x*-direction).

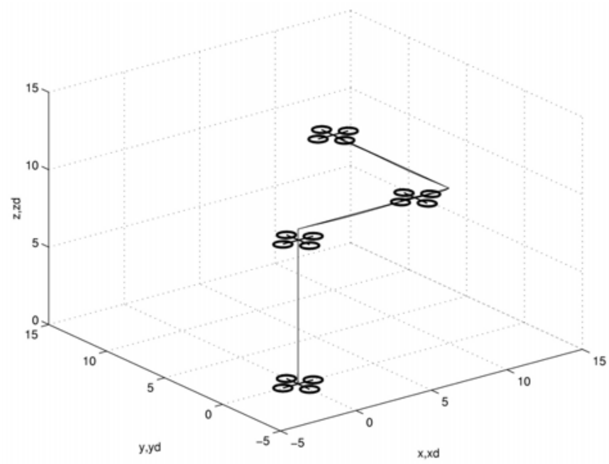

Fig. 12. Point to point steering control with motion planning and without perturbed model.

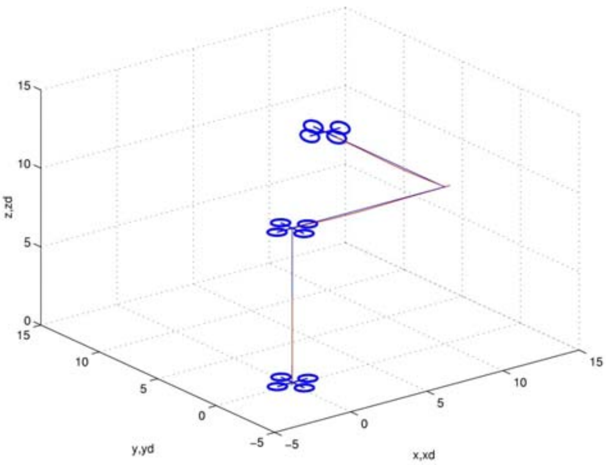

Fig. 13. Point to point steering control with motion planning and perturbed model.

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