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Tracking controllers in various navigation planes of an unmanned aerial blimp

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Abstract: One studies the tracking control problem in various navigation planes of an unmanned aerial vehicle of blimp type. The coupled dynamic equations of the blimp including the kinematics are distributed into the blimp's three geometric planes. The navigation control inputs take into account the effect of the added matrix extra-diagonal terms which are resulting from the non coincidence of the buoyancy and the gravity centres. With respect to the two geometrical planes, we prove that the dynamic of the blimp is flat and *flatness* based tracking controller is suggested. The *backstepping* techniques is followed in the horizontal plane to ensure the tracking of the blimp's trimmed flight. Simulations are presented to confirm the effectiveness of the tracking control schemes including drag forces.

Keywords: blimp; distributed flight dynamics; trimmed flight; nonlinear control

Biographical Notes:

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1 Introduction

The dynamic analysis and control of aerial vehicles is a challenging problem. Their capability is considerable in increasing the manoeuvrability for tasks such as transportation, surveillance and military applications, for airships, we quotes the escort of ships and also the detection of mines and under-sailors [14] [3] [6] [4] [13] [15]. However, heaviest that the air quickly beat lightest that the air. The ratio (distance, time of flight, altitude...) constitutes a state favorable for the planes that the airships. But there remain still applications for where airships always have an ecological and economic operational advantage. Nevertheless, the exploitation of the airships in the whole world is still very weak compared to the other air vehicles. They are even criticized to be creates only for promotional activities, such as for cruising or leisure flights. All the other awaited results depend strongly on the technological developments directed by the academic and industrial community which has as a common objective: the improvement of the airships in order to better exploit them in various fields.

In order ro characterize an airship and its needs in energy, various prototypes of airships were developed in several geometrical forms: such as the shapes of envelopes, cigar, hybrid and spherical wings. The airship is a means of transport which has the capacity to move very heavy loads with a minimum of mechanical default risk which is less critical for a plane. An airship is also more practical as the other flying vehicles because it can land practically anywhere.
Airships are member of family of under-actuated systems, because they have fewer inputs than degrees of freedom. The unactuated dynamics implies constraints on the accelerations. To maintain the balancing of the airship [9] [10], two principal propeller engines are used: the directional engines (propellers with tilt angle), assembled in a symmetrical way on two sides of the nacelle placed below the hull, which can control displacements in advance, back, the rise and the descent. There are also the ailerons which are placed in the back of the vehicle and have to control the yaw/pitch movement. In addition, for an aerostatic mode of fly (a slow flight which is sensitive to the external disturbances) and a quasi instantaneous rotation, a third propeller engine is placed on a vertical aileron acting as a rotor of tail (tail thruster).

In some studies such as [5] [6], motion is referenced to a system of orthogonal body axes located in the airship. The model used was written originally for a buoyant underwater vehicle [5]. It was modified later to take into account the specificity of the airship [6]. In this paper, we propose to control the model given in [2]. This dynamic model has the particularity that the origin of the airship fixed frame is located in the center of gravity, while in the cited works, it is located in the center of buoyancy. Recall that the center of buoyancy is the center of the airship volume.

There were some results in studying the control of the full nonlinear kinodynamic model of a blimp in term of the positioning control problem of both position and attitude to fixed constant values by Beji [7]. Complementarily, the tracking control of an helix, like an equilibrium trajectory for ascent and descent maneuvers, was studied by Beji [8]. Because of nonlinearities and highly coupled equations, the tracking of a general feasible trajectory, different from that of the equilibrium, led us to distribute the blimp kino-dynamic model into various planes. In some planes the controllability of the system is proved and the adequate tracking controller is proposed.

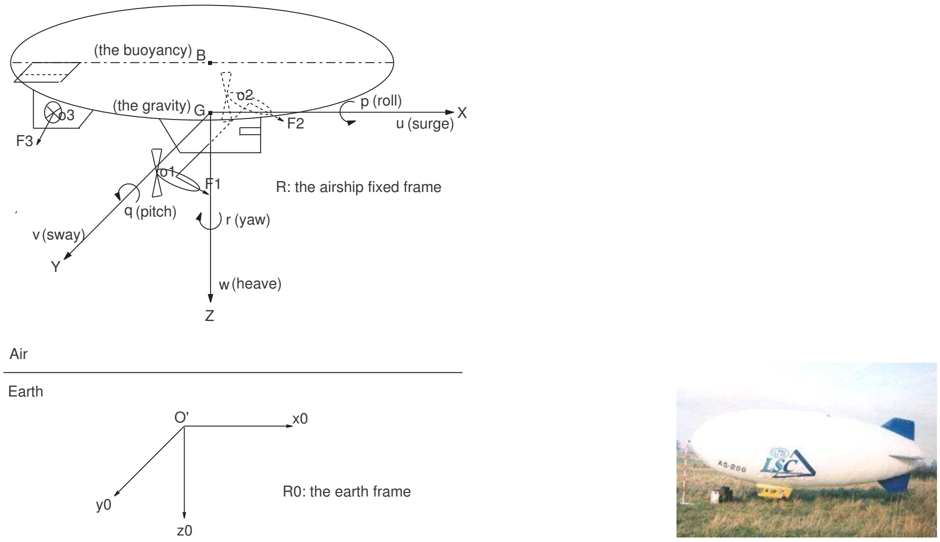

Figure 1 Airship fixed and earth frames

2 Flight decomposition of a blimp

In the aeronautic fields, it is common to decouple the flights of an aerial vehicle [1]. In order to reduce the highly coupled equations and nonlinearities, we chose to distribute the kino-dynamic model according to the three geometrical planes, called longitudinal, horizontal and lateral. Following this idea, it is possible to extract from the model of a blimp, three subsystems which correspond to the following analysis:

- In the longitudinal plane P_{XGZ} the dynamic of the blimp is described by the local velocity components u (longitudinal), w (lift) and q (pitch). The following state vectors are introduced $\eta_{long} = (x, \theta, z)^T$ and $\nu_{long} = (u, q, w)^T$. Hence, the dynamics associated to roll p, yaw r and lateral v are eliminated and considered like disturbances.
- The horizontal P_{XGY} dynamics is described by the longitudinal motion u, lateral v and yaw r. The considered states are $\eta_{hor} = (x, y, \psi)^T$ and $\nu_{hor} = (u, v, r)^T$. Consequently, the (p, w, q) motions are not considered in this plane.
- The motion in the lateral plane P_{YGZ} is subject to the lift w, lateral v and roll p components. The states $\eta_{lat} = (y, z, \phi)^T$ and $\nu_{lat} = (v, w, p)^T$ are taken into account to the control analysis in this plane. Hence, the dynamics associated to (u, q, r) are not considered.

The notation used for the airship are given in the following chart (see table 1).

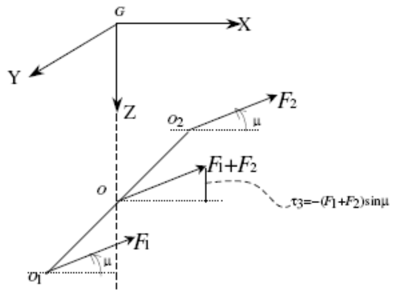

Figure 2 Presentation of inputs τ_1 and τ_3

where F_1 and F_2 input forces developed by the two electric actuators. Let $\tau_1 = (F_1 + F_2)cos\mu$ and $\tau_3 = -(F_1 + F_2)sin\mu$ the inputs which are necessary for both moving the blimp backward and forward also moving it up and down (figure 2). μ gives an oriented direction to actuators which is limited.

3 Trajectory tracking in the longitudinal plane $P_{(XGZ)}$

Following to the subsequent analysis, the dynamic of the blimp in the longitudinal plane $P_{(XGZ)}$ is reduced to the following model

$$\begin{aligned} \dot{x} &= u \\ \dot{\theta} &= q \\ \dot{z} &= w \\ m_x\dot{u} - X_{\dot{w}}\dot{w} &= X_u u + (F_B - F_G)\theta + \tau_1 \\ m_z\dot{w} - Z_{\dot{u}}\dot{u} &= Z_w w - (F_B - F_G) + \tau_3 \\ \dot{q} &= \frac{1}{J_y}(M_q q + F_B z_b \theta + GO_{1z}\tau_1) \end{aligned} \tag{1}$$

where we have supposed that the pitch attitude remains in a neighborhood of zero $\theta \in \vartheta(0)$ and a weak displacement in speeds of the blimp such that the

Table 1 Notations used for the blimp

	Linear and angular velocities	*Positions and Euler angles*
motion in the x-direction (surge)	u	x
motion in the y-direction (sway)	v	y
motion in the z-direction (heave)	w	z
rotation about the x-direction (roll)	p	ϕ
rotation about the y-direction (pitch)	q	θ
rotation about the z-direction (yaw)	r	ψ

.

quadratic terms in velocities (gyroscopics) are negligible. In system (1), the extra-diagonal terms due to the added mass matrix $X_{\dot{w}}$ and $Z_{\dot{u}}$ introduce coupling between the accelerations. Further, $m_x = m - X_{\dot{u}}$, $m_y = m - Y_{\dot{v}}$ and $m_z = m - Z_{\dot{w}}$ are the inertial terms including the inertia of air added masses. As soon as for $J_x = I_{GX}$, $J_y = I_{GY} - M_{\dot{q}}$ and $J_z = I_{GZ} - N_{\dot{r}}$ which represent the rotational part of inertias with I_{GX} is the principal element of inertia around the GX axis. $M_{\dot{q}}$ denotes the added mass element. m is the blimp's mass, F_B and F_G are the buoyancy and gravity forces, respectively. GO_{1z} denotes the O_1 position along the Z-axis. z_b is such that $GC = (0, 0, z_b)^T$, position vector of the buoyancy center w.r.t. G. All these standard notations can be found in [9] [5] [7].

Now, we introduce the following change of variables and consider $m_x m_z - X_{\dot{w}} Z_{\dot{u}} \neq 0$

$$\begin{aligned} U &= m_x u - X_{\dot{w}} w \\ W &= m_z w - Z_{\dot{u}} u \end{aligned} \tag{2}$$

Consequently, the system takes this form

$$\begin{aligned} \dot{x} &= \alpha U + \beta W \\ \dot{\theta} &= q \\ \dot{z} &= \alpha_1 U + \beta_1 W \\ \dot{U} &= X_u \dot{x} + (F_B - F_G)\theta + \tau_1 \\ \dot{W} &= Z_w \dot{z} + \bar{\tau}_3 \\ \dot{q} &= \frac{1}{J_y}(M_q q + F_B z_b \theta + GO_{1z}\tau_1) \end{aligned} \tag{3}$$

where the new controller to be proposed later is $\bar{\tau}_3 = \tau_3 - (F_B - F_G)$, $\alpha = \frac{m_z}{\Delta}$, $\beta = \frac{X_{\dot{w}}}{\Delta}$, $\alpha_1 = \frac{Z_{\dot{u}}}{\Delta}$, $\beta_1 = \frac{m_x}{\Delta}$ and $\Delta = m_x m_z - X_{\dot{w}} Z_{\dot{u}}$.

Lemma 1 *Under constant parameter values of the blimp, the linear dynamic model in $P_{(XGZ)}$ (3) is controllable and takes the Brunowsky's form, consequently the system is flat. Further the number of flat outputs is equal to the system inputs.*

Proof. First, we will test the controllability of the system which can be written in the form $\dot{\chi} = A\chi + B(\tau_1, \bar{\tau}_3)$ with $\chi = (x, \theta, z, U, W, q)^T$. The linear applications $A : IR^6 \to IR^6$ and $B : IR^6 \to IR^2$

$$A = \begin{pmatrix} 0 & 0 & 0 & \alpha & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \alpha_1 & \beta_1 & 0 \\ 0 & F_B - F_G & 0 & X_u\alpha & X_u\beta & 0 \\ 0 & 0 & 0 & Z_w\alpha_1 & Z_w\beta_1 & 0 \\ 0 & \frac{F_B z_b}{J_y} & 0 & 0 & 0 & \frac{M_q}{J_y} \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ \frac{GO_{1z}}{J_y} & 0 \end{pmatrix}$$

by writing the Commandability matrix $C = (B, AB, A^2B, \cdots, A^5B)$, one verifies that its rank is equal to six (number of states). Hence, Kalman's criterion is verified and the studied model is commandable using a continuous static state feedback law. Then, there exists the Brunovsky's form with two outputs (more details are in [12, 16]), said flat-output, which are equal in number to the systems's inputs (τ_1, τ_3) (see eq.(3)). For instance, recall the following theorem.

Theorem 1 *(Brunovsky's form)[12, 16]: For the system $\dot{\chi} = A\chi + Bu$, if the commandability matrix $C = (B, AB, \cdots, A^{n-1}B)$ is such that $rank(C) = dim(\chi) = n$ and if $rank(B) = dim(u) = m$ then there exists a transformation of the form $z = M\chi$ (M is an $n \times n$ invertible matrix) and a continuous static feedback law $u = Kz + Nv$ (N is an $m \times m$ invertible matrix), such that in term of (z, v), we have this form (m differential equations of order ≥ 1):*

$$y^{(\alpha_1)} = v_1, ..., y^{(\alpha_m)} = v_m$$

having like state

$$z = (y_1; y_1^{(1)}, ..., y_1^{(\alpha_1 - 1)}, ..., y_m; y_m^{(1)}, ..., y_m^{(\alpha_m - 1)})$$

The α_i being positive integers. The m quantities y_j are linear combinations of the state χ, are called Brunovsky's output.

Let us construct the Brunovsky's form and deduce the two flat-outputs of the system. The derivative of the first three equations from (3) with respect to time gives

$$\begin{aligned} \ddot{x} &= \alpha(X_u \dot{x} + (F_B - F_G)\theta + \tau_1) + \beta\tau_4 \\ \ddot{\theta} &= \frac{1}{J_y}(M_q\dot{\theta} + F_B z_b \theta + GO_{1z}\tau_1) \\ \ddot{z} &= \alpha_1(X_u \dot{x} + (F_B - F_G)\theta + \tau_1) + \beta_1\tau_4 \end{aligned} \tag{4}$$

where $\tau_4 = Z_w \dot{z} + \bar{\tau}_3$.

In order to make appear one input into each equation, we consider two regular transformations. The first one is given by

$$\begin{pmatrix} x_1 \\ \theta \\ z_1 \end{pmatrix} = \begin{pmatrix} 1 & -\alpha\frac{J_y}{GO_{1z}} & 0 \\ 0 & 1 & 0 \\ 0 & -\alpha_1\frac{J_y}{GO_{1z}} & 1 \end{pmatrix} \begin{pmatrix} x \\ \theta \\ z \end{pmatrix} \tag{5}$$

leading to

$$\begin{aligned} \ddot{x}_1 &= \alpha X_u \dot{x}_1 + D_1\theta + D_2\dot{\theta} + \beta\tau_4 \\ \ddot{\theta} &= \tau_\theta \\ \ddot{z}_1 &= \alpha_1 X_u \dot{x}_1 + D_3\theta + D_4\dot{\theta} + \beta_1\tau_4 \end{aligned} \tag{6}$$

with

$$\begin{aligned} D_1 &= \alpha(F_B - F_G) - \alpha\frac{F_B z_b}{GO_{1z}}; \quad D_2 = \frac{\alpha}{GO_{1z}}(\alpha X_u J_y - M_q) \\ D_3 &= \alpha_1(F_B - F_G) - \alpha_1\frac{F_B z_b}{GO_{1z}}; \quad D_4 = \frac{\alpha_1}{GO_{1z}}(\alpha X_u J_y - M_q) \\ \tau_\theta &= \frac{1}{J_y}(M_q\dot{\theta} + F_B z_b\theta + GO_{1z}\tau_1) \end{aligned}$$

The second regular transformation is incorporated in objective to have the input τ_4 in one equation. Hence,

$$\begin{pmatrix} x_1 \\ \theta \\ z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{\beta_1}{\beta} & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \theta \\ z_1 \end{pmatrix} \tag{7}$$

which permits to write this final basic system (7),

$$\begin{aligned} \ddot{x}_1 &= \tau_{x_1} \\ \ddot{\theta} &= \tau_\theta \\ \ddot{z}_2 &= \delta_1 \dot{x}_1 + \delta_2 \theta + \delta_3 \dot{\theta} \end{aligned} \tag{8}$$

with $\tau_{x_1} = \alpha X_u \dot{x}_1 + D_1 \theta + D_2 \dot{\theta} + \beta \tau_4$, $\delta_1 = \alpha_1 X_u - \frac{\beta_1}{\beta} \alpha X_u$, $\delta_2 = D_3 - \frac{\beta_1}{\beta} D_1$ and $\delta_3 = D_4 - \frac{\beta_1}{\beta} D_2$. τ_θ is given above. We construct the Brunovsky's outputs y_1, y_2 such that ($\delta_1, \delta_2, \delta_3$ are considered $\neq 0$).

$$\begin{aligned} y_1 &= -\delta_1 x_1 - \delta_3 \theta - \frac{\delta_2}{\delta_3} z_2 + \dot{z}_2 \\ y_2 &= 2\delta_1 x_1 + 2\delta_3 \theta + \frac{\delta_2}{\delta_3} z_2 - 2\dot{z}_2 \end{aligned} \tag{9}$$

Using relations (9) and the time derivative of (9) at order 3, in terms of the Brunovsky's outputs (flat-outputs), the system's (7) states and inputs take this form,

$$\begin{pmatrix} x_1 \\ \dot{x}_1 \\ \theta \\ \dot{\theta} \\ z_2 \\ \dot{z}_2 \\ \tau_{x_1} \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \frac{1}{\delta_1} & -\frac{\delta_3}{\delta_1\delta_2} & 0 & 0 & \frac{1}{\delta_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\delta_1} & -\frac{\delta_3}{\delta_1\delta_2} & 0 & 0 & \frac{1}{\delta_1} & 0 & 0 \\ 0 & -\frac{1}{\delta_2} & 0 & 0 & 0 & -\frac{1}{\delta_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{\delta_2} & 0 & 0 & 0 & -\frac{1}{\delta_2} & 0 \\ -2\frac{\delta_3}{\delta_2} & 0 & 0 & 0 & -\frac{\delta_3}{\delta_2} & 0 & 0 & 0 \\ 0 & -2\frac{\delta_3}{\delta_2} & 0 & 0 & 0 & -\frac{\delta_3}{\delta_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\delta_1} & -\frac{\delta_3}{\delta_1\delta_2} & 0 & 0 & \frac{1}{\delta_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{\delta_2} & 0 & 0 & 0 & -\frac{1}{\delta_2} \end{pmatrix} \begin{pmatrix} y_1 \\ \dot{y}_1 \\ \ddot{y}_1 \\ y_1^{(3)} \\ y_2 \\ \dot{y}_2 \\ \ddot{y}_2 \\ y_2^{(3)} \end{pmatrix} \tag{10}$$

In the following, one treats the tracking problem based on the Brunovsky's form. Having like inputs $y_1^{(3)}$ and $y_2^{(3)}$, system (10) is rearranged and transformed to

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ \dot{y}_1 \\ \ddot{y}_1 \\ y_2 \\ \dot{y}_2 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ \dot{y}_1 \\ \ddot{y}_1 \\ y_2 \\ \dot{y}_2 \\ \ddot{y}_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1^{(3)} \\ y_2^{(3)} \end{pmatrix} \tag{11}$$

Theorem 2 *For a given regular trajectories* (x_r, θ_r, z_r), *consequently for* (y_{1r}, y_{2r}) *and let* $e_{y_1} = y_1 - y_{1r}$, $e_{y_2} = y_2 - y_{2r}$, *the following choice for*

$$y_1^{(3)} = y_{1r}^{(3)} - k_1\ddot{e}_{y_1} - k_2\dot{e}_{y_1} - k_3 e_{y_1}$$
$$y_2^{(3)} = y_{2r}^{(3)} - \overline{k}_1\ddot{e}_{y_2} - \overline{k}_2\dot{e}_{y_2} - \overline{k}_3 e_{y_2} \tag{12}$$

consequently the following controllers τ_1 *and* τ_3

$$\begin{aligned}\tau_1 &= \frac{1}{GO_{1z}}(J_y(-\frac{1}{\delta_2}(y_{1r}^{(3)} - k_1\ddot{e}_{y_1} - k_2\dot{e}_{y_1} - k_3 e_{y_1}\\ &+ y_{2r}^{(3)} - \overline{k}_1\ddot{e}_{y_2} - \overline{k}_2\dot{e}_{y_2} - \overline{k}_3 e_{y_2})) - M_q\dot{\theta} - F_b z_b\theta)\\ \tau_3 &= \frac{1}{\beta}(\frac{1}{\delta_1}(\ddot{y}_1 + \ddot{y}_2 - \frac{\delta_3}{\delta_2}(y_{1r}^{(3)} - k_1\ddot{e}_{y_1} - k_2\dot{e}_{y_1} - k_3 e_{y_1}))\\ &- \alpha X_u\dot{x}_1 - D_1\theta - D_2\dot{\theta} - \beta Z_w\dot{z} + F_B - F_G)\end{aligned} \tag{13}$$

ensure the asymptotic tracking convergence of (x, θ, z) *to* (x_r, θ_r, z_r) *as time goto infinity. All the gain parameters are Hurwitz.*

Proof. One substitutes (12) into (11), we get

$$\frac{d}{dt}\begin{pmatrix} e_{y_1}\\ \dot{e}_{y_1}\\ \ddot{e}_{y_1}\\ e_{y_2}\\ \dot{e}_{y_2}\\ \ddot{e}_{y_2}\end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ -k_3 & -k_2 & -k_1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & -\overline{k}_3 & -\overline{k}_2 & -\overline{k}_1\end{pmatrix}\begin{pmatrix} e_{y_1}\\ \dot{e}_{y_1}\\ \ddot{e}_{y_1}\\ e_{y_2}\\ \dot{e}_{y_2}\\ \ddot{e}_{y_2}\end{pmatrix} \tag{14}$$

having like a polynomial characteristic $(\lambda^3+\lambda^2k_1+\lambda k_2+k_3)(\lambda^3+\lambda^2\overline{k}_1+\lambda\overline{k}_2+\overline{k}_3)$. This permits to apply the Hurwitz's criterion and deduce the $(k_i, \overline{k}_i)_{i=1,2,3}$ gains. Then $(e_{y_1}, e_{y_2}, e_{y_3})$ tends asymptotically to zero as time goto infinity. In order to verify the controller expressions given by (13), recall that

$$\begin{aligned}\tau_{x_1} &= \frac{1}{\delta_1}(\ddot{y}_1 + \ddot{y}_2 - \frac{\delta_3}{\delta_2}y_1^{(3)})\\ \tau_\theta &= -\frac{1}{\delta_2}(y_1^{(3)} + y_2^{(3)})\end{aligned} \tag{15}$$

let us substitute $y_1^{(3)}$ and $y_2^{(3)}$ given by (12) into (13), the backward computations permit easily to verify the adequate inputs given by (13). This ends the proof.

Simulation tests. With respect to $P_{(XGZ)}$, the reference trajectories are

$$\begin{aligned}x_r &= (C_1/k_0)e^{(k_0*t)} + L_1\\ z_r(t) &= (\frac{C_1}{X_{\dot{w}}k_0^2})(m_x k_0 - X_u)e^{k_0 t} + L_2\\ \theta_r &= 0\end{aligned} \tag{16}$$

with $C_1 = L_1 = L_2 = 1$, $k_0 = -0.6$. The blimp is initialized with $x_i = q_i = u_i = w_i = 0$, $\theta_i = 0.01$ and $z_i = 1$. The controller parameters are $\overline{k}_1 = 4.1$, $\overline{k}_2 = 4.4$, $\overline{k}_3 = 0.4$, $k_1 = 3.1$, $k_2 = 2.3$ and $k_3 = 0.2$. $X_{\dot{w}}$, m_x and X_u are from the blimp characteristics (see [8]).

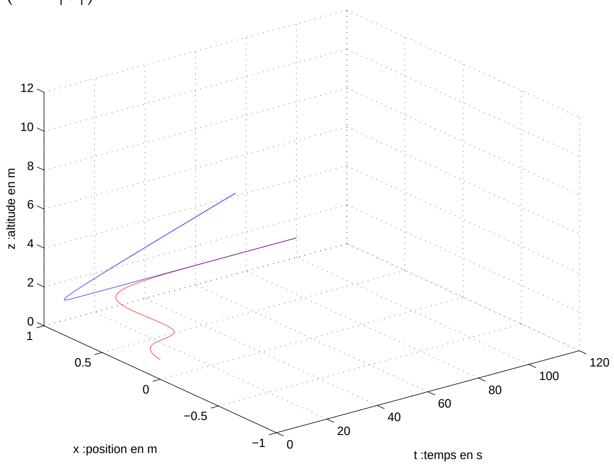

Figure 3 Blimp behaviour in $P_{(XGZ)}$.

4 Trajectory tracking in the lateral plane $P_{(YGZ)}$

Consider a motion with a low speed in this plane and $\phi \in \vartheta(0)$, consequently the model that describes the blimp's dynamic is given by

$$
\begin{aligned}
\dot{\phi} &= p; \quad \dot{y} = v; \quad \dot{z} = w \\
\dot{v} &= \frac{1}{m_y}(y_v v - (F_B - F_G)\phi + \tau_2) \\
\dot{w} &= \frac{1}{m_z}(Z_w w - (F_B - F_G) + \tau_3) \\
\dot{p} &= \frac{1}{J_x}(L_p p + F_B z_b \phi)
\end{aligned}
\tag{17}
$$

Let us consider $\tau_2^{'} = \frac{1}{m_y}(y_v v - (F_B - F_G)\phi + \tau_2)$ and $\tau_3^{'} = \frac{1}{m_z}(Z_w w - (F_B - F_G) + \tau_3)$.

The system (17) is controllable in the Kalman sense and has $y_1 = y$ and $y_2 = z$ as flat-outputs. Hence, the system takes the form

$$
\begin{aligned}
y_1 &= y; \quad \dot{y}_1 = \dot{y}; \quad \ddot{y}_1 = \tau_2^{'} \\
y_2 &= z; \quad \dot{y}_2 = \dot{z}; \quad \ddot{y}_2 = \tau_3^{'}
\end{aligned}
\tag{18}
$$

and the tracking controller inputs for (17), given by

$$
\begin{aligned}
\tau_2 &= m_y(\ddot{y}_{1r} - k_1(\dot{y}_1 - \dot{y}_{1r}) - k_2(y_1 - y_{1r})) - y_v v + (F_B - F_G)\phi \\
\tau_3 &= m_z(\ddot{y}_{2r} - k_1^{'}(\dot{y}_2 - \dot{y}_{2r}) - k_2^{'}(y_2 - y_{2r})) - Z_w w + (F_B - F_G)
\end{aligned}
\tag{19}
$$

ensure the convergence of (ϕ, y, z) to (ϕ_r, y_r, z_r) for the adequate choice of the controller's parameters.

Proof. With respect to time, the derivative of (y, z, ϕ) gives

$$\begin{aligned} \ddot{\phi} &= \frac{1}{J_x}[L_p\dot{\phi} + F_B z_b \phi] \\ \ddot{y} &= \tau_2^{'} \\ \ddot{z} &= \tau_3^{'} \end{aligned} \tag{20}$$

Then, the proof is straightforward.

Simulation tests. Figures 4-5 show the behaviour of the blimp into $P_{(YGZ)}$ with the real and the reference trajectories. The blimp is initialized at $x_i = z_i = 5m$, $y_i = 0m$, $\phi_i = 0.1$ and $v_i = w_i = p_i = 0$. The gain parameters are $k_1 = k_1^{'} = 10$ et $k_2 = k_2^{'} = 25$. The series of the blimp parameters used in simulation are detailed in [8]. In order to test the robustness of the proposed control laws, air drag forces are added to the model. In terms of amplitudes, in the local frame, these forces are taken equal to $(5v, 5w)$. The results are sketched in figure 5. The system is affected in the transient mode, but the tracking objective is achieved. This does not confirm the robustness of the control law and show the limit of stabilities in presence of perturbations. This was justified with the air drag forces superior to $(5v, 5w)$.

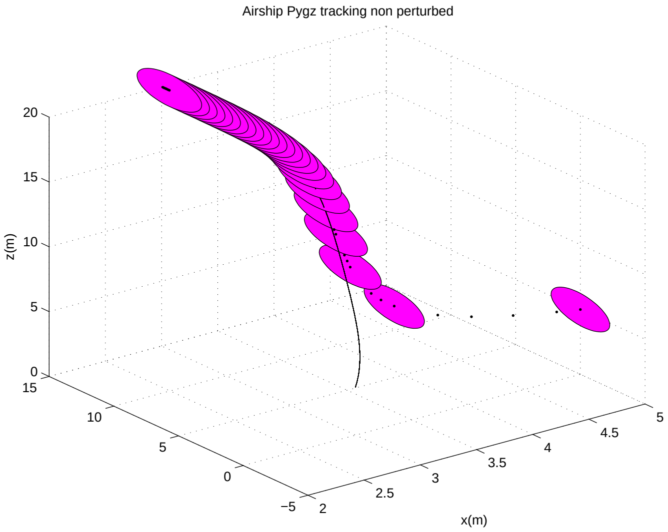

Figure 4 Blimp behaviour in $P_{(XGZ)}$ (non perturbed model).

5 Trajectory tracking in the horizontal plane $P_{(XGY)}$

In this navigation plane, the blimp is modeled by the following dynamic equations

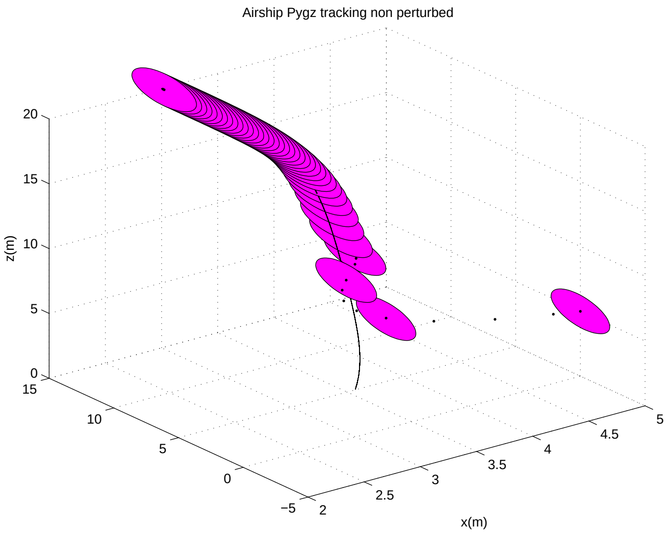

Figure 5 Blimp behaviour in $P_{(XGZ)}$ (perturbed model).

$$
\begin{aligned}
\dot{x} &= c_\psi u - s_\psi v \\
\dot{y} &= s_\psi u + c_\psi v \\
\dot{\psi} &= r \\
m_x \dot{u} &= X_u u + m_y v r + \tau_1 \\
m_y \dot{v} &= y_v v - m_x u r + \tau_2 \\
J_z \dot{r} &= (m_x - m_y) v u + N_r r - G o_{3x} \tau_2
\end{aligned} \tag{21}
$$

For a trajectory different from the trimmed flight, compared to the other two navigation planes, the blimp's dynamic tracking control in the horizontal plane is not straightforward. However, we can simplify it at the equilibrium, because some variables should not effect the equilibrium behaviour. To navigate in this plane, we can adopt a circle or a straight line like a trimmed flight. A helix trimmed flight was obtained and adopted for the ascent and descent manoeuvers (see Beji and Abichou [8]). Here, we solve the tracking problem related to a circle in the horizontal plane. Firstly, let us define the equilibrium or trimmed flight which can be formulated as following (r denotes the reference)

$$\dot{u}^r = \dot{v}^r = \dot{r}^r = 0 \tag{22}$$

This leads to this solution

$$
\begin{aligned}
x^r &= \frac{a_x}{\dot{\psi}_0^r} s_{\dot{\psi}_0^r t} + \frac{b_x}{\dot{\psi}_0^r} c_{\dot{\psi}_0^r t} \\
y^r &= -\frac{a_y}{\dot{\psi}_0^r} c_{\dot{\psi}_0^r t} + \frac{b_y}{\dot{\psi}_0^r} s_{\dot{\psi}_0^r t}
\end{aligned} \tag{23}
$$

with $a_x = u_0^r$, $b_x = v_0^r$, $a_y = a_x$ and $b_y = b_x$. The trajectory is a circle centered at $(0,0)$ with radius $r^r = \frac{\sqrt{u_0^2+v_0^2}}{\dot{\psi}_0^r}$.
To tackle the tracking problem, we introduce a new system of coordinates which is regular. Let

$$\begin{aligned} z_1 &= xc_\psi + ys_\psi \\ z_2 &= -xs_\psi + yc_\psi \\ z_3 &= \psi \end{aligned} \tag{24}$$

The model of blimp, as function of these coordinates, takes this form

$$\begin{aligned} \dot{z}_1 &= u + rz_2 \\ \dot{z}_2 &= v - rz_1 \\ \dot{z}_3 &= r \\ \dot{u} &= \frac{1}{m_x}(X_u u + m_y vr + \tau_1) \\ \dot{v} &= \frac{1}{m_y}(y_v v - m_x ur + \tau_2) \\ \dot{r} &= \frac{1}{J_z}((m_x - m_y)vu + N_r r - Go_{3x}\tau_2) \end{aligned} \tag{25}$$

Deducing the reference model from (26)

$$\begin{aligned} \dot{z}_1^r &= u^r + r^r z_2^r \\ \dot{z}_2^r &= v^r - r^r z_1^r \\ \dot{z}_3^r &= r^r \\ \dot{u}^r &= \dot{v}^r = \dot{r}^r = 0 \end{aligned} \tag{26}$$

Our interest is to construct the tracking controllers τ_1 and τ_2 such that the following system of errors converge asymptotically in neighborhood of 0_{IR^6}: $e_u = u - u^r$, $e_v = v - v^r$, $e_r = r - r^r$, $e_{z_1} = z_1 - z_1^r$, $e_{z_2} = z_2 - z_2^r$ and $e_{z_3} = z_3 - z_3^r$.

Proposition 1 *let us consider that the reference's movement is realizable with at low speed of flight and* $v^r = -\frac{N_r}{Go_{3x}y_v}r^r$. *After a preliminary feedback, the tracking problem analysis in* P_{XGY} *plane is reduced to*

$$\begin{aligned} \dot{e}_u &= \tau_1^{'} \\ \dot{e}_v &= \tau_2^{'} \\ \dot{e}_{r_1} &= D_1(t)e_u + D_2(t)e_v + D_3(t)e_{r_1} \\ \dot{e}_{z_1} &= e_u + z_2^r e_{r_1} + D_4(t)e_v + r^r e_{z_2} \\ \dot{e}_{z_2} &= D_5(t)e_v - z_1^r e_{r_1} - r^r e_{z_1} \\ \dot{e}_{z_3} &= e_{r_1} + D_6(t)e_v \end{aligned} \tag{27}$$

where

$$\tau_1^{'} = \frac{1}{m_x}(X_u u + m_y vr + \tau_1)$$
$$\tau_2^{'} = \frac{1}{m_y}(y_v e_v - m_x(r^r e_u + u^r e_r) + y_v v^r + \tau_2) \tag{28}$$

The $D_i(t)_{i=1,6}$ terms will be given thereafter.

Proof. The time derivative of each error is denoted by $\dot{e}_u = \dot{u} - \dot{u}^r$, etc. Therefore,

$$\begin{aligned}
\dot{e}_u &= \frac{1}{m_x}(X_u u + m_y vr + \tau_1)\\
\dot{e}_v &= \frac{1}{m_y}(y_v v - m_x ur + \tau_2)\\
\dot{e}_r &= \frac{1}{J_z}((m_x - m_y)vu + N_r r - Go_{3x}\tau_2)\\
\dot{e}_{z_1} &= e_u + z_2 e_r + r^r e_{z_2}\\
\dot{e}_{z_2} &= e_v - z_1 e_r - r^r e_{z_1}\\
\dot{e}_{z_3} &= e_r
\end{aligned} \tag{29}$$

One substitutes the transformation of type $v = e_v + v^r$ and the product $ur = (u^r + e_u)(r^r + e_r)$ into (30), we get

$$\begin{aligned}
\dot{e}_u &= \frac{1}{m_x}(X_u u + m_y vr + \tau_1)\\
\dot{e}_v &= \frac{1}{m_y}(y_v(e_v + v^r) - m_x(e_r e_u + u^r e_r + r^r e_u + u^r r^r) + \tau_2)\\
\dot{e}_r &= \frac{1}{J_z}((m_x - m_y)(e_v e_u + u^r e_v + v^r e_u + u^r v^r) + N_r(e_r + r^r) - Go_{3x}\tau_2)\\
\dot{e}_{z_1} &= e_u + e_{z_2} e_r + r^r e_{z_2} + z_2^r e_r\\
\dot{e}_{z_2} &= e_v - e_{z_1} e_r - r^r e_{z_1} - z_1^r e_r\\
\dot{e}_{z_3} &= e_r
\end{aligned} \tag{30}$$

Recall that, due to Hartman-Grobman theorem, the dynamic of a nonlinear system, topologically, is equivalent to its linearized tangent in the neighborhood of a hyperbolic equilibrium point. Further, the tracking is transformed to a stabilization problem. Consequently, the eliminating of the quadratic terms in the reference model and the introduction of $\tau_2 = \bar{\tau}_2 - y_v v^r$ with $v^r = -\frac{N_r}{Go_{3x} y_v} r^r$, leads to the following system

$$\begin{aligned}
\dot{e}_u &= \frac{1}{m_x}(X_u u + m_y v r + \tau_1) \\
\dot{e}_v &= \frac{1}{m_y}(y_v e_v - m_x(r^r e_u + u^r e_r) + \bar{\tau}_2) \\
\dot{e}_r &= \frac{1}{J_z}((m_x - m_y)(v^r e_u + u^r e_v) + N_r e_r - Go_{3x}\bar{\tau}_2) \\
\dot{e}_{z_1} &= e_u + z_2^r e_r + r^r e_{z_2} \\
\dot{e}_{z_2} &= e_v - z_1^r e_r - r^r e_{z_1} \\
\dot{e}_{z_3} &= e_r
\end{aligned} \tag{31}$$

By introducing this transformation $e_{r_1} = \frac{m_y Go_{3x}}{J_z} e_v + e_r$ and keeping the dynamic of v, the results in proposition 1 can be verified. The $D_i(t)_{i=1,6}$ terms are given by

$$\begin{aligned}
D_1 &= \frac{1}{J_z}((m_x - m_y)v^r - m_x GO_{3x} r^r) \\
D_2 &= \frac{1}{J_z}((m_x - m_y)u^r + y_v GO_{3x} - (N_r - m_x GO_{3x} u^r)\frac{m_y GO_{3x}}{J_z}) \\
D_3 &= \frac{1}{J_z}(N_r - m_x GO_{3x} u^r); \quad D_4 = -z_2^r \frac{m_y GO_{3x}}{J_z} \\
D_5 &= (1 + z_1^r \frac{m_y GO_{3x}}{J_z}); \quad D_6 = -\frac{m_y GO_{3x}}{J_z}
\end{aligned} \tag{32}$$

This ends the proof.

The next steps consist to apply the backstepping techniques to system (28). The following reduced system is obtained one takes $e_u \equiv \tau_1^{''}$ and $e_v \equiv \tau_2^{''}$ as virtual inputs in (28). Then

$$\begin{aligned}
\dot{e}_{r_1} &= D_1 \tau_1^{''} + D_2 \tau_2^{''} + D_3 e_{r_1} \\
\dot{e}_{z_1} &= \tau_1^{''} + z_2^r e_{r_1} + D_4 \tau_2^{''} + r^r e_{z_2} \\
\dot{e}_{z_2} &= D_5 \tau_2^{''} - z_1^r e_{r_1} - r^r e_{z_1} \\
\dot{e}_{z_3} &= e_{r_1} + D_6 \tau_2^{''}
\end{aligned} \tag{33}$$

First, one proposes this writing

$$\begin{pmatrix} e_{r_2} \\ e_{z_{11}} \\ e_{z_{21}} \\ e_{z_3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -\frac{D_2}{D_6} \\ 0 & 1 & 0 & -\frac{D_4}{D_6} \\ 0 & 0 & 1 & -\frac{D_5}{D_6} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{r_1} \\ e_{z_1} \\ e_{z_2} \\ e_{z_3} \end{pmatrix} \tag{34}$$

This permits to transform system (34) as

$$\begin{aligned}
\dot{e}_{r_2} &= D_1 \tau_1^{''} + L_1 e_{r_2} + L_2 e_{z_3} \\
\dot{e}_{z_{11}} &= \tau_1^{''} + r^r e_{z_{21}} + L_3 e_{r_2} + L_4 e_{z_3} \\
\dot{e}_{z_{21}} &= L_5 e_{r_2} - r^r e_{z_{11}} + L_6 e_{z_3} \\
\dot{e}_{z_3} &= e_{r_2} + L_7 e_{z_3} + D_6 \tau_2^{''}
\end{aligned} \tag{35}$$

with

$$L_1 = (D_3 - \frac{D_2}{D_6}); \quad L_2 = (D_3 - \frac{D_2}{D_6})\frac{D_2}{D_6}$$
$$L_3 = (z_2^r - \frac{D_4}{D_6}); \quad L_4 = ((z_2^r - \frac{D_4}{D_6})\frac{D_2}{D_6} + r^r\frac{D_5}{D_6})$$
$$L_5 = -(\frac{D_5}{D_6} + z_1^r); \quad L_6 = -(r^r\frac{D_4}{D_6} + (\frac{D_5}{D_6} + z_1^r)\frac{D_2}{D_6}); \quad L_7 = \frac{D_2}{D_6}$$

Moreover, through the following transformation

$$\begin{pmatrix} e_{r_3} \\ e_{z_{11}} \\ e_{z_{21}} \\ e_{z_3} \end{pmatrix} = \begin{pmatrix} 1 & -D_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{r_2} \\ e_{z_{11}} \\ e_{z_{21}} \\ e_{z_3} \end{pmatrix} \tag{36}$$

and while taking

$$\begin{aligned} \tau_3 &= \tau_1^{''} + r^r e_{z_{21}} + L_3 e_{r_2} + L_4 e_{z_3} \\ \tau_4 &= e_{r_2} + L_7 e_{z_3} + D_6 \tau_2^{''} \end{aligned} \tag{37}$$

one forces (36) to take this cascade form

$$\begin{aligned} \dot{e}_{r_3} &= -D_1 r^r e_{z_{21}} + S_1 e_{r_3} + S_2 e_{z_{11}} + S_3 e_{z_3} \\ \dot{e}_{z_{11}} &= \tau_3 \\ \dot{e}_{z_{21}} &= L_5 e_{r_3} + S_4 e_{z_{11}} + L_6 e_{z_3} \\ \dot{e}_{z_3} &= \tau_4 \end{aligned} \tag{38}$$

where in order to simplify the writing of this system, one considers

$$\begin{aligned} S_1 &= (L_1 - D_1 L_3); \quad S_2 = (L_1 - D_1 L_3) D_1 \\ S_3 &= (L_2 - D_1 L_4); \quad S_4 = (L_5 D_1 - r^r) \end{aligned} \tag{39}$$

Now, the new virtual controllers in (39) are $e_{z_{11}} \equiv \tau_3^{'}$ and $e_{z_3} \equiv \tau_4^{'}$. This leads to

$$\begin{aligned} \dot{e}_{r_3} &= -D_1 r^r e_{z_{21}} + S_1 e_{r_3} + S_2 \tau_3^{'} + S_3 \tau_4^{'} \\ \dot{e}_{z_{21}} &= L_5 e_{r_3} + S_4 \tau_3^{'} + L_6 \tau_4^{'} \end{aligned} \tag{40}$$

The system (40) can be decoupled under $S_2 L_6 - S_4 S_3 \neq 0$ and $L_6 \neq 0$ with

$$\begin{aligned} \tau_3^{'} &= \frac{L_6}{S_2 L_6 - S_4 S_3}(-k_5 e_{r_4} + D_1 r^r e_{z_{21}} - (S_1 - \frac{S_3 L_5}{L_6})(e_{r_4} + \frac{S_3}{L_6} e_{z_{21}})) \\ \tau_4^{'} &= \frac{1}{L_6}(-k_6 e_{z_{21}} - L_5(e_{r_4} - \frac{S_3}{L_6} e_{z_{21}}) - S_4 \tau_3^{'}) \end{aligned} \tag{41}$$

k_5 and k_6 are positive parameters. In closed loop,

$$\dot{e}_{r_4} = -k_5 e_{r_4}$$
$$\dot{e}_{z_{21}} = -k_6 e_{z_{21}} \tag{42}$$

Our stability result in the $P_{(XGY)}$ plane is regrouped in this main theorem.

Theorem 3 *Under (28), (41) and the following control feedback*

$$\begin{aligned} \tau_1^{'} &= -k_1(e_u - \tau_1^{''}) \\ \tau_2^{'} &= -k_2(e_v - \tau_2^{''}) \\ \tau_3 &= -k_3(e_{z_{11}} - \tau_3^{'}) \\ \tau_4 &= -k_4(e_{z_3} - \tau_4^{'}) \end{aligned} \tag{43}$$

with (k_1, k_2, k_3, k_4) *are strictly positive and large enough, the system of errors in (30) is locally exponentially stable. Further, we can deduce* τ_1 *and* τ_2 *in accordance with the above iterative results.*

Simulation tests. In the following figures, we show the behaviour of the blimp for a circle like a trimmed flight in $P_{(XGY)}$. The tracking controller parameters are $k_1 = k_2 = k_3 = 20$, $k_4 = 3$ and $k_5 = k_6 = 0.1$. The altitude in z is equal to $3meters$. The proposed control inputs ensure the stability of the engine when the equilibrium is reached. The blimp's parameters used in simulation are (International System Units)

$m_x = 10.2;$	$m_y = 16.32;$	$m_z = 16.32$
$X_u = -10;$	$y_v = -10$;	$Z_w = -10;$
$N_r = -10;$	$M_q = -10;$	$J_y = 27.73$
$GO_{3x} = 3;$	$GO_{1z} = 1;$	$J_z = 27.63$

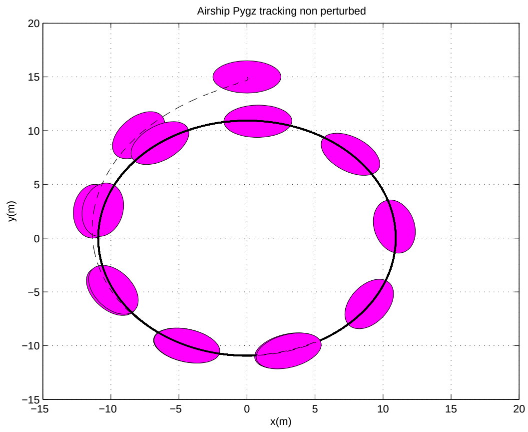

Figure 6 Tracking of a circle in the $P_{(XGY)}$ plane (without perturbation).

Figure 7 shows a perturbed circle tracking result under the air drag forces equal to $(0.05u, 0.05v)$. The proposed control law is robust for a low amplitude values of drag forces. A $(0.5u, 0.5v)$ introduces an important static tracking error and the robustness is blamed. More investigation in the conception of the control law is necessary with respect to variations of the blimp parameters.

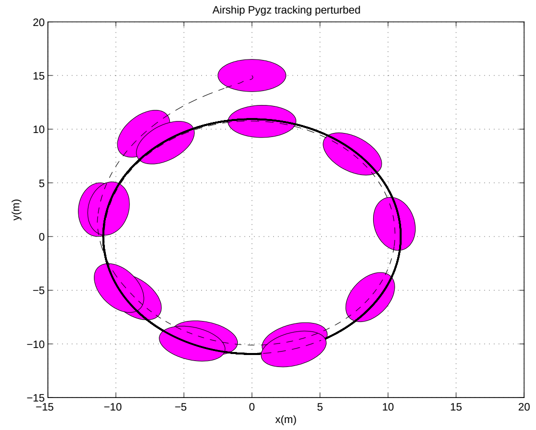

Figure 7 Tracking of a circle in the $P_{(XGY)}$ plane (with perturbation).

6 CONCLUSIONS

A various controllers were proposed to ensure the stability tracking of a blimp in a different geometric planes. The kino-dynamic of the blimp was distributed in each navigation plane and the controllability criterions were examined. Compared to some earlier works on blimps like the stabilization problem and the trimmed flight maneuvers, in this paper, we consider the effect of the generally neglected terms related to the added mass matrix. These terms introduce coupling in accelerations and affect here only the longitudinal dynamics. Hence, flatness-based control seems to be adequate to guide the blimp in this plane. Note that the proposed continuous tracking controller overcome the under-actuation which is present even if the dynamic was distributed. The problem that should be treated in the future is : how consider the maximum coupling in the model writing for an eventually flight manoeuver? For real tests, the blimp's dynamic model integrating parameter variation and navigation limits due to drag forces, are also in our future investigation.

References and Notes

1 Hygounenc, E. (2003) 'Modélisation et commande d'un dirigeable pour un vol autonome', *Phd from the University Paul Sabatier of Toulouse.*

2 Bestaoui, Y., Hamel, T. (2000) *Dynamic modeling of small autonomous blimps', Conference on Methods and models in automation and robotics, Miedzyzdroje, Poland, vol.2,pp.579-584.*

3 de Paiva, E. C., Bueno, S. S., Gomes, S. B. V. (1999) *A Control System Development Environment for AURORA's Semi-Autonomous Robotic Airship, Proc. IEEE Conference on Robotics Automation, Detroit. Michigan, pp. 2328-2335.*

4 Azinheira, J. R., de Paiva, E. C. (2001) *Lateral/directional control for an autonomous, unmanned airship, Aircraft Engineering and Aerospace Technology, Vol.73, No.5, pp. 453-459.*

5 Fossen, T.I. (1994) *Guidance and Control of Ocean Vehicles John Wiley Sons Ltd, Chichester.*

6 Khoury, G.A., Gillet, J.D. (1999) *eds. Airship technology, Cambridge university press.*

7 Beji, L., Abichou, A. and Bestaoui (2003), 'Position and attitude control of an underactuated autonomous airship', *Int. J. of Differential Equations*, Vol.8, 231-255.

8 Beji, L. and Abichou, A. (2005) 'Stabilization of Trim Trajectories of a Blimp in ascent and descent flight Conditions'. *International Journal of Control*, Vol.78, No.10/10, pp. 706-719.

9 Hygounenc, E., Soures, P., and Lacroix, S. (2000) *Modélisation d'un dirigeable: Etude de la cinématique et de la dynamique*, Technical report N 426, Lass-cnrs, France.

10 Hima, S. (2005) *Planification des Trajectoires d'un Dirigeable Autonome*, Phd from Université d'Evry Val d'Essonne, France.

11 Samaali, S., Abichou, A. and Beji, L. (2007) Longitudinal and horizontal stabilisation of an unmanned blimp, Int. J. Vehicle Autonomous Systems, Vol.5, No.1-2, pp. 138-157.

12 Laroche, B., Martin, P., Petit, N. (2006) *Commande par platitude, Equations différentielles ordinaires et aux drivèes partielles* Notes de cours, Cours B7-4 du Module Automatique Avancée. http://cas.ensmp.fr/ petit/ensta/index.html.

13 de Paiva, E. C. ; Moutinho, A., Azinheira, J. R. (2006) *Airship Hover Stabilization using a Backstepping Control Approach. Journal of Guidance,Control and Dynamics AIAA, New York, Vol.29, No.1, pp.903-914.*

14 Kulczycki, E., Joshi, S., Hess, R., Elfes, E. (2006) *Towards Controller Design for Autonomous Airships Using SLC and LQR Methods, AIAA Guidance, Navigation, and Control Conference, Keystone, CO.*

15 A. Nayler, A. (2000) Airship activity and development worldwide -2000. In 3rd International Airship Convention and Exhibition, Friedrichshafen (Germany).

16 Martin, P., and Rouchon, P. (1999) *Systèmes plats, planification et suivi de trajectoires*, Notes, Journée X-UPS. http://math.polytechnique.fr/xups/vol99.html.