POSITION AND ATTITUDE CONTROL OF AN UNDER-ACTUATED AUTONOMOUS AIRSHIP

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Abstract : A strategy to design a time-varying stabilizing controller of position and attitude of an under-actuated autonomous airship is proposed. The dynamic modelling of the airship involves six equations with only three inputs. This airship cannot be stabilized to a point using continuous pure-state feedback law. However, based on an averaging approach, the stabilization problem is solved with an explicit homogeneous time-varying control law. We prove that the origin of the system is locally exponentially stable.

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1 Introduction

The dynamic analysis and control of aerial vehicles is a challenging problem. Their capability is considerable in increasing the manoeuvrability for tasks such as transportation, surveillance and military applications [7] [9]. Airships are member of the family of under-actuated systems because they have fewer inputs than degrees of freedom.
In some studies such as [4] [9], motion is referenced to a system of orthogonal body axes fixed in the airship. The model used was written originally for a buoyant underwater vehicle [4]. It was modified later to take into account the specificity of the airship [9]. In this paper, we propose to control the model given in [8]. This dynamic model has the particularity that the origin of the airship fixed frame is the center of gravity, while in the cited works, it is located in the center of buoyancy. The center of buoyancy is the center of the airship volume.

Our objective is to solve the stabilizing control problem of both attitude and position for an under-actuated airship using only three available controls: the main and tail thrusters and the tilt angle of the propellers. The roll is totally unactuated. The same input controls both pitch and surge, while yaw and sway are related. The unactuated dynamics implies constraints on the accelerations. The dynamic positioning control problem consists of finding a feedback control law that asymptotically stabilizes both position and attitude to fixed constant values.

Homogeneous approximations and high gain feedback control have been applied to systems with drift. These applications can be found for instance in Morin and Samson [3] for the attitude stabilization only, or in Pettersen and Egeland [5] for the stabilization of both position and attitude but with four controls.

This paper is organized as follows. The dynamic model with kinematics resolution are addressed in section 2. The stabilization problem of the under-actuated airship are the subject of section 3. A periodic time-varying feedback law is developed. The feedback control law is derived using averaging theory and homogeneity properties. It is based on a quaternion representation of the orientation. We proved that it stabilizes asymptotically both the position and the attitude of the airship. Moreover, the convergence to the equilibrium point is proved to be exponential. The theoretical results are confirmed by simulation in section 4. In section 5 we present our concluding remarks.

Figure 1: The LSC'AS200 technology

2 Model of the Airship

The forces and moments are referred to a system of body-fixed axes, centered at the airship center of gravity. We assume that the earth fixed reference frame is inertial, the gravitational field is constant, the airship is supposed to be a rigid body, meaning that it is well inflated, the aero-elastic effects are ignored, the density of air is supposed to be uniform, and the influence of gust is considered as a continuous disturbance, ignoring its stochastic character.

2.1 Kinematics

Two reference frames are considered in the derivation of the kinematics and dynamics equations of motion. There are the Earth frame R_f and the airship fixed frame R_m. The origin C of R_m coincides with the center of gravity of the airship.
There are many ways to describe finite rotations. The usual minimal representation of orientation is given by a set of three Euler angles. However, to avoid the singularity inherent to this representation, we have chosen Euler parameters. They are unit quaternions and are represented by a normalized vector of four real numbers. Let e denote the Euler parameters which are expressed by the rotation axis n and the rotation angle δ about the axis as follows:

$$\eta_2 = \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos(\frac{\delta}{2}) \\ \sin(\frac{\delta}{2})n_x \\ \sin(\frac{\delta}{2})n_y \\ \sin(\frac{\delta}{2})n_z \end{pmatrix}, \quad 0 \leq \delta \leq 2\pi \tag{1}$$

Let us introduce $\eta = (\eta_1, \eta_2)^T$ where $\eta_1 = (x, y, z)^T$ is the position vector of the airship (expressed in the earth fixed frame), V as the linear velocity of the origin and Ω as the angular velocity (expressed in the airship fixed frame):

$$V = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

The kinematics of the airship can be expressed in the following way:

$$\begin{pmatrix} \dot{\eta_1} \\ \dot{\eta_2} \end{pmatrix} = \begin{pmatrix} R(\eta_2) & 0 \\ 0 & J(\eta_2) \end{pmatrix} \nu \tag{2}$$

where $\nu = (V^T, \Omega^T)^T$.
The orientation matrices R and J are as [5]:

$$R(\eta_2) = \begin{pmatrix} 1 - 2(e_2^2 + e_3^2) & 2(e_1e_2 - e_3e_0) & 2(e_1e_3 + e_2e_0) \\ 2(e_1e_2 + e_3e_0) & 1 - 2(e_1^2 + e_3^2) & 2(e_2e_3 - e_1e_0) \\ 2(e_1e_3 - e_2e_0) & 2(e_2e_3 + e_1e_0) & 1 - 2(e_1^2 + e_2^2) \end{pmatrix} \tag{3}$$

$$J(\eta_2) = \frac{1}{2} \begin{pmatrix} -e_1 & -e_2 & -e_3 \\ e_0 & -e_3 & e_2 \\ e_3 & e_0 & -e_1 \\ -e_2 & e_1 & e_0 \end{pmatrix} \tag{4}$$

We assume in the control section that $|\delta| < \pi$, i.e. $e_0 > 0$.

2.2 Dynamics of the airship

Here, the dynamics model is defined as the set of ordinary differential equations relying the situation of the vehicle in its position, velocity and acceleration to the control vector. The translational part is separated from the rotational part. As the blimp displays a very large volume, its virtual mass and inertia properties cannot be neglected. The dynamic model is expressed in the airship fixed frame, centered at the airship center of gravity, as:

$$\begin{aligned} \dot{X} &= -\Omega \times X + V \\ M\dot{V} &= -\Omega \times MV - D_V V + (mg - B) R^T e_z + F_1 + F_2 \\ \dot{R} &= R sk(\Omega) \\ I\dot{\Omega} &= -\Omega \times I\Omega - V \times MV - D_\Omega \Omega + \left(R^T e_z \times \overline{BG}\right) B - F_1 \times \overline{P_1G} - F_2 \times \overline{P_2G} \end{aligned} \tag{5}$$

where

$$X = R^T \eta_1 \tag{6}$$

X represents the position of the airship,in the airship fixed frame.
m: mass of the airship, the propellers and the actuators,
M: this (3×3) mass matrix includes both the airship's actual mass as well as the virtual mass elements associated with the dynamics of buoyant vehicles,
I: this (3×3) inertia matrix includes both the airship's actual inertia as well as the virtual inertia elements associated with the dynamics of buoyant vehicles, with respect to G,
D_V: a (3×3) aerodynamic forces diagonal matrix,
D_Ω: a (3×3) aerodynamic moments diagonal matrix,
F_1 and F_2: Vectors of the propulsion forces,
$e_z = (0 \ \ 0 \ \ 1)^T$: a unit vector,
$B = \rho \Gamma g$: represents the magnitude of the buoyancy force. Γ is the volume of the envelope, ρ the difference between the density of the ambient atmosphere ρ_{air} and the density of the helium ρ_{helium} in the envelope, g is the constant gravity acceleration.

$sk(\Omega)$: represents the skew-symmetric matrix representation of the vector Ω in space x, y and z.
$\bar{P_i G}$: represents the position of the ith propeller.

The terms $\Omega \times MV$ and $\Omega \times I\Omega$ show the centrifugal and Coriolis components.

Added mass should be understood as pressure - induced forces and moments due to a forced harmonic motion of the body which are proportional to the acceleration of the body. In order to allow the vehicle to pass through the air, the fluid must move aside and then close behind the vehicle. As a consequence, the fluid passage possesses kinetic that it would lack if the vehicle was not in motion. The mass of the dirigible is assumed to be concentrated in the center of gravity. Then if we consider the plane XZ as a plane of symmetry, the mass and inertia matrices can be written as :

$$M = \begin{pmatrix} m + X_x & 0 & X_z \\ 0 & m + Y_y & 0 \\ Z_x & 0 & m + Z_z \end{pmatrix} \tag{7}$$

where X_x, Y_y and Z_z are the virtual mass terms of X, Y and Z axes respectively.

$$I = \begin{pmatrix} I_x + K_x & 0 & -I_{xz} + K_z \\ 0 & I_y + M_y & 0 \\ -I_{xz} + N_x & 0 & I_z + N_z \end{pmatrix} \tag{8}$$

K_x, M_y and N_z are the virtual inertia terms of X, Y, Z about GX, GY and GZ axes respectively.

The mass and inertia matrices are positive definite. We will assume that the added mass coefficients are constant. They can be estimated from the inertia ratios and the airship weight and dimension parameters.

The aerodynamic force can be resolved into two component forces, one parallel and the other perpendicular to the direction of motion.

$$D_V = diag(-X_u, -Y_v, -Z_w) \tag{9}$$

$$D_\Omega = diag(-L_p, -M_q, -N_r) \tag{10}$$

The gravitational force vector is given by the difference between the airship weight and the buoyancy acting upwards on it:

$$(mg - B)R^T e_z = (mg - B) \begin{pmatrix} 2(e_1 e_3 - e_2 e_0) \\ 2(e_2 e_3 + e_1 e_0) \\ 1 - 2(e_1^2 + e_2^2) \end{pmatrix} \tag{11}$$

and the gravitational and buoyant moments are given by:

$$\begin{aligned} & B\left(R^T e_z \times \bar{BG}\right) = \\ & B \begin{pmatrix} 2z_B(e_2 e_3 + e_1 e_0) - y_B(1 - 2(e_1^2 + e_2^2)) \\ x_B(1 - 2(e_1^2 + e_2^2)) + 2z_B(e_1 e_3 - e_2 e_0) \\ 2y_B(e_1 e_3 - e_2 e_0) - 2x_B(e_2 e_3 + e_1 e_0) \end{pmatrix} \end{aligned} \tag{12}$$

where $\bar{BG} = (x_B,\ y_B,\ z_B)$ represents the position of the center of buoyancy with respect to the airship-fixed frame. The term buoyancy is used in hydrodynamics while the term static lift is used in aerodynamics. The center of buoyancy is the center of gravity of the displaced fluid. It is the point through which the static lift acts. The center of gravity is the point through which the weight of the object is acting. The relationship between the center of gravity and the center of buoyancy is an important parameter. For the airship to remain statically level (aerodynamics and thrust effects are ignored here), the center of gravity should be directly below the center of buoyancy. If the center of gravity sits below the center of buoyancy, then $\bar{BG} = (0\ 0\ z_B)^T$. Any horizontal offset will result in the airship adopting a pitch angle. The vertical separation between these two centers affects the handling characteristics of the airship.

Actuators provide the means for maneuvering the airship along its course. A airship is propelled by thrust. Propellers are designed to exert thrust to drive the airship forward.

An airship is an under-actuated system with two types of control: forces generated by thrusters and angular inputs controlling the direction of the thrusters (μ is the tilt angle of the main propeller) :

$$F_1 = \begin{pmatrix} T_m \sin\mu \\ 0 \\ T_m \cos\mu \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 \\ T_t \\ 0 \end{pmatrix} \tag{13}$$

where T_m and T_t represent respectively the main and tail thrusters.
Thus in building the non linear six degrees of freedom mathematical model, the additional following assumptions are made:

$$\bar{P_1G} = \begin{pmatrix} 0 \\ 0 \\ P_1^3 \end{pmatrix}, \quad \bar{P_2G} = \begin{pmatrix} -P_2^1 \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

3 Stabilization of the airship

With the previous assumptions, the dynamics and kinematics of a small airship can be written in the following compact form:

$$M_\nu \dot{\nu} + C_\nu(\nu)\nu + D_\nu(\nu)\nu + g_\nu(\eta_2) = B_\tau \tau \tag{15}$$

$$\dot{\eta} = J(\eta_2)\nu \tag{16}$$

where M_ν is the inertia matrix which is block-diagonal and constant matrix (symmetric and definite positive):

$$M_\nu = \begin{pmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{11} & 0 & I_{13} \\ 0 & 0 & 0 & 0 & I_{22} & 0 \\ 0 & 0 & 0 & I_{13} & 0 & I_{33} \end{pmatrix} \tag{17}$$

and the centrifugal and Coriolis matrix:

$$C_\nu(\nu) = \begin{pmatrix} 0 & -m_{22}r & m_{33}q & 0 & 0 & 0 \\ m_{11}r & 0 & -m_{33}p & 0 & 0 & 0 \\ -m_{11}q & m_{22}p & 0 & 0 & 0 & 0 \\ 0 & (Y_v - Z_z)\omega & 0 & I_{13}q & -I_{22}r & I_{33}q \\ 0 & 0 & (Z_z - X_u)u & -I_{13}p - I_{11}r & 0 & I_{33}p + I_{13}r \\ (X_u - Y_v)v & 0 & 0 & -I_{11}q & I_{22}p & -I_{13}q \end{pmatrix} \tag{18}$$

The constant definite positive damping matrix D_ν takes the following form:

$$D_\nu(\nu) = diag(D_V, D_\Omega) \tag{19}$$

The gravitational vector is

$$g_\nu(\eta_2) = \begin{pmatrix} 2(B-mg)(e_1e_3 - e_0e_2) \\ 2(B-mg)(e_2e_3 + e_0e_1) \\ (B-mg)(1-2(e_1^2+e_2^2)) \\ -2Bz_b(e_0e_1 + e_2e_3) \\ 2Bz_b(e_0e_2 - e_1e_3) \\ 0 \end{pmatrix} \tag{20}$$

and the constant matrix B_τ in (15) represents the directions in which the torques are applied :

$$B_\tau = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ P_1^3 & 0 & 0 \\ 0 & -P_2^1 & 0 \end{pmatrix} \tag{21}$$

For the following the control parameters are taken as :

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} T_m \sin(\mu) \\ T_t \\ T_m \cos(\mu) \end{pmatrix} \tag{22}$$

Remark 1 *Notice that the transformation* $(\tau_1, \tau_2, \tau_3) \mapsto (T_m, \mu, T_t)$ *is a diffeomorphism. Then we can maintain* (τ_1, τ_2, τ_3) *as a control vector for the airship.*

3.1 A stabilizing feedback law

We will show first, that it is not possible to stabilize the airship, using a feedback law that is continuous function of the state only. This follows from results by Brockett [6], Coron and Rosier [2]. The problem is then not solvable using linearization and linear control theory or classical nonlinear control theory like feedback linearization. Thus, we propose a continuous periodic time-varying feedback law that stabilizes the airship using only the three available inputs.

Proposition 1 *The system (15)-(16) cannot be stabilized by a time invariant smooth pure-state feedback law.*

Proof. Let us consider $\epsilon = (\epsilon_1, 0)^T$, from equation (16) we will have $\nu = 0$ since $J^T J$ is invertible. Therefore, equation (15) leads to:

$$B_\tau \tau - g(\eta_2) = M_\nu \epsilon_1 \tag{23}$$

Then if we take $\epsilon_1 = (0, \epsilon_0, 0, 0, 0, 0)^T$ with $\epsilon_0 \neq 0$, we will obtain the following system:

$$\begin{aligned}
\tau_1 - 2(B - mg)(e_1e_3 - e_0e_2) =& 0 \\
\tau_2 - 2(B - mg)(e_2e_3 + e_0e_1) =& m_{22}\epsilon_0 \\
\tau_3 - (B - mg)(1 - 2(e_1^2 + e_2^2)) =& 0 \\
2Bz_b(e_0e_1 + e_2e_3) =& 0 \\
P_1^3\tau_1 + 2Bz_b(e_1e_3 - e_0e_2) =& 0 \\
-P_2^1\tau_2 =& 0
\end{aligned}$$

We can deduce from the last equation that $\tau_2 = 0$. Further, the fourth equation implies $e_0e_1 + e_2e_3 = 0$. As a result: $m_{22}\epsilon_0 = 0$ which is impossible since $\epsilon_0 \neq 0$. Therefore, we cannot stabilize the airship by a continuous pure-state feedback (Brokett's necessary condition [6]). However Coron theorem proves that time periodic continuous feedback is sufficient to stabilize the system to a point. We develop in the sequel a continuous time-varying feedback law. The main result is given by the following proposition.

Proposition 2 *Consider the function*

$$\begin{aligned}
p_d =& - k^r r - k^{e_3} e_3 - k^{e_1} e_1 + \frac{k^v v + k^y y}{\sqrt{|v| + |y|}} \sin(\frac{t}{\varepsilon}) \\
w_d =& - k^z z + 2\sqrt{|v| + |y|} \sin(\frac{t}{\varepsilon}) \\
q_d =& - k^{e_2} e_2 - k^x x - k^u u
\end{aligned} \tag{24}$$

furthermore, consider the following time-varying feedback law:

$$\tau_1(\nu, \eta, t) = \frac{1}{P_1^3}\left((I_{22}k^q - M_q)q - I_{22}k^q q_d + 2Bz_b e_2\right) \tag{25}$$

$$\begin{aligned}
&\tau_2(\nu, \eta, t) = \\
&\frac{1}{P_2^1 I_{13}}\left(-(\Delta k^p + L_p I_{33})p + \Delta k^p p_d - 2Bz_b e_1 I_{33}\right) + \frac{N_r}{P_2^1} r
\end{aligned} \tag{26}$$

$$\tau_3(\nu, \eta, t) = (m_{33}k^w - Z_w)w - m_{33}k^w w_d + (B - mg) \tag{27}$$

with $\Delta = I_{13}^2 - I_{11}I_{33}$. Then, for a suitable choice of the positive parameters k^r, k^{e_3}, k^{e_1}, k^z, k^{e_2}, k^x, and k^u there exists ε_0 such that for any $\varepsilon \in (0, \varepsilon_0]$ and large enough k^q, k^p and k^w the feedback (25)-(27) stabilizes locally exponentially the system (15)-(16). ε is a parameter that we need to adjust.

Proof. Let consider the following dilation

$$\chi_\lambda^\alpha(\nu, \eta, t) = (\lambda u, \lambda^2 v, \lambda w, \lambda p, \lambda q, \lambda r, \lambda x, \lambda^2 y, \lambda z, \lambda e_1, \lambda e_2, \lambda e_3, t) \tag{28}$$

The initial system (15)-(16) can be rewritten as

$$\begin{pmatrix} \dot{\nu} \\ \dot{\eta} \end{pmatrix} = f(\nu, \eta, t) + g(\nu, \eta, t) \tag{29}$$

with

$$f(\nu, \eta, t) = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B - mg)e_2 + \tau_1) \\ \frac{1}{m_{22}}(Y_v v - 2(B - mg)e_1 + m_{33}pw + \tau_2) \\ \frac{1}{m_{33}}(Z_w w - (B - mg) + \tau_3) \\ \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2Bz_b e_1 I_{33} - P_2^1 I_{13} \tau_2) \\ \frac{1}{I_{22}}(M_q q - 2Bz_b e_2 + P_1^3 \tau_1) \\ \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2I_{13} Bz_b e_1 + P_2^1 I_{11} \tau_2) \\ u \\ v \\ w \\ -\frac{1}{2}(e_1 p + e_2 q + e_3 r) \\ \frac{1}{2}p \\ \frac{1}{2}q \\ \frac{1}{2}r \end{pmatrix} \tag{30}$$

and $g(\nu, \eta, t)$ is the remaining terms.

As the functions τ_1, τ_2 and τ_3 are homogeneous of degree one with respect to the dilation and continuous for (ν , η)$\neq$ 0, they are continuous at zero. Further, one easily verifies that $f(\nu, \eta, t)$ defines a periodic, continuous homogeneous of degree zero with respect to the dilation. Also, the function $g(\nu, \eta, t)$ is continuous and defines a sum of homogeneous vector field of degree strictly positive with respect to the dilation.
To prove the stability it is well known (see [1]) that it is sufficient to show that the origin of the unperturbed system:

$$\begin{pmatrix} \dot{\nu} \\ \dot{\eta} \end{pmatrix} = f(\nu, \eta, t) \tag{31}$$

is locally asymptotically stable.
To this purpose, let us consider the following reduced system obtained from (31), by tacking $q = q_d$, $p = p_d$, and $w = w_d$ as new control variables, and where we have removed the equation corresponding to e_0 as it is uniquely defined by e_1, e_2, and e_3 since the Euler parameters satisfy the equation: $e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$ and we have assumed that $e_0 > 0$.
We have obtained the following resulting system:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \\ \dot{r} \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2) + \frac{1}{P_1^3 m_{11}}(-M_q q_d + 2Bz_b e_2) \\ \frac{1}{m_{22}}\left(Y_v v - 2(B-mg)e_1 + m_{33}p_d w_d\right) \\ -\frac{1}{P_2^1 I_{13} m_{22}}\left(-L_p I_{33} p_d + N_r I_{13} r - 2Bz_b e_1 I_{33}\right) \\ \frac{1}{I_{13}}\left(L_p p_d + 2Bz_b e_1\right) \\ u \\ v \\ w_d \\ \frac{1}{2}p_d \\ \frac{1}{2}q_d \\ \frac{1}{2}r \end{pmatrix} \tag{32}$$

The controls q_d, p_d, and w_d are given by (24). One verifies by application of theorem 3.11. [1] that the origin of the closed loop system is asymptotically stable. Indeed, the vector field associated with right-hand side of the closed loop system is continuous periodic and homogeneous of degree zero with respect to the dilation. Due to the periodic time-variant control, the resulting system is a periodic time-varying system, which can be written in the form:

$$\begin{pmatrix} \dot{\nu} \\ \dot{\eta} \end{pmatrix} = h(\nu, \eta, t/\varepsilon) \tag{33}$$

We approximate this system by an averaged system which is autonomous. The averaged system is defined as $(\dot{\nu}\ \dot{\eta})^T = h_0(\nu, \eta)$ where $h_0(\nu, \eta) = \frac{1}{T_T}\int_0^{T_T} h(\nu, \eta, t/\varepsilon)dt$ (T_T is the period). Now, the corresponding averaged system is given by:

$$\begin{pmatrix} \dot{u} \\ \\ \dot{v} \\ \\ \dot{r} \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2) + \frac{1}{P_1^3 m_{11}}(-M_q(-k^{e_2}e_2 - k^x x - k^u u) \\ +2Bz_b e_2) \\ \frac{1}{m_{22}}\left(Y_v v - 2(B-mg)e_1 - m_{33}k^z z(-k^r r - k^{e_3}e_3 - k^{e_1}e_1)\right) \\ -\frac{1}{P_2^1 I_{13} m_{22}}\left(-L_p I_{33}(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) + N_r I_{13} r - 2Bz_b e_1 I_{33}\right) \\ \frac{1}{I_{13}}\left(L_p(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) + 2Bz_b e_1\right) \\ u \\ v \\ -k^z z \\ \frac{1}{2}(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) \\ \frac{1}{2}(-k^{e_2}e_2 - k^x x - k^u u) \\ \frac{1}{2}r \end{pmatrix} \tag{34}$$

The linearization of the system (34) about the origin is:

$$\begin{pmatrix} \dot{u} \\ \\ \dot{v} \\ \\ \dot{r} \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2) \\ +\frac{1}{P_1^3 m_{11}}(-M_q(-k^{e_2}e_2 - k^x x - k^u u) + 2Bz_b e_2) \\ \frac{1}{m_{22}}(Y_v v - 2(B-mg)e_1) - \frac{1}{P_2^1 I_{13} m_{22}}(-L_p I_{33}(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) \\ +N_r I_{13} r - 2Bz_b e_1 I_{33}) \\ \frac{1}{I_{13}}(L_p(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) + 2Bz_b e_1) \\ u \\ v \\ -k^z z \\ \frac{1}{2}(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) \\ \frac{1}{2}(-k^{e_2}e_2 - k^x x - k^u u) \\ \frac{1}{2}r \end{pmatrix} \tag{35}$$

The stability study of this system can be reduced to the following subsystems:

$$\begin{pmatrix} \dot{u} \\ \\ \dot{x} \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2) \\ +\frac{1}{P_1^3 m_{11}}(-M_q(-k^{e_2}e_2 - k^x x - k^u u) + 2Bz_b e_2) \\ u \\ \frac{1}{2}(-k^{e_2}e_2 - k^x x - k^u u) \end{pmatrix} \tag{36}$$

and

$$\begin{pmatrix} \dot{v} \\ \\ \dot{r} \\ \dot{y} \\ \dot{e}_1 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{m_{22}}(Y_v v - 2(B-mg)e_1) \\ -\frac{1}{P_2^1 I_{13} m_{22}}(-L_p I_{33}(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) + N_r I_{13} r - 2Bz_b e_1 I_{33}) \\ \frac{1}{I_{13}}(L_p(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) + 2Bz_b e_1) \\ v \\ \frac{1}{2}(-k^r r - k^{e_3}e_3 - k^{e_1}e_1) \\ \frac{1}{2}r \end{pmatrix} \tag{37}$$

Now, it is clear that for a suitable gain parameters, the origin of the subsystems (36), (37) is obviously asymptotically stable. Therefore, the origin of the system (35) is locally asymptotically stable. Consequently, the origin of the system (34) is asymptotically stable. The asymptotic stability of the origin of the system (32) follows by direct application of corollary 1. [3]. After noticing that the functions q_d, p_d,and w_d are homogeneous of degree one with respect to the dilation, and of class C^1 on $\{\Re^6 \times \Re^3 - (0,0)\} \times \Re$, this ends the proof.

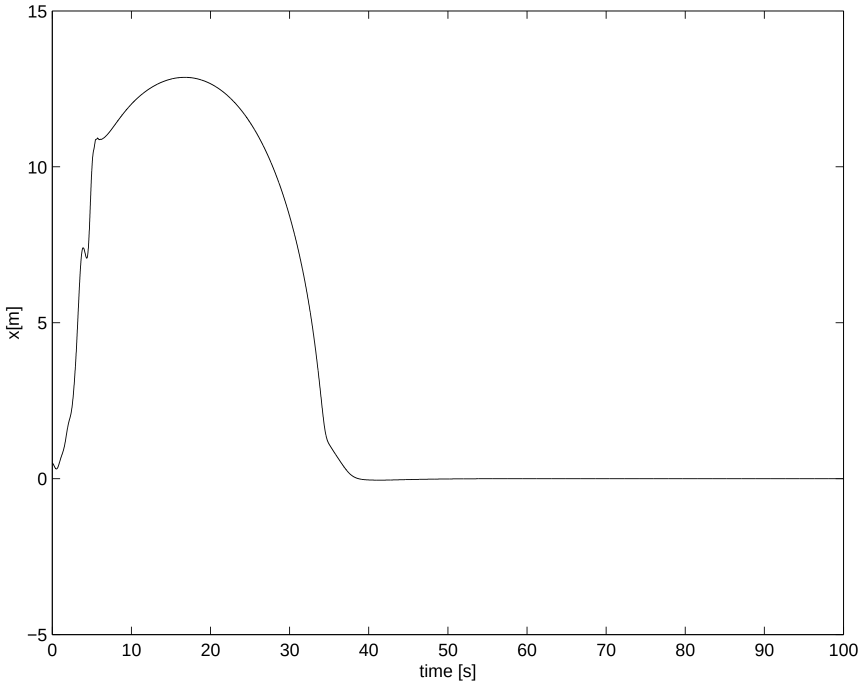

Figure 2: The x behavior (meter)

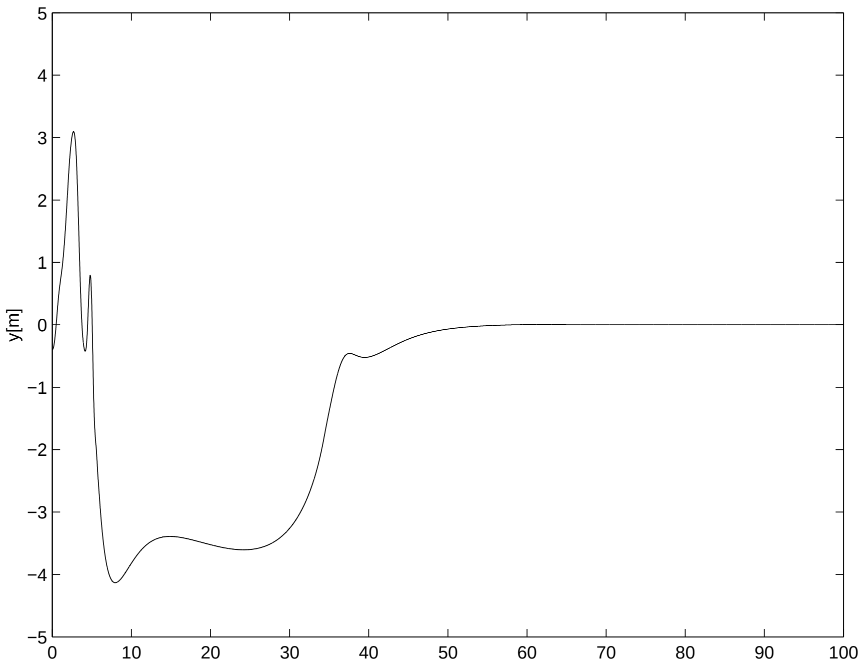

Figure 3: The y behavior (meter)

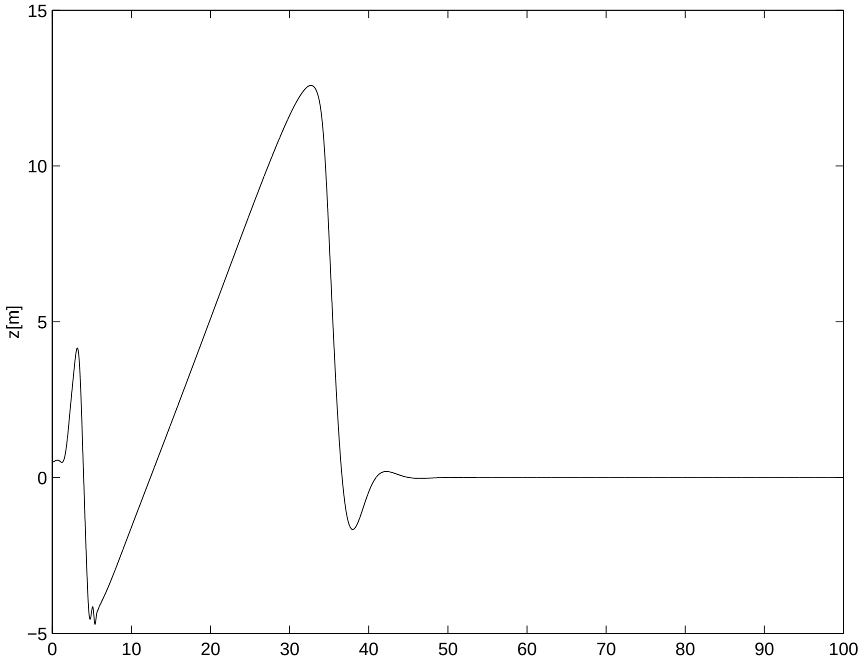

Figure 4: The z behavior (meter)

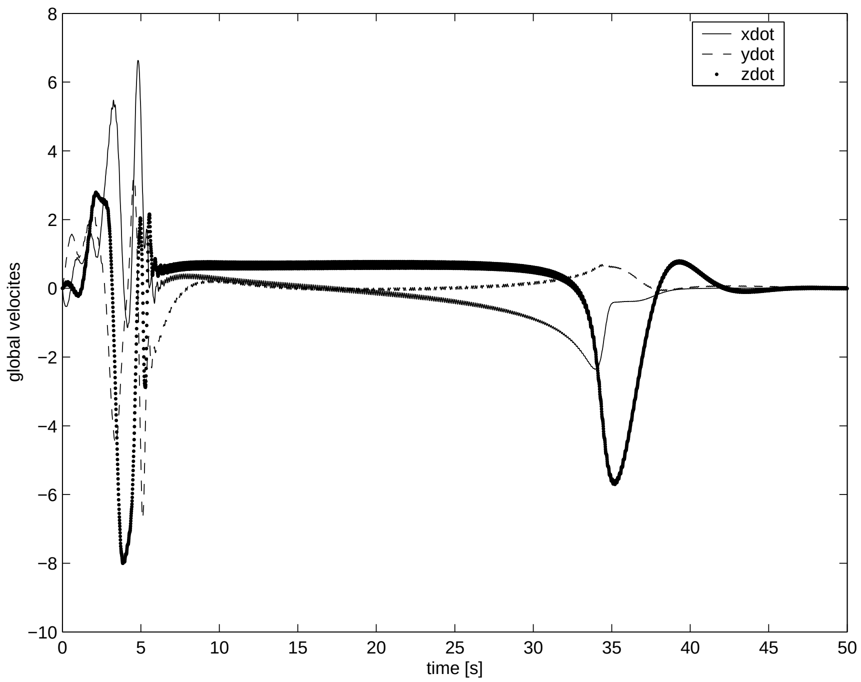

Figure 5: The global velocities

4 Simulation results and discussions

The lighter than air platform used for simulations is the AS200 by Airspeed Airships. It is a remotely piloted airship designed for remote sensing. It is a non rigid 6m long, 1.4m diameter and 7.6m3 volume airship. In this section, we present some simulation results. Guided by linear control theory applied to the linearization, we have chosen the following control parameters: $k_p = 0.35$, $k_q = 0.01$, $k_w = 0.2$, $k_v = 1.5$, $k_r = 2$, $k_{e_2} = 1$, $k_x = 1$, $k_u = 0.7$, $k_{e_1} = 0.1$, $k_{e_3} = 1.1$, $k_y = 1.48$, $k_z = 0.2$ and $\epsilon = 0.0002$. The initial position and orientation of the airship are taken as: $[x(0), y(0), z(0), e_1(0), e_2(0), e_3(0]^T$=$[4, 5, 0, 0.1, 0.1, 0.1]^T$ Initially, the airship was at rest. Figures 2, 3 and 4 show the trajectory along the x, y and z axis, respectively. The velocities with respect to the inertial frame (global) are given in figure 5.

The simulations show that the airship converges to the origin. The airship experience oscillations about stationary errors. We notice that the gravitational force and moment are important for stabilizability properties of the airship.

5 Conclusions

Airships offer a control challenge as they have non zero drift. Their linearization at zero velocity is not controllable. We proved that an airship represented by our model is not stabilizable by continuous state feedback. We have discussed the problem of stabilization of an airship and used the fact that the input vector fields are homogeneous of degree one with respect to some dilation. A feedback that is a homogeneous function of degree one makes the closed loop vector field homogeneous of order zero. In this paper, we have derived an explicit smooth time varying continuous feedback by using time-averaging technique. This feedback being uniformly stabilizing in time than each state may be bounded by a decaying exponential envelope. However, proper modelling of the other aerodynamic effects must be adopted. In this paper, we have studied only local properties. In our future work, we will use the fact that asymptotic stability for the averaged system implies semi-global practical asymptotic stability for the actual system.

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