



## The Kinematics and the Full Minimal Dynamic Model of a 6-DOF Parallel Robot Manipulator \*

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**Abstract.** In this paper we present a particular architecture of parallel robots which has six-degrees-of-freedom (6-DOF) with only three limbs. The particular properties of the geometric and kinematic models with respect to that of a classical parallel robot are presented. We show that inverse problems have an analytical solution. However, to solve the direct problems, an efficient numerical procedure which needs to inverse only a  $3 \times 3$  passive Jacobian matrix is proposed. In a second step, dynamic equations are derived using the Lagrangian formalism where the joint variables are passive and active joint coordinates. Based on the geometrical properties of the robot, the equations of motion are derived in terms of only nine coordinates related by three kinematic constraints instead of 18 joint coordinates. The computational cost of the dynamic model obtained is reduced by using a minimum set of base inertial parameters.

**Keywords:** Parallel robot, kinematic resolutions, Lagrangian formalism, minimal dynamic model.

### Nomenclature

The principal notations used to formulate the dynamic model are the following:

- $a_j$  = a unit vector carried by the joint axis:  ${}^j a_j = (0 \ 0 \ 1)^t$ ,
- $g$  = the gravity vector:  ${}^B g = (0 \ 0 \ g_z)^t$ , referring to the base,
- $M_j$  = the mass of body  $j$ ,
- $MS_j$  = a constant vector referring to frame  $j$ , giving the position of the center of mass of body  $j$ :  
 ${}^j MS_j \equiv MS_j = (MX_j \ MY_j \ MZ_j)^t$ ,
- $V_j$  = the linear velocity of body  $j$ ,
- ${}^j \omega_j$  = the angular velocity of body  $j$ , referring to frame  $j$ ,
- ${}^j J_j$  = a constant matrix giving the inertia matrix of body  $j$  referring to the frame  $j$ :  
 ${}^j J_j \equiv J_j = \begin{pmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{pmatrix}$ .

### 1. Introduction

In recent years, a large amount of research has been carried out on the kinematics and dynamic modeling of parallel robot manipulators [1–7]. With respect to serial manipulators, parallel robots are stiffer and lighter and able to perform accurate and fast displacements. However, due to the increasing number of loops in parallel robots, their reachable workspaces are limited [3].

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