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Two inertial models of X4-flyers dynamics, motion planning and control

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Abstract. Two models of mini-flying robots with four rotors, called X4-flyer, presenting and studying the stabilization/tracking with and without motion planning are proposed in this paper. So the first model is called bidirectional X4-flyer and the second one is called conventional X4-flyer. The impact of the planning of the trajectory for the control of the engines, consequently economy in energy, is shown. The stabilizing (tracking) feedback control used with and without motion planning is based on receding horizon point to point steering. The developed control algorithm of the X4-flyer is based on the Lyapunov method and is obtained using the backstepping techniques. This enabled to stabilize the engine in hovering and to generate its trajectory. All the forces developed by the two models are studied and simulated. Finally, results of simulations are given for the two models.

1. Introduction

Recently an increasing interest for mini-helicopter vehicles is observed. This is due to the growing number of civil and military applications of UAV (Unmanned Aerial Vehicles). They initially relate to the fields of safety (monitoring of the airspace, of the urban and interurban traffic), the natural risk management (monitoring of the activity of the volcanoes), the environmental protection (measurement of the air pollution, monitoring of the forests), the intervention in hostile sites (radioactive mediums, mine clearance of the grounds without human intervention), the management of the great infrastructures (stoppings, high-tension lines, pipelines), agriculture (detection and treatment of the cultures). All these missions require a powerful control of the apparatus and consequently of precise information on its absolute and/or relative state to its environment. The control of aerial robots requires the knowledge of a dynamic model. The effects of gravity and the aerodynamic loads are the principal causes. These systems, for which the number of control inputs is lower than the number of degrees of freedom, known as underactuated.

Modelling and controlling aerial vehicles (blimps, mini rotorcraft) are the principal preoccupation of the IBISC-group. Within this optic, who attracted the contest of the DGA-ONERA[1] was the X4 Stationnary Flyer (XSF) project which consists of a drone with revolving aerofoils [4], (Fig. 1). It is equipped with four rotors where two are directionals, what we call in the text bidirectional X4-flyer. In fact, the study of quadrotor vehicles is not recent. However, combination of revolving aerofoils and directional rotors were attractive for the contest. In this topic, a mini-UAV was constructed by the IBISC-group taking into account industrial constraints. The areal flying engine could not exceed 2 kg in mass, and 70cm of scale with approximatively 30 mn of flying-time. Compared to helicopters, named quad-rotor [1], the four-rotor rotorcraft has some advantages [12,20]: given that two motors rotate counterclockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend, in trimmed flight, to cancel. Vertical motion is controlled by collectively increasing or decreasing the power for all motors.

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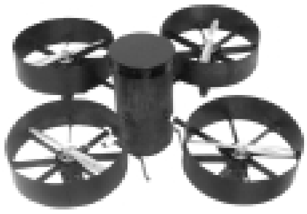

Fig. 1. Conceptual form of the four rotors rotorcraft.

The XSF is an engine of 68 cm $\times$ 68 cm of total size. It is designed in a cross form and made of carbon fiber. Each tip of the cross has a rotor including an electric brushless motor, a speed controller and a two-blade ducted propeller. In the middle one can find a central cylinder enclosing electronics namely Inertial Measurement Unit, on board processor, Global Positing System (GPS), radio transmitter, cameras and ultrasound sensors, as well as the LI-POLY batteries. The operating principle of the XSF can be presented thus: two rotors turn clockwise, and the two other rotors turn counterclockwise to maintain the total equilibrium in yaw motion. The equilibrium of angular velocities of all rotors done, the UAV is either in stationary position, or moving vertically (changing altitude). A characteristic of the XSF compared to the existing quadrotors, is the swiveling of the supports of the motors 1 and 3 around the pitching axis thanks to two small servomotors (Fig. 2). This per mits a more stabilized horizontal flight and a suitable cornering.

Several recent work was completed for the design and control in pilot-less aerial vehicles domain such that quadrotor [1–3,8,9,21], X4-flyer [4,12,13], blimp [23] and classical helicopter [11]. Also, related models for controlling the Vertical Take-Off and Landing (VTOL) aircraft are studied by Hauser et al. [14]. A model for the dynamic and configuration stabilization of quasi-stationary flight conditions of a four rotors Vertical Take-Off and Landing was studied by Hamel et al. [12] where the dynamic motor effects are incorporating and a bound of perturbing errors was obtained for the coupled system. The stabilization problem of a four rotors rotorcraft is also studied and tested by Castillo [10] where the nested saturation algorithm is used, the input/output linearization procedure [14], sliding-mode and LQR techniques [15,22] and application of the theory of flat systems by Beji et al. [6,7].

In this paper, the backstepping controllers and motion planning are combined to stabilize the helicopter by using the point to point steering stabilization. After having presented the study of modeling and the description of the configuration in the second section.

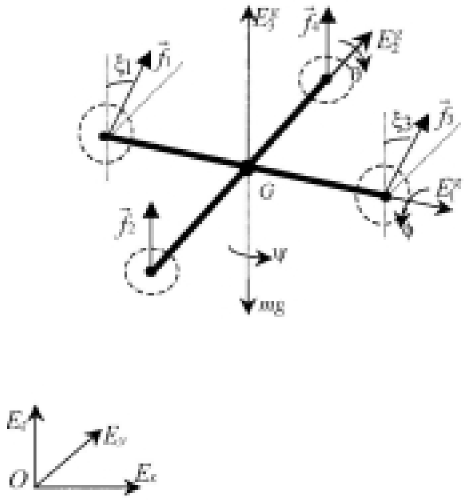

Fig. 2. 3D X4-flyer model.

The third section describes the dynamics of the system which treats the two models bidirectional and conventional X4-flyer. Backstepping controllers is described for two models of the X4-flyer in the fourth section. All the forces developed are studied in the fifth section. A strategy to solve the tracking problem through point to point steering is shown the sixth section. In the seventh section simulation results are introduced for two models. Finally, conclusion and future work are given in the last section.

2. Configuration description and modeling

Unlike regular helicopters that have variable pitch angles, an engine has fixed pitch angle rotors and the rotor speeds are controlled to produce the desired lift forces. Basic motions of the four rotors rotorcraft can described using the Fig. 2. Vertical motion is controlled by collectively increasing or decreasing the power for all motors. Lateral motion, in x direction or in y direction, is not achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations (case of the X4-flyer). But, two engines of direction are used to permute between the x and y motion. The conventional and the bidirectional XSF are a system consisting of four individual electrical fans attached to a rigid cross frame. We consider a local reference airframe $\Re_G = \{G, E_1^g, E_2^g, E_3^g\}$ at G (mass center) while the inertial frame is denoted by $\Re_O = \{O, E_x, E_y, E_z\}$ such that the vertical direction E_z axis is pointing up-

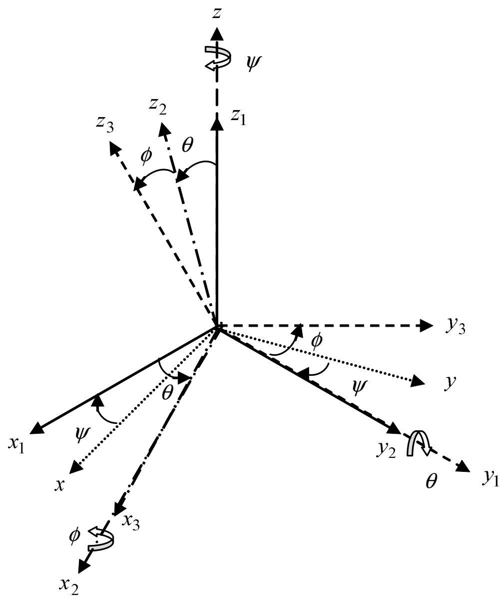

Fig. 3. Euler's angle definition.

wards. Let the vector $X = (x, y, z)$ denotes the G position with respect to $\Re_O$. The rotation of the rigid body is defined by $R_{\phi,\theta,\psi}$: $\Re_O \longrightarrow \Re_G$, where $R_{\phi,\theta,\psi} \in SO(3)$ is an orthogonal rotation matrix which is defined by the Euler angles, θ (pitch), ϕ (roll) and ψ (yaw), regrouped in $\eta = (\phi, \theta, \psi)$. A parametrization form is sketched in Fig. 3. An Euler angle representation given in Eq. (1) has been chosen [17].

$$R = \tag{1}$$
$$\begin{pmatrix} C_\psi C_\theta & C_\theta S_\psi & -S_\theta \\ S_\phi C_\psi S_\theta - S_\psi C_\phi & S_\theta S_\psi S_\phi + C_\psi C_\phi & C_\theta S_\phi \\ S_\theta C_\psi C_\phi + S_\psi S_\phi & C_\phi S_\theta S_\psi - C_\psi S_\phi & C_\theta C_\phi \end{pmatrix}$$

Where for example C_θ and S_θ represent $\cos\theta$ and $\sin\theta$, respectively.

The four rotors generate the forces and moments, that will be expressed as function of accelerations, linear and angular velocities. This will be established by Newton laws and the kinetics moment theorem (see Fig. 4), one denotes by:

- $\omega_{1,4}$: the angular velocity resulting from rotor 1 to 4,
- ξ_1 and ξ_3: the swiveling of the actuators supports 1 and 3 around the axis of pitching,
- F and τ: the force and the torque developed by the the four rotors,
- x, y, z, θ, ϕ and ψ: the six output of the system.

There are four/five input forces and six output states $(x, y, z, \theta, \phi, \psi)$ therefore the X4-flyer is an under-actuated system. The rotation direction of two of the rotors are clockwise while the other two are counter-clockwise, in order to balance the moments and produce yaw motions as needed.

In the present work, two X4-flyer models are presented, the first is called the bidirectional X4-flyer, the second one is the conventional X4-flyer. For the conventional X4-flyer, the rotors 2 and 4 are actuated in clockwise direction, the remain rotors, the rotors 1 and 3 are in the contrary actuated in the inverse direction in order to guarantee total balance in yaw (Fig. 5 (right)).

The main feature of the presented X4-flyer (called the XSF) in comparison with the existing quadrirotors, is the swiveling of the actuators supports 1 and 3 around the axis of pitching (angles ξ_1 and ξ_3). This swiveling ensures either the horizontal rectilinear motion or the rotational movement around the yaw axis or a combination of these two movements which gives the turn (see the Fig. 5 (left)), as well as the direction of rotation of the rotors implies that rotors 1 and 2 turn clockwise and rotors 3 and 4 turn counterclockwise.

3. Governing system of differential equations of the motion dynamics

We consider the translation motion of $\Re_G$ with respect to (wrt) $\Re_O$. The position of the center of mass wrt $\Re_O$ is defined by $\overline{OG} = (x\ y\ z)^T$, its time derivative gives the velocity wrt to $\Re_O$ such that $\frac{d\overline{OG}}{dt} = (\dot{x}\ \dot{y}\ \dot{z})^T$, while the second time derivative permits to get the acceleration $\frac{d^2\overline{OG}}{dt^2} = (\ddot{x}\ \ddot{y}\ \ddot{z})^T$. In the following, the bidirectional X4-flyer is described, after that the conventional X4-flyer is given.

3.1. Bidirectional X4-flyer

Currently, the model is a simplified one's. The constraints as external perturbation and the gyroscopic torques are neglected. The aim is to control the engine vertically (z) axis and horizontally according to x and y axis. The dynamics of the vehicle, represented on Fig. 2, is modelled by the system of Eqs (2) [4,5].

$$\begin{aligned} m\ddot{x} &= S_\psi C_\theta u_2 - S_\theta u_3 \\ m\ddot{y} &= (S_\theta S_\psi S_\phi + C_\psi C_\phi)\, u_2 + C_\theta S_\phi u_3 \\ m\ddot{z} &= (S_\theta S_\psi C_\phi - C_\psi S_\phi)\, u_2 \\ &\quad + C_\theta C_\phi u_3 - mg \end{aligned} \tag{2}$$

Where m is the total mass of the vehicle. The vector u_2 and u_3 combines the principal non conservative forces applied to the engine airframe including forces

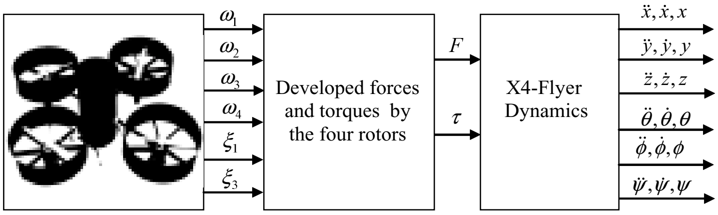

Fig. 4. Input-output parameters.

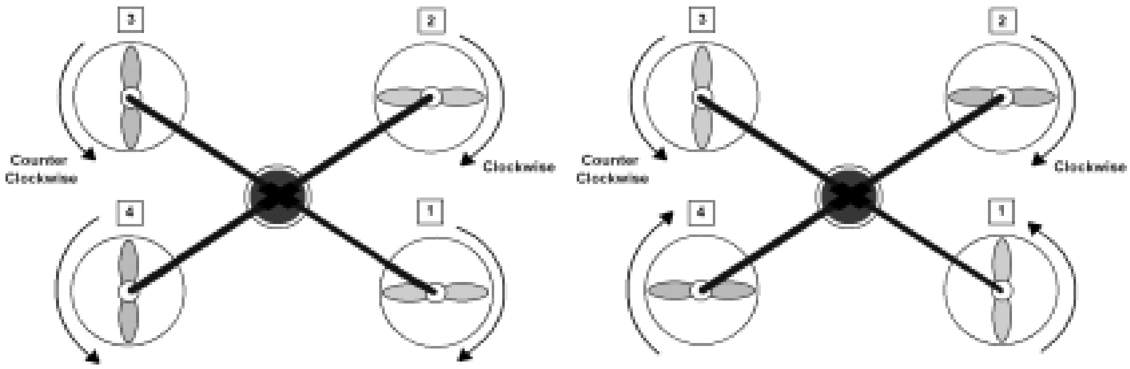

Fig. 5. Rotor rotations: The bidirectional (left) and conventional (right).

generated by the motors and drag terms. Drag forces and gyroscopic due to motors effects will be not considered in this work. The lift (collective) force u_3 and the direction input u_2 are such that

$$\begin{pmatrix} 0 \\ u_2 \\ u_3 \end{pmatrix} = f_1 \acute{e}_1 + f_2 e_2 + f_3 \acute{e}_3 + f_4 e_4 \tag{3}$$

with $f_i = K_T \omega_i^2$ where $K_T = 10^{-5} N.s^2$ and ω_i is the angular speed resulting of motor i. Let

$$\begin{array}{l} \acute{e}_1 = \begin{pmatrix} 0 \\ S_{\xi_1} \\ C_{\xi_1} \end{pmatrix}_{\Re_G} ; \acute{e}_3 = \begin{pmatrix} 0 \\ S_{\xi_3} \\ C_{\xi_3} \end{pmatrix}_{\Re_G} ; \\ e_2 = e_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{\Re_G} \end{array} \tag{4}$$

Then we deduce:

$$\begin{array}{l} u_2 = f_1 S_{\xi_1} + f_3 S_{\xi_3} \\ u_3 = f_1 C_{\xi_1} + f_3 C_{\xi_3} + f_2 + f_4 \end{array} \tag{5}$$

ξ_1 and ξ_3 are the two internal degree of freedom of rotors 1 and 3, respectively. These variables are controlled by dc-motors and bounded $-20^o \leqslant \xi_1, \xi_3 \leqslant +20^o$. e_2 and e_4 are the unit vectors along E_3^g which imply that rotors 2 and 3 are identical of that of a classical quadrotor (not directional).

3.2. Rotational motion of the bidirectional X4-flyer

The rotational motion of the X4 bidirectional flyer will be defined wrt to the local frame but expressed in the inertial frame.

Where the inertia elements I_{xx}, I_{yy} and I_{zz} are of the inertia matrix I_G expressed in G, then $I_G = diag\,(I_{xx}, I_{yy}, I_{zz})$.

$$\begin{array}{l} \ddot{\theta} = \frac{1}{I_{xx} C_\phi} (\tau_\theta + I_{xx} S_\phi \dot{\phi} \dot{\theta}) \\ \ddot{\phi} = \frac{1}{I_{yy} C_\theta C_\phi} \\ \quad (\tau_\phi + I_{yy} S_\phi C_\theta \dot{\phi}^2 + I_{yy} S_\theta C_\phi \dot{\theta} \dot{\phi}) \\ \ddot{\psi} = \frac{\tau_\psi}{I_{zz}} \end{array} \tag{6}$$

With the three inputs in torque

$$\begin{array}{l} \tau_\theta = l\,(f_2 - f_4) \\ \tau_\phi = l\,(f_1 C_{\xi_1} - f_3 C_{\xi_3}) \\ \tau_\psi = l\,(f_1 S_{\xi_1} - f_3 S_{\xi_3}) \\ \quad + \frac{K_M}{K_T}\,(f_1 C_{\xi_1} - f_3 C_{\xi_3} + f_4 - f_2) \end{array} \tag{7}$$

where l is the distance from G to the rotor i and $K_M = 9.10^{-6} N.m.s^2$. The equality from Eq. (6) is ensured, meaning that:

$$\ddot{\eta} = \Pi_G\,(\eta)^{-1}\,[\tau - \dot{\Pi}_G\,(\eta)\,\dot{\eta}] \tag{8}$$

With $\tau = (\tau_\theta, \tau_\phi, \tau_\psi)^T$ as an auxiliary inputs.

And

$$\Pi_G(\eta) = \begin{pmatrix} I_{xx}C_\phi & 0 & 0 \\ 0 & I_{yy}C_\phi C_\theta & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \quad (9)$$

As a first step, the model given above can be input/output linearized by the following decoupling feedback laws

$$\begin{aligned} \tau_\theta &= -I_{xx}S_\phi\dot{\phi}\dot{\theta} + I_{xx}C_\phi\tilde{\tau}_\theta \\ \tau_\phi &= -I_{yy}S_\phi C_\theta\dot{\phi}^2 - I_{yy}S_\theta C_\phi\dot{\theta}\dot{\phi} \\ &\quad + I_{yy}C_\theta C_\phi\tilde{\tau}_\phi \\ \tau_\psi &= I_{zz}\tilde{\tau}_\psi \end{aligned} \quad (10)$$

and the decoupled dynamic model of rotation can be written as

$$\ddot{\eta} = \tilde{\tau} \quad (11)$$

with $\tilde{\tau} = (\tilde{\tau}_\theta\tilde{\tau}_\phi\tilde{\tau}_\psi)^T$.

Using the system of Eqs (2) and (11), the dynamic of the system is defined by

$$\begin{aligned} m\ddot{x} &= S_\psi C_\theta u_2 - S_\theta u_3 \\ m\ddot{y} &= (S_\theta S_\psi S_\phi + C_\psi C_\phi)\, u_2 + C_\theta S_\phi u_3 \\ m\ddot{z} &= (S_\theta S_\psi C_\phi - C_\psi S_\phi)\, u_2 \\ &\quad + C_\theta C_\phi u_3 - mg \\ \ddot{\theta} &= \tilde{\tau}_\theta; \quad \ddot{\phi} = \tilde{\tau}_\phi; \quad \ddot{\psi} = \tilde{\tau}_\psi \end{aligned} \quad (12)$$

3.3. *Conventional X4-flyer*

We follows the same steps as the bidirectional X4-flyer and finally we finds for the dynamics of the conventional X4-flyer:

$$\begin{aligned} m\ddot{x} &= -S_\theta u_3 \\ m\ddot{y} &= C_\theta S_\phi u_3 \\ m\ddot{z} &= C_\theta C_\phi u_3 - mg \end{aligned} \quad (13)$$

3.4. *Rotational motion of the conventional X4-flyer*

The three inputs in torque are given by:

$$\begin{aligned} \tau_\theta &= l\,(f_2 - f_4) \\ \tau_\phi &= l\,(f_1 - f_3) \\ \tau_\psi &= lk\,(f_1 - f_2 + f_3 - f_4) \end{aligned} \quad (14)$$

The vertical controller is: $u_3 = f_1 + f_3 + f_2 + f_4$.

Using the translational and rotational motions (13) and (14), equations of the dynamic are detailed by

$$\begin{aligned} m\ddot{x} &= -S_\theta u_3 \\ m\ddot{y} &= C_\theta S_\phi u_3 \\ m\ddot{z} &= C_\theta C_\phi u_3 - mg \\ \ddot{\theta} &= \tilde{\tau}_\theta;\ \ddot{\phi} = \tilde{\tau}_\phi;\ \ddot{\psi} = \tilde{\tau}_\psi \end{aligned} \quad (15)$$

Remark: As shown in the system (2), the three inputs torque (see the Eq. (7)), the yaw τ_ψ is equal to zero if we take $\xi_1 = \xi_3 = 0$. Then, with the proposed sense of rotations (see Fig. 5 (left)), we can not generate yaw motions if rotors 1 and 3 are not oriented. With $\xi_1 = \xi_3 = 0$, to obtain yaw motions, the rotor sense of rotations is identical of that of the quadrotor.

Then rotors 1 and 3 are with the same sense of rotations, while rotors 2 and 4 are in opposite sense (see Fig. 5 (right)).

Conventional or bidirectional X4-flyer, the rotational part can be easily linearized with static feedback control laws. Then, we get

$$\begin{aligned} \ddot{\theta} &= u_4 \\ \ddot{\phi} &= u_5 \\ \ddot{\psi} &= u_6 \end{aligned} \quad (16)$$

with

$$\begin{aligned} u_4 &= \tfrac{1}{I_{xx}C_\phi}(\tau_\theta + I_{xx}S_\phi\dot{\phi}\dot{\theta}) \\ u_5 &= \tfrac{1}{I_{yy}C_\theta C_\phi}(\tau_\phi + I_{yy}S_\phi C_\theta\dot{\phi}^2 \\ &\quad + I_{yy}S_\theta C_\phi\dot{\theta}\dot{\phi}) \\ u_6 &= \tfrac{1}{I_{zz}}\tau_\psi \end{aligned} \quad (17)$$

4. Backstepping based controller

Backstepping controllers are especially useful when some states are controlled through other states. As it was observed in the previous section, in order to control the x and y motion of the X4-flyer, tilt angles need to be controlled. Therefore a backstepping controller has been developed in this section. Similar ideas of using backstepping with visual serving have been developed for a traditional helicopter by Hamel and Mahony [13]. As well as the backstepping controllers was applied for quadrotor by Altug et al. [1–3]. Figure 6, shows the simplified block diagram for the control of the X4-flyer. Where e: the difference between the set point and the process output, u: the control input, y_d: desired output, y: effective output value.

4.1. *"Backstepping" application to the conventional X4-flyer*

4.1.1. *Altitude and yaw control*

The altitude and the yaw on the other hand, can be controlled by a feedforward controller. The z movement equation is given by:

$$m\ddot{z} = C_\theta C_\phi u_3 - mg \quad (18)$$

The control of the vertical position (altitude) can be obtained considering the following control input

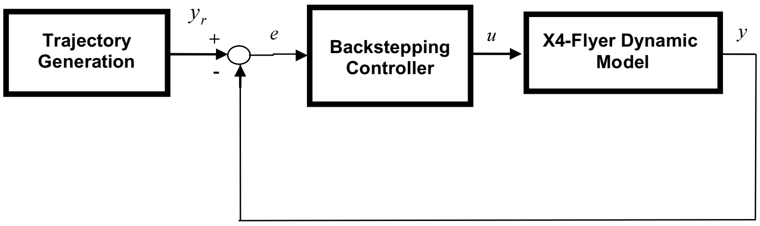

Fig. 6. Simplified block diagram for X4-flyer controller.

$$u_3 = m(g + \ddot{z}_r - k_z^1(\dot{z} - \dot{z}_r) - k_z^2(z - z_r)) \quad (19)$$

with

$$\ddot{z} = \ddot{z}_r - k_z^1(\dot{z} - \dot{z}_r) - k_z^2(z - z_r) \quad (20)$$

z_r is the desired altitude. The yaw attitude can be stabilized to a desired value with the following tracking feedback control

$$u_6 = \ddot{\psi}_r - k_\psi^1(\dot{\psi} - \dot{\psi}_r) - k_\psi^2(\psi - \psi_r) \quad (21)$$

where k_z^1, k_z^2, k_ψ^1, k_ψ^2 are the coefficients of stable polynomial. The simulated parameters are $k_z^1 = k_z^2 = 20$.

4.1.2. Roll control (ϕ, y)

First we notice that motion in the y direction can be controlled through the changes of the roll angle. These variables are related by the cascade system

$$\begin{cases} m\ddot{y} = C_\theta S_\phi u_3 \\ \ddot{\phi} = u_5 \end{cases} \quad (22)$$

This leads to a backstepping controller for y, ϕ which is given by

$$u_5 = \frac{1}{u_3 C_\theta C_\phi}(5y + 10\dot{y} + u_3\Theta_{\theta,\phi}) \quad (23)$$

where

$$\Theta_{\theta,\phi} = \Big(9S_\phi C_\theta + 4\dot{\phi}C_\phi C_\theta - \dot{\phi}^2 S_\phi C_\theta - 2\dot{\theta}S_\phi S_\theta + \dot{\phi}\dot{\theta}C_\phi S_\theta - \dot{\theta}\dot{\phi}C_\phi S_\theta + \dot{\theta}^2 S_\phi C_\theta\Big) \quad (24)$$

4.1.3. Pitch control (θ, x)

To develop a controller for motion along the x axis, similar analysis is needed. The equation of motion of the X4-flyer on x is given as

$$\begin{cases} m\ddot{x} = -S_\theta u_3 \\ \ddot{\theta} = u_4 \end{cases} \quad (25)$$

This leads to a backstepping controller for x, θ which is given by

$$u_4 = \frac{1}{u_3 C_\theta}(-5x - 10\dot{x} + u_3\Theta_\theta) \quad (26)$$

where

$$\Theta_\theta = 9S_\theta + 4\dot{\theta}C_\theta - \dot{\theta}^2 S_\theta \quad (27)$$

4.2. Bidirectional X4-flyer

4.2.1. Control input for (z, y) motions

We propose to control motion along y and z directions through u_3 and u_2, respectively. So we have the proposition (28).

$$\begin{pmatrix} \ddot{y} \\ \ddot{z} \end{pmatrix} = \frac{1}{m} H \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} - \begin{pmatrix} 0 \\ g \end{pmatrix} \quad (28)$$

where

$$H = \begin{pmatrix} S_\psi S_\theta S_\phi + C_\psi C_\phi & C_\theta S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi & C_\theta C_\phi \end{pmatrix} \quad (29)$$

For the given conditions in ψ and θ, the 2 by 2 matrix (29) is invertible. Then a nonlinear decoupling feedback permits to write the following decoupled linear dynamics

$$\begin{aligned} \ddot{y} &= \nu_y \\ \ddot{z} &= \nu_z \end{aligned} \quad (30)$$

Then we can deduce from Eq. (30) the linear controller

$$\begin{aligned} \nu_y &= \ddot{y}_r - k_y^1(\dot{y} - \dot{y}_r) - k_y^2(y - y_r) \\ \nu_z &= \ddot{z}_r - k_z^1(\dot{z} - \dot{z}_r) - k_z^2(z - z_r) \end{aligned} \quad (31)$$

With the k_y^1, k_y^2, k_z^1 and k_z^2 are the coefficients of a polynomial of Hurwitz. The simulated parameters are $k_y^1 = k_z^1 = 30$ and $k_y^2 = k_z^2 = 10$.

Proposition: Consider

$$(\psi, \theta) \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\quad (32)$$

with the controllers (33) and (34)

$$u_2 = \frac{C_\phi}{C_\psi} m\ddot{y} - \frac{S_\phi}{C_\psi} m(\ddot{z} + g) \quad (33)$$

$$u_3 = \frac{-S_\psi S_\theta C_\phi + C_\psi S_\phi}{C_\theta C_\psi} m\ddot{y} + \frac{S_\psi S_\theta S_\phi + C_\psi C_\phi}{C_\theta C_\psi} m(\ddot{z} + g) \quad (34)$$

The dynamic of y and z are linearly decoupled and exponentially-asymptotically stable with the appropriate choice of the gain controller parameters.

4.2.2. *Control input for the x motion*

To control the movement along the x axis, the backstepping controller is used. The noted controller x, θ is given by the Eq. (35):

$$\begin{cases} m\ddot{x} = S_\psi C_\theta u_2 - S_\theta u_3 \\ \ddot{\theta} = u_4 \end{cases} \quad (35)$$

One supposes it exists a time T_f^1 such that $\forall\ |t| \in \left[T_0, T_f^1\right]$, $u_3 > 0$, then the dynamic of x is decoupled under the following controller

$$u_4 = \frac{1}{u_3 C_\theta + u_2 S_\theta S_\psi} (-5x - 10\dot{x} + u_3\Theta_\theta + u_2\Theta_{\theta,\psi}) \quad (36)$$

where

$$\Theta_\theta = 9S_\theta + 4\dot{\theta}C_\theta - \dot{\theta}^2 S_\theta \quad (37)$$

and

$$\Theta_{\theta,\psi} = \Big(9S_\psi C_\theta + 4\dot{\theta}C_\psi C_\theta - \dot{\psi}^2 S_\psi C_\theta - 2\dot{\psi}S_\psi S_\theta + \dot{\psi}\dot{\theta}C_\psi S_\theta - \dot{\theta}\dot{\psi}C_\psi S_\theta + \dot{\theta}^2 S_\psi C_\theta\Big) \quad (38)$$

5. Developed input in forces

5.1. *Conventional X4-flyer*

In this paragraph, we incorporate relations between torques, motor velocities and command references positioning. Recall that the X4-flyer equipped with four brushless dc-motors which are commanded in voltages (currents) and not directly in torques. Brushless motors deliver high rate, largely boarded on miniflying machines. The variation of current permits to adjust speeds and forces.

Using the system of Eqs (15) and (16) permit to write

$$\begin{pmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} = Q \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{S_\phi\dot{\phi}\dot{\theta}}{C_\phi} \\ \frac{S_\phi\dot{\phi}^2}{C_\phi} + \frac{S_\theta\dot{\phi}\dot{\theta}}{C_\theta} \\ 0 \end{pmatrix} \quad (39)$$

where

$$Q = \quad (40)$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{l}{I_{xx}C_\phi} & 0 & -\frac{l}{I_{xx}C_\phi} \\ \frac{l}{I_{yy}C_\phi C_\theta} & 0 & -\frac{l}{I_{yy}C_\phi C_\theta} & 0 \\ \frac{l}{I_{zz}} & -\frac{l}{I_{zz}} & \frac{l}{I_{zz}} & -\frac{l}{I_{zz}} \end{pmatrix}$$

Let

$$I = \begin{pmatrix} 0 \\ \frac{S_\phi\dot{\phi}\dot{\theta}}{C_\phi} \\ \frac{S_\phi\dot{\phi}^2}{C_\phi} + \frac{S_\theta\dot{\phi}\dot{\theta}}{C_\theta} \\ 0 \end{pmatrix}; \ U = \begin{pmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} \quad (41)$$

As Q is a regular matrix, we calculate the input in forces by

$$F = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = Q^{-1}(U - I) \quad (42)$$

5.2. *Bidirectional X4-flyer*

Using the system of Eqs (5) and (7), the system follows the following equation:

$$\begin{aligned} u_2 &= f_1 S_{\xi_1} + f_3 S_{\xi_3} \\ u_3 &= f_1 C_{\xi_1} + f_3 C_{\xi_3} + f_2 + f_4 \\ u_4 &= l(f_2 - f_4) \\ u_5 &= l(f_1 C_{\xi_1} - f_3 C_{\xi_3}) \\ u_6 &= l(f_1 S_{\xi_1} - f_3 S_{\xi_3}) \\ &\quad + \tfrac{K_M}{K_T}(f_1 C_{\xi_1} - f_3 C_{\xi_3} + f_4 - f_2) \end{aligned} \quad (43)$$

Consider

$$B = \begin{pmatrix} S_{\xi_3} & 0 & S_{\xi_3} & 0 \\ C_{\xi_3} & 1 & C_{\xi_3} & 1 \\ 0 & l & 0 & -l \\ lC_{\xi_3} & 0 & -lC_{\xi_3} & 0 \\ k_1 & -k & k_2 & k \end{pmatrix};$$

$$U = \begin{pmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} \quad (44)$$

where

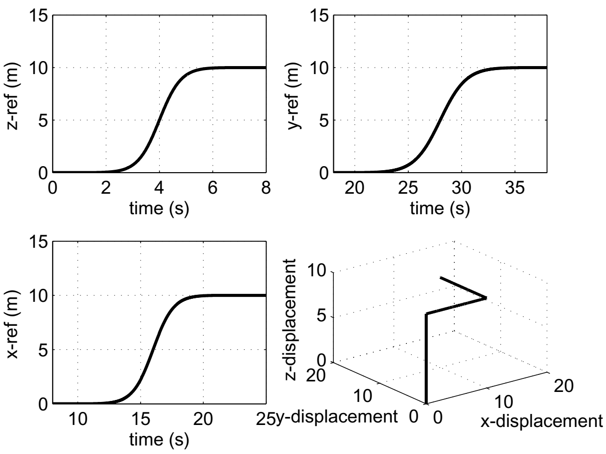

Fig. 7. Motion planning with $h_d = 10$ m.

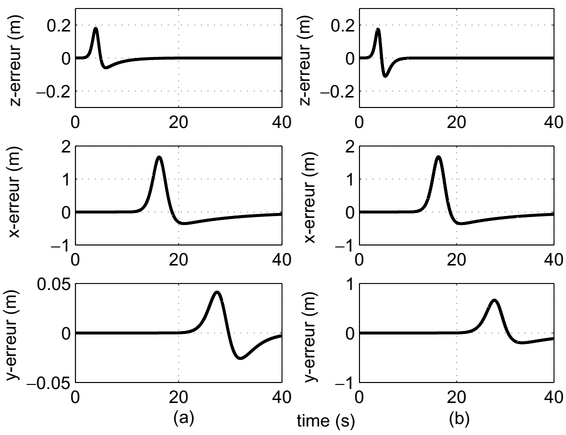

Fig. 8. Displacement errors: (a) bidirectional X4-flyer, (b) conventional X4-flyer.

$$k = \frac{K_M}{K_T} \tag{45}$$

$$k_1 = lS_{\xi_3} + kC_{\xi_3} \tag{46}$$

$$k_2 = -lS_{\xi_3} - kC_{\xi_3} \tag{47}$$

For $-20^o \leqslant \xi_1, \xi_3 \leqslant +20^o$, the matrix $(B^T B)$ is regular

$$F = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \left(B^T B\right)^{-1} B^T U \tag{48}$$

6. Trajectory generation and point to point steering

Due to the structure limit of the X4-flyer, motion can be asserted only in straight line along the x, y and z directions. In our case, that is sufficient to navigate in a region. Otherwise, an other version of the engine is under study by the group [7]. The version flyer is to make easy manoeuvres in corners with arc of circle. In the following, we solve the tracking problem as point to point steering one over a finite interval of time. Then we take each ending point with its final time as a new starting point. Figure 7 illustrates the reference trajectory along the x, y and z directions.

As we see, the X4-flyer fly in the z direction followed by the x motion and the y motion. The reference trajectory is parameterized as:

$$z^r(t) = h_d \frac{t^5}{t^5 + (T_f^1 - t)^5} \tag{49}$$

where h_d is the desired altitude and (T_f^1) the final time. In order to solve the point to point steering control, the end point of the trajectory (49) can be adopted as initial point to move along x, then we have

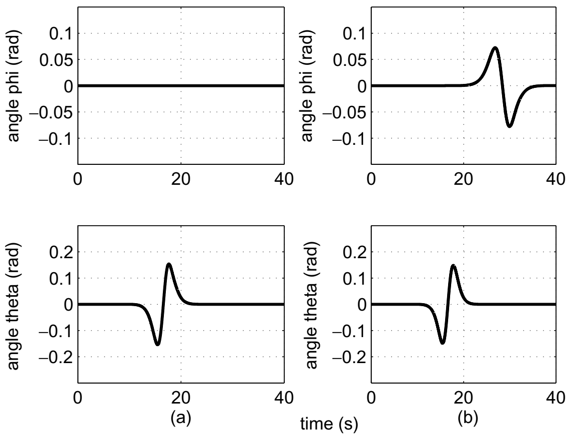

Fig. 9. The pitch θ and the roll ϕ: (a) bidirectional X4-flyer, (b) conventional X4-flyer.

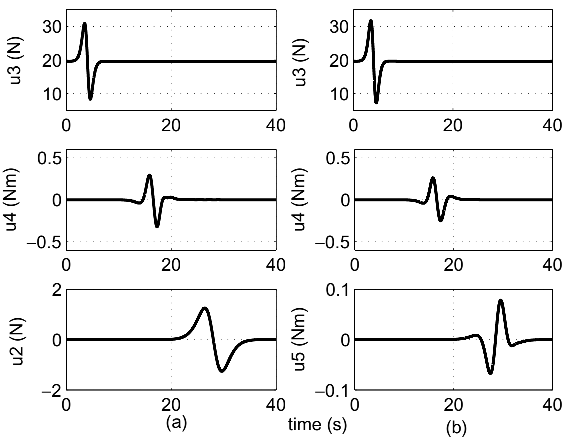

Fig. 10. Inputs u_2, u_3, u_4 and u_5 for the xyz displacement: (a) bidirectional X4-flyer, (b) conventional X4-flyer.

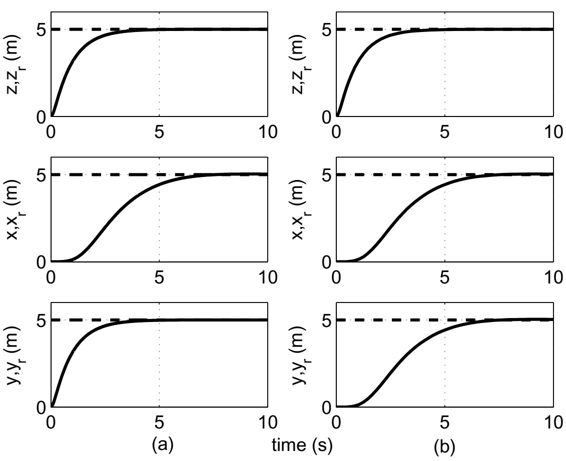

Fig. 11. Without motion planning with $h_d = 5$ m: (a) bidirectional X4-flyer, (b) conventional X4-flyer.

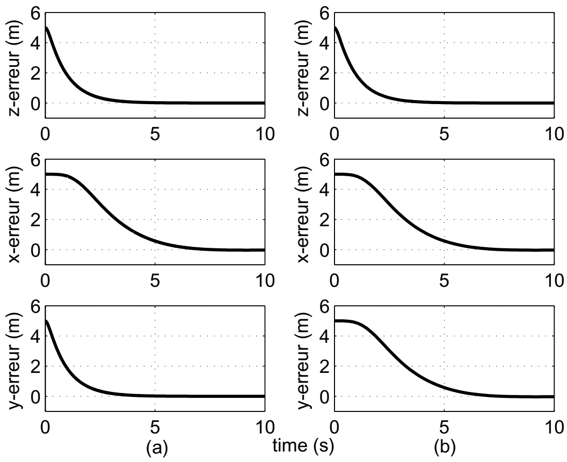

Fig. 12. Tracking errors without motion planning ($z_r = x_r = y_r = 5$ m): (a) bidirectional X4-flyer, (b) conventional X4-flyer.

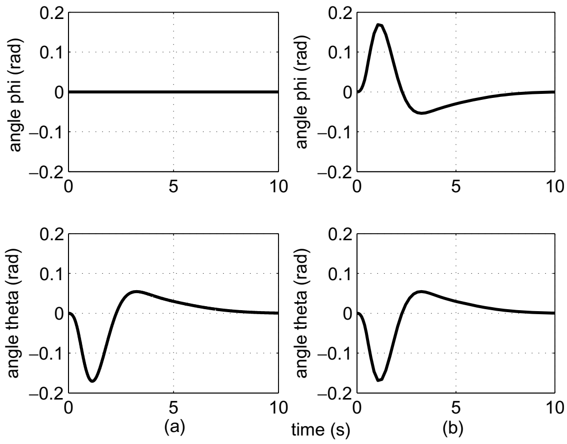

Fig. 13. The pitch θ and the roll ϕ for the vehicle without motion planning: (a) bidirectional X4-flyer, (b) conventional X4-flyer.

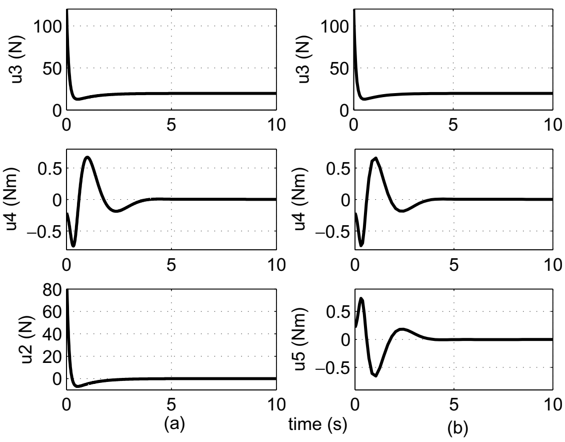

Fig. 14. Inputs u_2, u_3, u_4 and u_5 for the vehicle without motion planning: (a) bidirectional X4-flyer, (b) conventional X4-flyer.

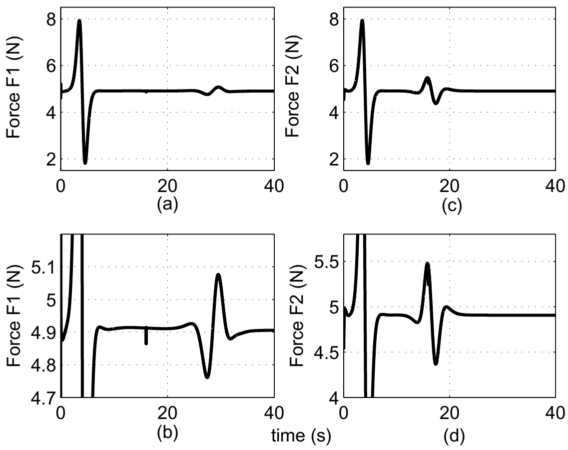

Fig. 15. Forces applied to rotors 1–2: conventional X4-flyer.

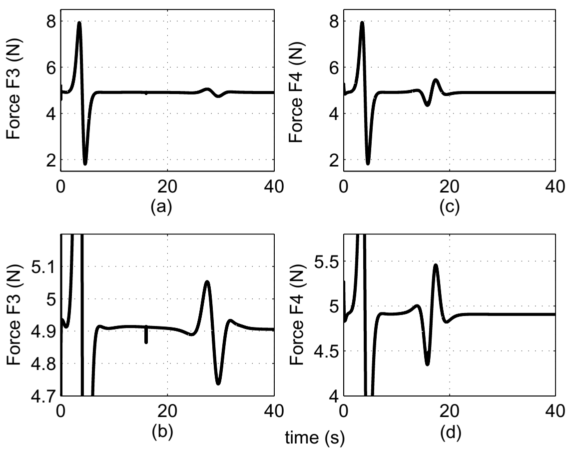

Fig. 16. Forces applied to rotors 3–4: conventional X4-flyer.

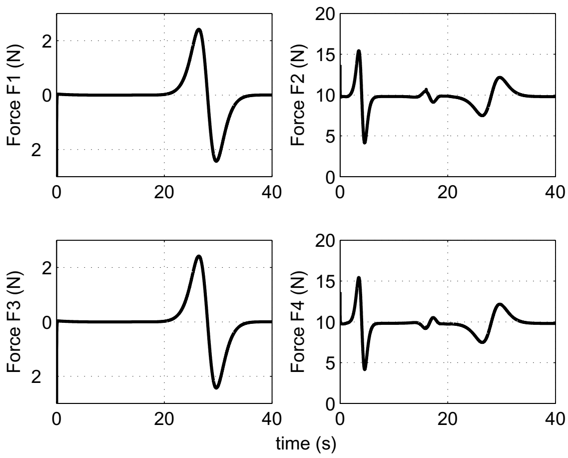

Fig. 17. Forces in the case of the bidirectional X4-flyer.

$$x^r(t) = h_d \frac{(t - T_f^1)^5}{(t - T_f^1)^5 + (T_f^2 - (t - T_f^1))^5} \quad (50)$$

As soon as for $y^r\ (t)$

$$y^r(t) = h_d \frac{(t - T_f^2)^5}{(t - T_f^2)^5 + (T_f^3 - (t - T_f^2))^5} \quad (51)$$

The constraints to perform these trajectories are such that:

$$\begin{cases} z^r(0) = x^r(T_f^1) = y^r(T_f^2) = 0 \\ z^r(T_f^1) = x^r(T_f^2) = y^r(T_f^3) = h_d \\ \dot{z}^r(0) = \dot{x}^r(T_f^1) = \dot{y}^r(T_f^2) = 0 \\ \dot{z}^r(T_f^1) = \dot{x}^r(T_f^2) = \dot{y}^r(T_f^3) = 0 \\ \ddot{z}^r(0) = \ddot{x}^r(T_f^1) = \ddot{y}^r(T_f^2) = 0 \\ \ddot{z}^r(T_f^1) = \ddot{x}^r(T_f^2) = \ddot{y}^r(T_f^3) = 0 \end{cases} \quad (52)$$

Minimizing the time of displacement implies that the X4-flyer accelerates at the beginning and decelerates at the arrival.

7. Simulation results

Two engine models were studied and controlled using the backstepping technique which (a) present the bidirectional X4-flyer and (b) the conventional X4-flyer.

Figure 8 shows displacement errors according to all the directions for the conventional and bidirectional X4-flyer. It is noticed that the error thus tends to zero towards the desired positions.

In Fig. 9, we notices that the angles θ and ϕ control the engine for displacements along the axes x and y. These angles tend to zero value. It is also shown in Fig. 11(a) that we can stabilize the system to make a following movement by the swivelling of the engine actuators 1 and 3.

According to the Fig. 10, which represent our vehicle input, we remark that the input $u_3 = mg$ at the equilibrium state is always verified. The inputs u_2, u_4 and u_5 tend to zero after having carried out the desired orientation of the vehicle. These figures show also the effectiveness of the used controllers laws.

Figures 11, 12, 13 and 14, show the system without motion planning. Motion in different directions z, x and y is also tested and shown by Fig. 11. In addition we show that the behavior of errors, given by Fig. 12 is verified. At the equilibrium, attitudes of θ and ϕ are equal to zero (Fig. 13).

Without motion planning, the amplitude of controllers is important (Fig. 14) and a maximum of energy is asserted which is requested for flying vehicles.

In the conventional X4-flyer, the engine forces are such as $f_1 = f_3$ and $f_2 = f_4$ (Figs 15 and 16). The Figs (15b), (15d), (16b) and (16d) give the respective zooms of the Figs (15a), (15c), (16a) and (16c). In the time interval 10 s to 20 s, the force f_2 is compensatory to f_4 to have navigation according to x. As too, in the time interval 22 s to 32 s, the force f_1 is compensatory to f_3 to have navigation according to y.

Figure 17 shows the four forces applied to the engine (bidirectional X4-flyer) where $f_2 = f_4$ and $f_1 = f_3$. The equality $f_1 = f_3$ represents the forces of navigation producing the movement according to y.

8. Conclusion

The study of the stabilization with and without a predefined trajectory of the mini-flying robot with four rotors (X4-flyer) was discussed in this paper. The importance of the trajectory generation and its consequences with respect to amplitude of the used controller, was highlited. With the proposed motion planning, actuator saturations can be overcomed. Consequently, economy in energy of batteries can be asserted during the fly. The backstepping technique was successfully applied and enabled us to design control algorithms ensuring the vehicle displacement from an initial position to a desired position. The backstepping approach used requires the well knowledge of the system model and parameters. Future works, will addressed essentially the development of a fuzzy controller based algorithm (which not require the good knowledge of the model) [16,18,19], to make the comparison of both controllers. Test control inputs permit to perform the tracking objectives; flying road with straight and round corners like connection and the capability of engines to fly with rounded intersections and crossroads. A realization of a control system based on engine sensors information is under studying.

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