Longitudinal and horizontal stabilization of an unmanned blimp

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Abstract: In this paper, we will present the stabilization of an underactuated autonomous blimp in the neighborhood of a desired point in both plans the longitudinal P_{xz}, and the horizontal P_{xy}. First, we will present a control design that stabilizes asymptotically the blimp in the neighborhood of a desired point in P_{xz}. The expression of the proposed control is based on the use of the iterative backstepping technique and the Lyapunov function, second we will try to locally stabilize the airship in the neighborhood of a desired point in P_{xy}, using the method of homogeneity and averaging theorem. Accordingly, we will try to define a continuous time-varying command which stabilizes the blimp in the neighborhood of a defined point. All that has been mentioned in the second part can only be achieved after proving that the orientation of the airship in a horizontal plane, cannot be stabilized with a continuous feedback only as a function of the state. However, stabilization is still possible with a continuous time dependent feedback. Simulation results are presented to show the validity of the proposed control scheme.

Keywords: airship; nonlinear control; backstepping technique; averaging theorem; homogeneity.

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1 Introduction

The control of underactuated vehicles, i.e. vehicles where the control vector has a lower dimension than the configuration vector, is a field of increasing interest. Underactuated systems regroup blimps (airships)[6], the four-rotor Vertical Take Off and Landing (VTOL) named as X4-flyer [9-12], the Planar Vertical Take Off and Landing (PVTOL) aircraft, the surface spacecraft [8], and Unmanned Underwater Vehicles (UUV's)[11].
The class of underactuated systems, has been examined by Beji [6] who presented a homogeneous periodic time-varying control that asymptotically stabilizes an underactuated autonomous airship. Homogeneous approximations and feedback control have been applied successfully to systems without drift by Morin and Samson [8], they presented a smooth time varying feedback law that asymptotically stabilizes an underactuated surface spacecraft. Pound et al [9] authors present the X4-flyer and they developed the dynamic model, but Hamel et al [12] proposes a new control design of the X4-flyer in which they separated the rigid body dynamics from the motor dynamics in order to obtain a strong practical stability of the complete system. Pettersen and Egeland [5] have proved that a large class of underactuated vehicles cannot be stabilized either by a continuous or a discontinuous state feedback. They have considered a system with drift vector (surface vessel) as a case study and they have shown that a continuous periodic time-varying feedback law can stabilize it.
In this paper we will highlight the study of the control of the dynamic blimp (see figure (1)) which has already been used by Beji and Abichou in [6-7]. Given that the model is hard to be used under its original form, we will focus on studying the movement of the blimp in both planes-longitudinal and horizontal, separately. The problem here is to find in each plane a control law that asymptotically stabilizes the airship in the neighborhood of a desired point.
This paper consist of three sections:
- The first section deals with the study of the asymptotic stabilization in the neighborhood of a desired point in the longitudinal plane. The study was achieved in two basic steps: we began with presenting the modified mathematical model, which describes the movement of the blimps in the longitudinal plane, relying on the equivalence of the behavior of the non linear system and its linearized tangent. We restrict ourselves to the study of the asymptotic stability of the linearized tangent of our mathematical model. In the second step, we noticed that the achieved study

system can be divided into two under-systems which are completely disconnected at the level of the variables. Consequently the problem remains to explicitly construct the expressions of the commands that asymptotically stabilize each under-system in the neighborhood of a desired point.
- The second section deals with the study of the asymptotic stabilization in the neighborhood of a desired point in the horizontal plane. It consists of four steps which are organized as follows: The first step deals with the airship kinematic and dynamic models. The form of the stabilizing controller is given in the second step. The third step presents a coordinate transformation and the controllability properties of the airship. A continuous periodic time varying feedback law is developed in step four where one stabilizes the airship in the neighborhood of a desired point.
- Finally, the third section deals with numerical simulations to prove the efficiency of the controller.

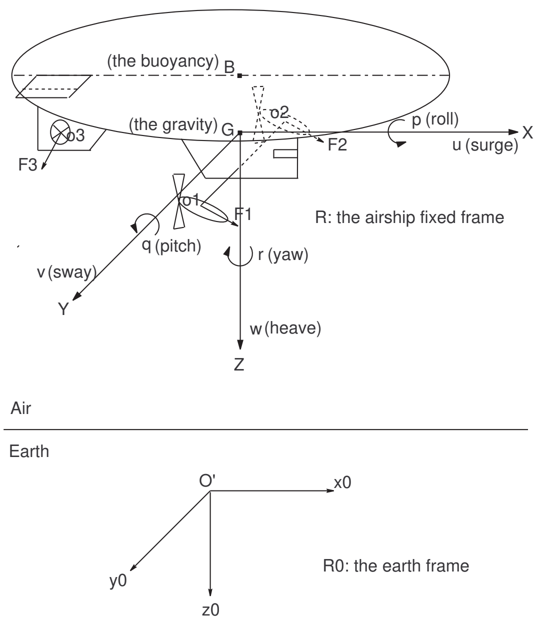

Figure 1 Airship fixed and earth reference frames

2 LONGITUDINAL ASYMPTOTIC STABILIZATION

2.1 Presentation of the study model

The notation used for the airship are given in the following chart (see table 1)

The longitudinal model is described in (XGZ) plane. The dynamic of the blimp in this case introduces the states $\eta_{long} = [x, \theta, z]^T$ and $\nu_{long} = [u, q, w]^T$. Therefore, we consider the following dynamic model

$$
\begin{aligned}
\dot{\eta}_{long} &= H_\theta \nu_{long} \\
m_x \dot{u} &= -m_z wq + x_u u + (F_B - F_G) sin\theta + \tau_1 \\
m_z \dot{w} &= m_x uq + z_w w - (F_B - F_G) cos\theta + \tau_3 \\
J_y \dot{q} &= (m_z - m_x) uw + M_q q + F_B z_b sin\theta + GO_{1z}\tau_1
\end{aligned}
\tag{1}
$$

with H_θ is the *pitch* rotation matrix, given by

$$
H_\theta = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \tag{2}
$$

with $\tau_1 = (F_1 + F_2)cos\mu$ and $\tau_3 = -(F_1 + F_2)sin\mu$ the commands which are necessary for both moving the blimp back word and forward also moving it up and down $(see figure(2))$

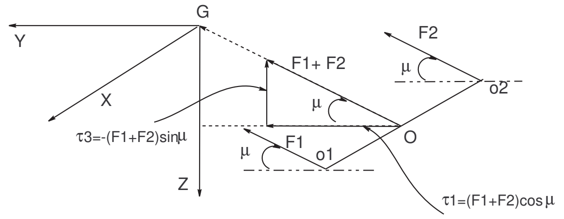

Figure 2 Presentation of commands τ_1 et τ_3

The development of the model has led us to the following expression

$$
\begin{cases}
\dot{x} = cos\theta u + sin\theta w \\
\dot{\theta} = q \\
\dot{z} = -sin\theta u + cos\theta w \\
\dot{u} = \frac{1}{m_x}(-m_z wq + x_u u + (F_B - F_G) sin\theta + \tau_1) \\
\dot{w} = \frac{1}{m_z}(m_x uq + z_w w - (F_B - F_G) cos\theta + \tau_3) \\
\dot{q} = \frac{1}{J_y}((m_z - m_x) uw + M_q q + F_B z_b sin\theta + GO_{1z}\tau_1)
\end{cases}
\tag{3}
$$

Table 1 Notation used for air vehicles

	Linear and angular velocity	*Positions and Euler angles*
motion in the x-direction (surge)	u	x
motion in the y-direction (sway)	v	y
motion in the z-direction (heave)	w	z
rotation about the x-direction (roll)	p	ϕ
rotation about the y-direction (pitch)	q	θ
rotation about the z-direction (yaw)	r	ψ

.

Then, relying on the equivalent behavior of a nonlinear system and its tangent linearized equivalent (the approximation in the first order in the neighborhood of a desired point) [1], we simplify the equations of the system (3) to:

$$(4)\qquad \begin{cases} \dot{x} = u \\ \dot{\theta} = q \\ \dot{z} = w \\ \dot{u} = \frac{1}{m_x}(x_u u + (F_B - F_G)\theta + \tau_1) \\ \dot{w} = \frac{1}{m_z}(z_w w - (F_B - F_G) + \tau_3) \\ \dot{q} = \frac{1}{J_y}(M_q q + F_B z_b \theta + GO_{1z}\tau_1) \end{cases}$$

Correspondingly, the problem refers to construct the commands τ_1 and τ_3 in order to asymptotically stabilize the tangent linearized in the neighborhood of a desired point $(x_d, 0, z_d, 0, 0, 0)$.

2.2 Controller investigation

Before starting the development of the stabilized controller, one notices that we can divide the linearized tangent system into two underactuated subsystems. Each subsystem is totally disconnected in terms of the variables where τ_3 and τ_1 are the inputs, respectively.

$$(5)\qquad \begin{cases} \dot{w} = \frac{1}{m_z}[z_w w - (F_B - F_G) + \tau_3] \\ \dot{z} = w \end{cases}$$

$$(6)\qquad \begin{cases} \dot{u} = \frac{1}{m_x}[x_u u + (F_B - F_G)\theta + \tau_1] \\ \dot{q} = \frac{1}{J_y}[M_q q + F_B z_B \theta + GO_{1z}\tau_1] \\ \dot{x} = u \\ \dot{\theta} = q \end{cases}$$

The two subsystems given below will be asymptotically stabilized taking $(0, z_d)$ and $(0, 0, x_d, 0)$ like objectives.

At this stage, let us announce this theorem, largely concerns the following class of systems (see [4])

$$(7)\qquad \begin{cases} \dot{x}_1 = f_1(x_1, x_2) \\ \dot{x}_2 = u \end{cases}$$

with $(x_1, x_2) \in (IR^{n_1}, IR^m) = IR^n$ represent the state variables, and $u \in IR^m$ is the control.

Theorem 1. *Assume that f_1 is of class C^1 and that the control system*

$$(8)\qquad \dot{x}_1 = f_1(x_1, v)$$

where the state is $x_1 \in IR^{n_1}$ *and the control* $v \in IR^m$*, can be globally asymptotically stabilized by means of a stationary feedback law of class* C^1*. Then control system* (7) *can be globally asymptotically stabilized by means of a continuous stationary command having the form* $u = -k(x_2 - v)$ *with* k *is a real positive constant.*

a- Stabilization of the first subsystem

Theorem 2.
The first under-system (5) *is asymptotically stable in the neighborhood of a desired point* $(0, z_d)$ *with a stationary continuous feedback of form*

$$\tau_3 = m_z[-k_2^{'}(w + k_1^{'}(z - z_d))] - z_w w + F_B - F_G$$

with $k_1^{'}$ *and* $k_2^{'}$ *are large enough and strictly positive.*

Proof.
<u>1^{st} step</u>: Study of the controllability
Before starting to construct the stabilized controller, we will study the commendability of the mentioned subsystem.

We consider the following form of write

$$\frac{d}{dt}\begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{m_z} z_w & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tau_3^{'}$$

with $\tau_3^{'} = \frac{1}{m_z}[-(F_B - F_G) + \tau_3]$. Consider the rank of the commendability matrix C which can be calculated by $C = [B, AB]$ [1], we can easily prove that the rang of C is equal to two. Then, the system is controllable by a stationary continuous feedback.

<u>2^{nd} step</u>: Feedback stabilization
Let us define the error $\xi_1 = z - z_d$ where z denotes the blimp altitude and z_d is the desired constant altitude.

Consider $\tau_3 = m_z \tau_3^{''} - z_w w + (F_B - F_G)$. ($\tau_3^{''}$ is the new control to be calculated in such away if we replace it by its expression in the first subsystem (5), we get

$$\begin{cases} \dot{w} = \tau_3^{''} \\ \dot{\xi}_1 = w \end{cases} \tag{9}$$

It will be written according to model (7). Then, relying on the Backstepping theorem, it is enough to stabilize asymptotically the following reduced system

$\dot{\xi}_1 = \tau_3^{'''}$ with the virtual controller $\tau_3^{'''} \equiv \tau_3^{''}$. It is easy to see that with $\tau_3^{'''} = -k_1^{'}\xi_1$ ($k_1^{'} > 0$), the reduced subsystem is stabilized exponentially. Furthermore, according to **theorem 1** $\tau_3^{''} = -k_2^{'}(w + k_1^{'}\xi_1)$ ($k_2^{'} > 0$ and large enough)

b- Stabilization of the second subsystem

Theorem 3
The second subsystem (6) *is asymptotically stable in the neighborhood of a desired*

point $(0,0,x_d,0)$ *with a stationary continuous controller* τ_1 *(see the end of the proof)* k_1,k_2 *and* k_3 *are large enough strictly positive gains and* $k_4 > 0$.

Proof.
1^{st} step: Study of the controllability
The second subsystem (6) takes the linear form $\dot{X} = AX + B\tau_1$ with $X = (u, q, x, \theta)$;

$$A = \begin{bmatrix} \frac{1}{m_x} x_u & 0 & 0 & \frac{1}{m_x}(F_B - F_G) \\ 0 & \frac{1}{J_y} M_q & 0 & \frac{1}{J_y} F_B z_b \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} \frac{1}{m_x} \\ \frac{GO_{1z}}{J_y} \\ 0 \\ 0 \end{bmatrix}$$

such that the rank the controllability matrix $C = [B, AB, A^2B, A^3B]$ is equal to 4. So we can conclude that it's asymptotic stability can be asserted by a linear feedback.

2^{st} step: Feedback stabilizing controller
We consider the behavior of the blimp error according to (GX) axis by $\xi_2 = x - x_d$. x denotes the position of the blimp centre of gravity and x_d is the desired position. To make the subsystem (6) in cascade, we will propose the following change of variable $q_1 = q - \frac{m_x}{J_y} GO_{1z} u$ and to better visualize the subsystem in cascade, we propose $\tau_{11} = \frac{1}{m_x}[x_u u + (F_B - F_G)\theta + \tau_1]$.

As a result, the subsystem (6) in the new variable takes the form

$$\begin{cases} \dot{u} = \tau_{11} \\ \dot{q_1} = \frac{1}{J_y}[M_q q_1 + (F_B z_B - GO_{1z}(F_B - F_G))\theta + D_1 u] \\ \dot{\xi_2} = u \\ \dot{\theta} = q_1 + D_2 u \end{cases} \tag{10}$$

where D_1 and D_2 are two constants, given by

$$\begin{cases} D_1 = (\frac{m_x}{J_y} M_q - x_u) GO_{1z} \\ D_2 = \frac{m_x}{J_y} GO_{1z} \end{cases}$$

Taking $v_1 \equiv \tau_{11}$ as a first virtual command. Using the Backstepping technique, we get the reduced subsystem (11) as below,

$$(11)\qquad \begin{cases} \dot{q_1} = \frac{1}{J_y}[M_q q_1 + (F_B z_b - GO_{1z}(F_B - F_G))\theta + D_1 v_1] \\ \dot{\xi_2} = v_1 \\ \dot{\theta} = q_1 + D_2 v_1 \end{cases}$$

where $\tau_{11} = -k_1(u - v_1)$. k_1 is large enough strictly positive gain.

1^{st} ***iteration***:
To make the system (11) in cascade, we will propose the following two change of variables

$$(12)\qquad \begin{cases} q_2 = q_1 - \frac{D_1}{J_y}\xi_2 \\ \theta_1 = \theta - D_2\xi_2 \end{cases}$$

thanks to these changes, the subsystem (11) take the following form

$$(13)\qquad \begin{cases} \dot{q_2} = \frac{1}{J_y}[M_q q_2 + (F_B z_b - GO_{1z}(F_B - F_G))\theta_1 + D_3\xi_2] \\ \dot{\xi_2} = v_1 \\ \dot{\theta_1} = q_2 + \frac{D_1}{J_y}\xi_2 \end{cases}$$

with D_3 is constant and takes the expression $D_3 = \frac{D_1}{J_y}M_q + D_2(F_B z_b - GO_{1z}(F_B - F_G))$.

Applying the **backstepping** technique again, we will get the reduced subsystem below which is obtained by taking $v_2 \equiv v_1$ as a second virtual controller

$$(14)\qquad \begin{cases} \dot{q_2} = \frac{1}{J_y}[M_q q_2 + (F_B z_b - GO_{1z}(F_B - F_G))\theta_1 + D_3 v_2] \\ \dot{\theta_1} = q_2 + \frac{D_1}{J_y}v_2 \end{cases}$$

where $v_1 = -k_2(\xi_2 - v_2)$, $k_2 > 0$ and large enough.

2^{st} ***iteration***:
In order to put the subsystem (14) in cascade form, we will suggest the following change of variable $q_3 = q_2 - \frac{D_3}{D_1}\theta_1$.
then (14) takes this new form

$$(15)\qquad \begin{cases} \dot{q_3} = \frac{1}{J_y}[(M_q - \frac{D_3 J_y}{D_1})q_3 + D_4\theta_1] \\ \dot{\theta_1} = q_3 + \frac{D_3}{D_1}\theta_1 + \frac{D_1}{J_y}v_2 \end{cases}$$

D_4 is a constant with this expression $D_4 = (F_B z_b - GO_{1z}(F_B - F_G)) + \frac{D_3}{D_1}(M_q - \frac{D_3 J_y}{D_1})$ and to better visualize this form in cascade, we assume $v_3 = q_3 + \frac{D_3}{D_1}\theta_1 + \frac{D_1}{J_y}v_2$.

Taking this suggestion the system (15) becomes

(16)
$$\begin{cases} \dot{q}_3 = \frac{1}{J_y}[(M_q - \frac{D_3 J_y}{D_1})q_3 + D_4\theta_1] \\ \dot{\theta}_1 = v_3 \end{cases}$$

Applying the backstepping technique, we get the reduced subsystem (17). The last is formed by a unique equation obtained by taking $v_4 \equiv v_3$ as a third virtual controller

(17)
$$\begin{cases} \dot{q}_3 = \frac{1}{J_y}[(M_q - \frac{D_3 J_y}{D_1})q_3 + D_4 v_4] \end{cases}$$

$v_3 = -k_3(\theta_1 - v_4)$, whith $k_3 > 0$ and large enough.

Finally the problem is reduced to stabilizing the subsystem $\dot{q}_3 = v_5$ with $v_5 = \frac{1}{J_y}[(M_q - \frac{D_3 J_y}{D_1})q_3 + D_4 v_4]$.

We propose that $v_5 = -k_4 q_3$ $(k_4 > 0)$.

Consider the Lyapunov function $V = \frac{1}{2}q_3^2 > 0$ which is definite positive.

The drift of V time derivative is $\dot{V} = -k_4 q_3^2$ which is negative definite.

The choice of V guarantees the exponential asymptotic stabilization of the subsystem (17). In addition (6) is asymptotically stable in the neighborhood of a desired point $(0, 0, x_d, 0)$ by taking τ_1 as

$$\begin{aligned}
\tau_1 = & -[(k_1 m_x + x_u) + k_1 k_2 m_x \frac{J_y}{D_1}(\frac{k_3}{D_4}(M_q - \frac{D_3 J_y}{D_1}) \\
& + k_3 k_4 \frac{1}{D_4} J_y + 1)\frac{m_x}{J_y} G O_{1z}]u \\
& - [k_1 k_2 m_x \frac{J_y}{D_1}(\frac{k_3}{D_4}(M_q - \frac{D_3 J_y}{D_1}) + k_3 k_4 \frac{1}{D_4} J_y \\
& + 1)]q - [(F_B - F_G) + k_1 k_2 m_x \frac{J_y}{D_1}(\frac{D_3}{D_1} + k_3) \\
& - k_1 k_2 m_x \frac{J_y}{D_1}(\frac{k_3}{D_4}(M_q - \frac{D_3 J_y}{D_1}) + k_3 k_4 \frac{1}{D_4} J_y \\
& + 1)\frac{D_3}{D_1}]\theta - [k_1 k_2 m_x - k_1 k_2 m_x \frac{J_y}{D_1}(\frac{D_3}{D_1} + k_3) D_2 \\
& + k_1 k_2 m_x \frac{J_y}{D_1}(\frac{k_3}{D_4}(M_q - \frac{D_3 J_y}{D_1}) + k_3 k_4 \frac{1}{D_4} J_y \\
& + 1)(-\frac{D_1}{J_y} + \frac{D_3}{D_1} D_2)](x - x_d)
\end{aligned}$$

k_1, k_2, k_3 are large enough strictly positive gains and $k_4 > 0$.

3 ASYMPTOTIC STABILIZATION IN HORIZONTAL PLANE

3.1 The studied model

The horizontal motion of the blimp is usually described by the motion components in surge, sway and yaw. Then, we choose the following states $\nu_{hor} = [u, v, r]$ and $\eta_{hor} = [x, y, \psi]$. Which implies that the dynamics associated with the motion in heave, roll and pitch don't have a big influence on the blimp movement. Hence, we considered the following hypotheses $(w = p = q = 0)$.
The horizontal configurations are described by

$$
\begin{cases} \dot{\eta}_{hor} &= H_\psi \nu_{hor} \\ M\dot{\nu}_{hor} &= -C\nu_{hor} - D\nu_{hor} + B\tau \end{cases} \tag{18}
$$

with

$$
H_\psi = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, M = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & J_z \end{bmatrix}
$$

$$
C = \begin{bmatrix} 0 & 0 & -m_y v \\ 0 & 0 & m_x u \\ m_y v & -m_x u & 0 \end{bmatrix}, D = \begin{bmatrix} -x_u & 0 & 0 \\ 0 & -y_v & 0 \\ 0 & 0 & -N_r \end{bmatrix}
$$

and

$$
B = \begin{bmatrix} \frac{1}{m_x} & 0 & 0 \\ 0 & \frac{1}{m_y} & \frac{-GO_{3x}}{J_z} \end{bmatrix}^T
$$

where H_ψ describes the rotation of the blimp, M is the inertia matrix, $C(\nu_{hor})$ regroups the Coriolis and centripetal terms, D is the damping matrix , B implies the thruster configurations, and $\tau = (\tau_1, \tau_2)^T$ is the control vector. τ_1 is the control force in surge and τ_2 is the control torque in yaw.
So, we get the following development

$$
\begin{cases} \dot{x} = cos\psi u - sin\psi v \\ \dot{y} = sin\psi u + cos\psi v \\ \dot{\psi} = r \\ \dot{u} = \frac{1}{m_x}[x_u u + m_y v r + \tau_1] \\ \dot{v} = \frac{1}{m_y}[y_v v - m_x u r + \tau_2] \\ \dot{r} = \frac{1}{J_z}[(m_x - m_y)vu + N_r r - Go_{3x}\tau_2] \end{cases} \tag{19}
$$

Given that our basic problem is the stabilization in neighborhood of a desired point $(x = x_d, y = y_d)$, we introduce the error in position $\xi_1 = x - x_d$ and $\xi_2 = y - y_d$. Under conditions given by system (19), we get

$$(20)\qquad \begin{cases} \dot{\xi}_1 = cos\psi u - sin\psi v \\ \dot{\xi}_2 = sin\psi u + cos\psi v \\ \dot{\psi} = r \\ \dot{u} = \frac{1}{m_x}[x_u u + m_y vr + \tau_1] \\ \dot{v} = \frac{1}{m_y}[y_v v - m_x ur + \tau_2] \\ \dot{r} = \frac{1}{J_z}[(m_x - m_y)vu + N_r r - Go_{3x}\tau_2] \end{cases}$$

3.2 Study of the nature of the stabilizing controller

Let's first start by remind the following necessary condition for **Brockett**'s stabilizability by stationary continuous feedback $[4 - 10]$

Theorem.4: *If the control system C*

$$\begin{cases} \dot{x} = f(x, u) \\ f(0, 0) = 0 \end{cases}$$

where $x \in IR^n$ *and* $u \in IR^m$ *can be locally asymptotically stabilized by means of feedback laws, then the image of any neighborhood of* $(0, 0) \in IR^n \times IR^m$ *is a neighborhood of* $0 \in IR^n$.

Based on Theorem 4, we show that we cannot stabilize the system by means of a stationary continuous feedback.

In fact, we will prove that any point under the form $X_c = (0, 0, 0, 0, 0, \varepsilon)$ in the neighborhood of 0_{IR^6}, where $\varepsilon \neq 0$ is not a part of the image set.

Furthermore, notice that the system (20) is affine relative to the controller, it takes the following form

$$(21)\qquad [\dot{\eta}_{hor}, \dot{\nu}_{hor}]^t = F(X) + G(X)(\tau_1, \tau_2)^t$$

with $X = [\eta_{hor}, \nu_{hor}]$.

So, we confirm the the system cannot be stabilized by a stationary discontinuous controller.

Given that the system (20) does not verify Brockett's conditions, its stabilization in the neighborhood of a desired point cannot be achieved with the help of a stationary command whether it is continuous or discontinuous. We give a solution through a continuous time-varying stabilizing feedback.

3.3 Controllability study of the stabilized controller

Before studying the system commendability and in order to achieve a simple polynomial kinematic equations and to eliminate ψ , we will conduct the coordinate

transformation, which is a global diffeomorphism.
Let

$$\left\{\begin{array}{l} z_1 = cos\psi\xi_1 + sin\psi\xi_2 \\ z_2 = -sin\psi\xi_1 + cos\psi\xi_2 \\ z_3 = \psi \end{array}\right. \tag{22}$$

The resulting model is then

$$\left\{\begin{array}{ll} \dot{z}_1 = rz_2 + u & \\ \dot{z}_2 = -rz_1 + v & \\ \dot{z}_3 = r & \\ \dot{u} & = \frac{1}{m_x}[x_u u + m_y vr + \tau_1] \\ \dot{v} & = \frac{1}{m_y}[y_v v - m_x ur + \tau_2] \\ \dot{r} & = \frac{1}{J_z}[(m_x - m_y)vu + N_r r - Go_{3x}\tau_2] \end{array}\right. \tag{23}$$

From the above paragraph we have shown that the underactuated airship cannot be asymptotically stabilized by either continuous or discontinuous stationary command. However, we show after that the airship still checks the necessary and sufficient condition of a small time locally controllable (STLC). Furthermore, by introducing explicit time dependence in the controller, the asymptotic stability can be achieved.

Let

$$f_0 = \begin{pmatrix} rz_2 + u \\ -rz_1 + v \\ r \\ \frac{x_u u + m_y vr}{m_x} \\ \frac{y_v v - m_x ur}{m_y} \\ \frac{(m_x - m_y)vu + N_r r}{J_z} \end{pmatrix}; f_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_x} \\ 0 \\ 0 \end{pmatrix}$$

and

$$f_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{m_y} \\ \frac{GO_{3x}}{J_z} \end{pmatrix} \tag{24}$$

Then, the system (23) can be written as

$$[\dot{\eta}_{hor}, \dot{\nu}_{hor}]^t = f_0(x) + f_1\tau_1 + f_2\tau_2 \tag{25}$$

Lemma 1. The STLC necessary condition is verified by the blimp model (23) $\forall(\eta_{hor}, \nu_{hor}) \in IR^6$.

Proof. The vectors f_1,f_2,$[f_0, f_1]$,$[f_0, f_2]$,$[f_2, [f_0, f_1]]$ and $[[f_2, [f_0, f_1]], f_0]$ are linearly independent and span $Lie\{f_0, f_1, f_2\}$, thus we have

$$\{ g(0); g \in Lie \{ f_0, f_1, f_2 \} \} = IR^6$$

.

Hence, the STLC necessary condition is satisfied.

Proposition 1. The STLC sufficient condition is satisfied by model (23).

Proof.
The model (23) satisfies the condition of the total rank for $(z_1, z_2, z_3, u, v, r) \in IR^6$. In addition any bracket of a degree superior than 4 can be written as a linear combination of a lower order bracket. We also notice that the degree of the the bad bracket must be odd. The bad bracket of degree one, is f_0 but it disappears at any equilibrium point. The bad brackets of degree three are $[f_2, [f_0, f_2]]$ and $[f_1, [f_0, f_1]]$ but they are identically equal to zero. Consequently, the system (23) satisfies the **Sussmann** Theorem $[3-4]$. So we can say that it is locally commendable for a small time.

In the following, we show that a time-varying control stabilizes asymptotically the underactuated airship. Given that the system (23) satisfies the Lie algebra rank condition and the Sussmann Theorem, we can also construct a periodic time-varying feedback law which is a function τ in $C^0(IR^6 \times IR; IR^2)$ of class C^∞ on $(IR^6 \setminus \{0\}) \times IR$ that satisfies

$$\tau(0, t) = 0, \forall\, t \in IR$$

$$\tau(\eta, \nu, t + T) = \tau(\eta, \nu, t), \forall\, t \in IR, \forall\, (\eta, \nu) \in IR^n,$$

Proposition 2. The underactuated blimp is locally asymptotically stabilizable in small time by means of an almost smooth periodic time-varying feedback law.

3.4 The blimp stabilization

In order to facilitate the feedback control law construction, we suggest the following variable transformation (global diffeomorphism) τ_1, τ_2 appear only one time in the system.

Let

$$d = r + \frac{GO_{3x} m_y}{J_z} v \tag{26}$$

The state equations are then given by

$$(27) \qquad \begin{cases} \dot{z}_1 = u + z_2 d - \frac{GO_{3x} m_y}{J_z} z_2 v \\ \dot{z}_2 = v - z_1 d + \frac{GO_{3x} m_y}{J_z} z_1 v \\ \dot{z}_3 = d - \frac{GO_{3x} m_y}{J_z} v \\ \dot{u} \ = \frac{1}{m_x}[x_u u + m_y v d - \frac{GO_{3x} m_y^2}{J_z} v^2 + \tau_1] \\ \dot{v} \ = \frac{1}{m_y}[y_v v - m_x u d + \frac{GO_{3x} m_y m_x}{J_z} v u + \tau_2] \\ \dot{d} \ = \frac{1}{J_z}[(m_x - m_y + \frac{GO_{3x}^2 m_y m_x}{J_z}) v u \\ \quad - m_x GO_{3x} u d + N_r d + (GO_{3x} y_v - \frac{N_r GO_{3x} m_y}{J_z}) v] \end{cases}$$

Theorem 5. Consider the following functions

$$(28) \qquad \begin{cases} u^{'} \ = -k_1 z_1 - \rho sin(\frac{t}{\epsilon}) \\ v^{'} \ = -k_2 z_2 + k_3 \frac{d}{\rho} sin(\frac{t}{\epsilon}) \end{cases}$$

with the homogeneous norm $\rho = |z_1| + |z_2|^{1/2} + |d|^{1/3}$, associated with the dilation $\delta_\lambda^r(X,t) = (\lambda z_1, \lambda^2 z_2, \lambda z_3, \lambda u, \lambda^2 v, \lambda^3 d, t)$, and the following time varying continuous feedback

$$(29) \qquad \begin{cases} \tau_1 = m_x \tau_1^{'} - x_u u \\ \tau_2 = m_y \tau_2^{'} - y_v v \end{cases}$$

with

$$(30) \qquad \begin{cases} \tau_1^{'} = -k_4(u - u^{'}) \\ \tau_2^{'} = -k_5(v - v^{'}) \end{cases}$$

Then for a good choice of positive parameters k_1 , k_2 and k_3, there exists $\epsilon_0 > 0$, such that for any $\epsilon \in (0, \epsilon_0)$ and large enough parameters $k_4 > 0$ and $k_5 > 0$, the feedback (29) locally asymptotically and exponentially stabilizes the equilibrium point $(0, 0, c_1, 0, 0, 0)$.

Poof. Let us consider the following dilation

$$\delta_\lambda^r(X,t) = (\lambda z_1, \lambda^2 z_2, \lambda z_3, \lambda u, \lambda^2 v, \lambda^3 d, t)$$

The system (27) can be rewritten as:

$$(31) \qquad \dot{X} = f(X,t) + g(X,t)$$

with

$f(X,t) = (u, v, 0, \frac{1}{m_x}[x_u u + \tau_1], \frac{1}{m_y}[y_v v + \tau_2], \frac{1}{J_z}[(m_x - m_y + \frac{GO_{3x}^2 m_y m_x}{J_z}) v u + N_r d + (GO_{3x} y_v - \frac{N_r GO_{3x} m_y}{J_z}) v])^T$

and

$$g(X,t) = \begin{pmatrix} z_2 d - \frac{GO_{3x} m_y}{J_z} z_2 v \\ -z_1 d + \frac{GO_{3x} m_y}{J_z} z_1 v \\ d - \frac{GO_{3x} m_y}{J_z} v \\ \frac{1}{m_x}[m_y v d - \frac{GO_{3x} m_y^2}{J_z} v^2] \\ \frac{1}{m_y}[-m_x u d + \frac{GO_{3x} m_y m_x}{J_z} v u] \\ \frac{1}{J_z}[-m_x GO_{3x} u d] \end{pmatrix} \tag{32}$$

It is straightforward to verify that $f(X,t)$ defines a continuous T-periodic homogeneous vector field of zero degree with respect to the dilation $\delta_\lambda^r(X,t)$. $g(X,t)$ is continuous and defines a sum of homogeneous vector fields of degree strictly positive with respect to $\delta_\lambda^r(X,t)$. Thus, according to [12, $Prop2$], the origin $(\eta,\nu) = (0,0)$ of system (27) is locally exponentially stable with respect to δ_λ^r if the equilibrium $(\eta,\nu) = (0,0)$ of $\dot{X} = f(X,t)$ is locally asymptotically stable.

To this purpose, let us first consider the following reduced system, obtained from $\dot{X} = f(X,t)$ by taking $u^{'} = u$ and $v^{'} = v$ as control variables. Moreover, we have neglected the term in v $((GO_{3x} y_v - \frac{N_r GO_{3x} m_y}{J_z}))$ since it has very small value. This leads to

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{d} \end{pmatrix} = \begin{pmatrix} u^{'} \\ v^{'} \\ 0 \\ \frac{1}{J_z}[(m_x - m_y + \frac{GO_{3x}^2 m_y m_x}{J_z}) v^{'} u^{'} + N_r d] \end{pmatrix} \tag{33}$$

Remark. Notice that $\dot{z}_3$ in system (27) is equal to zero, which implies that z_3 tends to a constant c_1 if t tends to infinity.

We introduce controls given by (29) in system (27), we obtain a periodic time-varying system, which can be written in the form $\dot{\xi} = g(\xi, t/\epsilon)$ where $\xi = (\dot{z}_1, \dot{z}_2, \dot{d})$. We approximate this system by an averaged system which is autonomous. The averaged system is defined as $\dot{\xi} = g_0(\xi)$ where $g_0(\xi) = (1/T)\int g(\xi, t/\epsilon) dt$.

The linear approximation of system $\dot{\xi} = g_0(\xi)$ around the origin can be written in matrix form as follow

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{d} \end{pmatrix} = A \begin{bmatrix} z_1 \\ z_2 \\ d \end{bmatrix} \tag{34}$$

with

$$A = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & \frac{1}{J_z}[(m_x - m_y + \frac{GO_{3x}^2 m_y m_x}{J_z})(-\frac{k_3}{2}) + N_r] \end{bmatrix}$$

Now, it is clear that for a suitable gain parameters the origin of (27) is obviously asymptotically stable. The asymptotic stability of the origin of $\dot{X} = f(X,t)$ is followed after noticing that the functions $u^{'}$ and $v^{'}$ are of class C^1 on $((IR^3 \times IR\{0\}) \times IR)$.

4 SIMULATION RESULTS

In this section, we will test the efficiency of the proposed controller to reach the desired position in each of the two plans (longitudinal and horizontal).

Simulations has been effectuated for a blimp of mass $m = 9.0718Kg$ with the following parameters (USI)

$$\begin{array}{ccc} m_x = 10.2; & m_y = 16.32; & m_z = 16.32 \\ x_u = -10; & y_v = -10\ ; & z_w = -10; \\ N_r = -10; & M_q = -10; & J_y = 27.73 \\ GO_{3x} = 3; & GO_{1z} = 1; & J_z = 27.63 \end{array}$$

4.1 Longitudinal motion

In order to test the effects τ_1 and τ_3 in (XGZ), we take the neighborhood of a target point $(x_d, 0, z_d, 0, 0, 0)$. The following gain controller parameters are considered

$$\begin{array}{ccc} k_1^{'} = 10; & k_2^{'} = 0.5; & k_1 = 10; \\ k_2 = 5; & k_3 = 0.01\ ; & k_4 = 5 \end{array}$$

The initial values are as follows

$$[x, \theta, z, u, w, q] = [18, 0, -18, 0, 0.5, 0]$$

The desired configurations are $x_d = 20$ and $z_d = -20$.

Figure (3) presents the response of linear speeds (u, w) and the angular speed q. Speeds oscillate in 20 seconds in time before to reach a null values. At the same time, the positions (x, z) reach the desired configuration $(x_d = 20, y_d = -20)$.

Figure (4) shows the time evolution of the controllers τ_1 and τ_3.

4.2 Horizontal motion

The control parameters were chosen to be

$$\begin{array}{ccc} k_1 = 0.3; & k_2 = 0.3; & k_3 = 0.3; \\ k_4 = 5; & k_5 = 1\ ; & \epsilon = 1 \end{array}$$

and the homogeneous norm used in the feedback law is

$$\rho = |z_1| + |z_2|^{1/2} + |d|^{1/3}$$

the initial values were

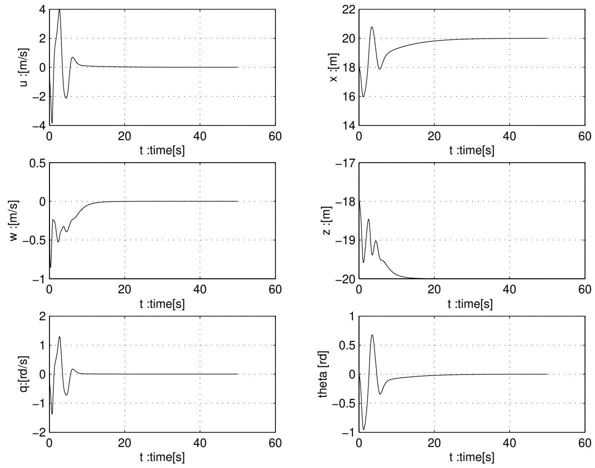

Figure 3 The time evolution of the states

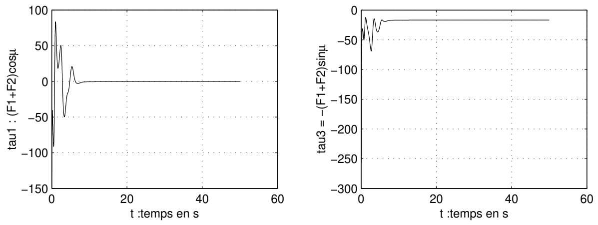

Figure 4 The time evolution of the commands τ_1 and τ_3

$$[x, y, \psi, u, v, r] = [0, 0, 0, 0, 0, 0]$$

with the desired position $x_d = 1$ and $y_d = 1$.

Figure (5) presents the response of the linear speeds $(u, v)$and that of the angular speed, under the action of the selected controls. The speeds oscillate in 10 seconds, then they reach a null speed. At the same time, the positions (x, y) reach the desired point $(x_d = 1, y_d = 1)$.

In figure (6) the response of the movement of the blimp in the plane (XGY) under the influence of the proposed control is plotted. We notice that it starts from the position $(x = 0, y = 0)$ and reaches the desired position $(x_d = 1, y_d = 1)$.

We see in figure (7) the time evolution of the controls.

5 CONCLUSIONS

In this article we have studied the positioning problem of an autonomous under-actuated airship in the neighborhood of a desired point in both plans longitudinal and horizontal. So, firstly we are interested in positioning the blimp in the longitudinal plane. With the help of a continuous stationary command we have succeeded in stabilizing the blimp in the neighborhood of a desired point. Secondly, we have

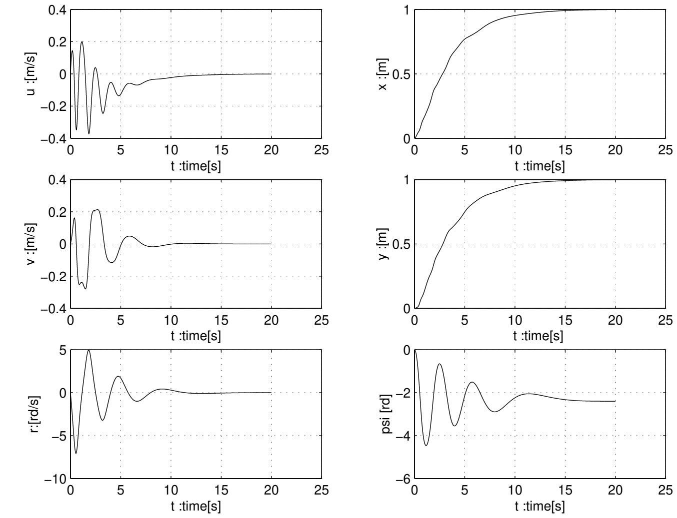

Figure 5 The time evolution of the state variables

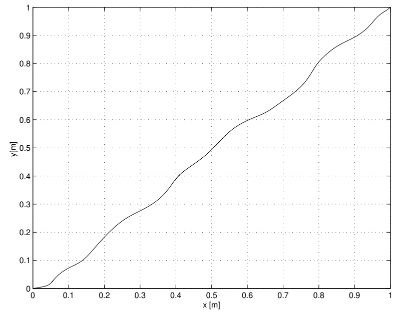

Figure 6 The trajectory of the airship in xy-space

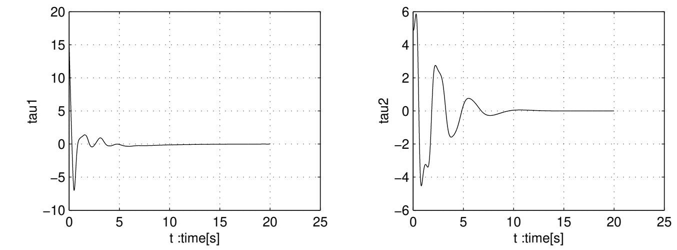

Figure 7 The time evolution of the commands τ_1 and τ_2

proposed a time varying continuous periodic command that asymptotically stabilizes the airship in the neighborhood of a chosen point in the horizontal plane , after proving that the blimp cannot be stabilized by a continuous command which only depends on state variables. Then, we have presented some numerical results that showed the validity of the chosen controls, thanks to the convergence of the airship toward the desired position.

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