3rd US-European Competition and Workshop on Micro Air Vehicle Systems (MAV07) European Micro Air Vehicle Conference and Flight Competition (EMAV2007), 17-21 September 2007, Toulouse, France

B-UAV tracking control integrating planned yaw and longitudinal/lateral inputs

R. Mlayeh* and L. Beji†

Polytechnic school of Tunisia, LaMarsa, 2078, Tunisia

Evry Val d'Essonne Univesity, Evry, 91020, France

A. Abichou‡

Polytechnic school of Tunisia, LaMarsa, 2078, Tunisia

In this study the tracking controller solution for the cartesian position and orientation (yaw) of the IBISC Bidirectional-Unmanned Arial Vehicle (B-UAV) is addressed in the cartesian coordinates. With respect to velocities based control, the cartesian acceleration-based model involves some difficulties in the conception of the tracking controller. Controlling the vehicle velocities (typical example from mobile robots) leads to a stabilizing/tracking of vehicle's positions. However, the problem is not straightforward when one considers acceleration-based motion. The aim of this work was to steer the B-UAV using the yaw attitude and two inclined rotor forces. The tracking control problem considers the dynamic model in accelerations and integrates some kinematic transformations. In neighborhood of the reference path, the transformed model in errors is linearized. Hence, the tracking results are local of nature but lead to a satisfactory simulation tracking tests. The planned yaw and longitudinal/lateral inputs are also considered in the tracking control design.

Nomenclature

X	Cartesian position vector
η	Euler angles vector
$\Re_G$	Local frame attached to G
$\Re_O$	Inertial frame
m	Mass, kg
u	Collective forces vector
$\tilde{\tau}$	Torques vector

I. Introduction

Unmanned Air Vehicles (UAV) are envisioned in many applications, including terrain exploration, military/civil surveillance and scientific research, see for example[3–5,10] and the references therein. The UAV may differ considerably regarding size and power consumption, as well as motion and sensing capabilities. In order to enable complex autonomous behaviors, it is important as a basic functionality to be able to move the UAV in a partially unknown environment and in an autonomous manner. One notes that UAV

*PhD Student, LIM Laboratory, Ecole Polytechnique de Tunisie, BP 743, 2078 La Marsa, Tunisia.

†Associate Professor, IBISC CNRS-FRE2873 Laboratory, Evry University, 40 rue du Pelvoux, 91020 Evry Cedex, France.

‡Professor, LIM Laboratory, Ecole Polytechnique de Tunisie, BP 743, 2078 La Marsa, Tunisia.

includes Autonomous Unmanned Helicopter (AUH)[2] which is a versatile machine that can perform aggressive maneuvers. Compared to helicopters,[6–8] the UAV X4-flyer (with four rotors) has some advantages:[9,10] given that two motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend, in trimmed flight (constant rotor velocities), to be canceled.

A model for the dynamic and configuration stabilization of quasi-stationary fight conditions of a four rotor vertical take-off and landing (VTOL) was studied in[10] where the dynamic motor effects are incorporating and a bound of perturbing errors was obtained for the coupled system. The stabilization problem of a four rotor rotorcraft is also presented in[11] where the nested saturation algorithm is considered. With the intent to stabilize aircrafts that are able to take-off vertically as helicopters, the control problem was solved for the planar vertical take-off and landing (PVTOL) with the input/output linearization procedure[12] and theory of flat systems.[13–15]

An B-UAV operates as follows: vertical motion is controlled by collectively increasing or decreasing the power of all motors. longitudinal motion, in x-direction or in y-direction, is not achieved by differentially controlling the motors generating a pitch/roll motion of the airframe that inclines the collective thrust (producing horizontal forces, case of the X4-flyer). In the B-UAV case, two engines of direction are used to permute between the x/y displacement. The tracking problem using a smooth variable structure control was presented in.[5]

II. System Modeling

One presents the dynamic model for the engine able to realize a fast flight of advance, hovering and quasi-stationary motion. Such a model can be achieved in a local reference frame related to the vehicle, known as *local model*, or in a supposed fixed frame, known as *global model*. Many authors consider the dynamics from a rigid body associated the fuselage to approach the modeling to which is added the aerodynamic forces, generated by the rotors. We quote for the work of Chriette with Hamel on the helicopters,[1] Castello with Lozano on X4[11] and Beji with Abichou[3] on the bidirectional X4 flyer. The model that one studies is different in structure due to the orientation of their axes compared to the conventional model.[5] Let G denotes the center of mass attached to the vehicle, let $\Re_G = \{G, E_1^g, E_2^g, E_3^g\}$ (see figure 1) be the local frame attached in G. The global fixed frame, known as the inertial frame, is denoted by $\Re_O = \{O, E_x, E_y, E_z\}$. Consider the vector $X = (x, y, z)$ of vehicle's G position and one uses the Euler angles $\eta = (\theta, \phi, \psi)$ to define the attitude, such that $(R : \Re_G \to \Re_O)$ and $R \in SO(3)$.

The objectif is to propel the aerial vehicle through the two servo-rotors and not through the orientation of the engine and to carry out the turn movement (movement coupled the horizontal motion to the yaw attitude). This idea proves its interest in the control of displacements by the yaw angle. This concept adds two servo-motors, consequently a disadvantage with respect to the embarked mass. The two internal degree of freedom are denoted by $(\xi_1, \xi_3) \in (-20^o, 20^o)$. Hence, the two supports of the engines can, either to swivel in the same direction to create a horizontal component likely to propel the X4 flyer in translation, or to swivel in opposite direction to create a yaw without translation. One deduces the following model:[3]

$$\begin{aligned} m\ddot{x} =& S_\psi C_\theta u_2 - S_\theta u_3 \\ m\ddot{y} =& \left(S_\theta S_\psi S_\phi + C_\psi C_\phi\right) u_2 + C_\theta S_\phi u_3 \\ m\ddot{z} =& \left(S_\theta S_\psi C_\phi - C_\psi C_\phi\right) u_2 + C_\theta C_\phi u_3 - mg \\ \ddot{\theta} =& \tilde{\tau}_\theta; \quad \ddot{\phi} = \tilde{\tau}_\phi; \quad \ddot{\psi} = \tilde{\tau}_\psi \end{aligned} \tag{1}$$

With respect to the conventional X4 flyer, we get the following inputs: $u_2 = f_1 S_{\xi_1} + f_3 S_{\xi_3}$ and the collective force is $u_3 = f_1 C_{\xi_1} + f_3 C_{\xi_3} + f_2 + f_4$. In the following, we deal with this inputs like the control feedback for the system and we reduce our analysis to the not trivial problem of the planar motion in acceleration. Let:

$$\begin{aligned} \ddot{x} =& u \sin(\psi) \\ \ddot{y} =& u \cos(\psi) \\ \ddot{\psi} =& \tau_\psi \end{aligned} \tag{2}$$

in system (1) our attention is to consider that u and ψ are the inputs. Hence, the last second order dynamic

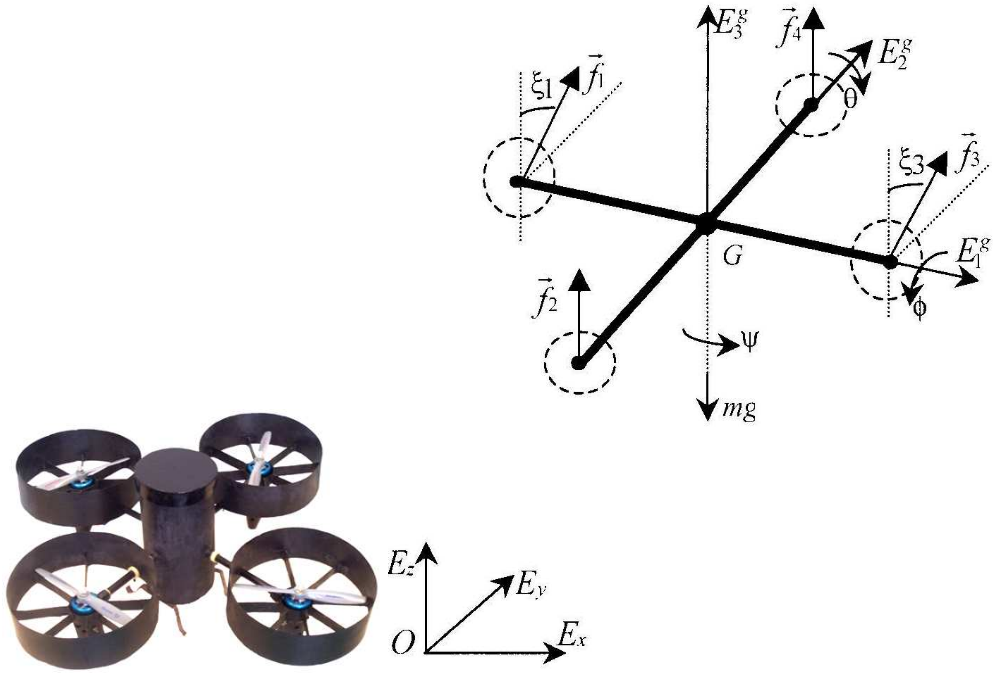

Figure 1. The B-UAV test bed and its parametrization.

of ψ will be omitted. τ_ψ will be designed such that Vehicle yaw converges to the input.
In the kinematics change of variables carries out according to:

$$\begin{aligned} z_1 &= \dot{x}\sin(\psi) + \dot{y}\cos(\psi) \\ z_2 &= -\dot{x}\cos(\psi) + \dot{y}\sin(\psi) \end{aligned} \tag{3}$$

by derivation and using (1) one obtains:

$$\begin{aligned} \dot{z}_1 &= u - \dot{\psi} z_2 \\ \dot{z}_2 &= \dot{\psi} z_1 \end{aligned} \tag{4}$$

Let us introduce the reference model according to

$$\begin{aligned} \ddot{x}^r &= u^r \sin(\psi^r) \\ \ddot{y}^r &= u^r \cos(\psi^r) \end{aligned} \tag{5}$$

we have from (2)

$$\begin{aligned} z_1^r &= \dot{x}^r \sin(\psi^r) + \dot{y}^r \cos(\psi^r) \\ z_2^r &= -\dot{x}^r \cos(\psi^r) + \dot{y}^r \sin(\psi^r) \end{aligned} \tag{6}$$

we can simplify system (5) by holding account of $\dot{x}^r = \dot{y}^r \tan(\psi^r)$ which can be reduce to

$$\begin{aligned} z_1^r &= \frac{\dot{y}^r}{\cos(\psi^r)} \\ z_2^r &= 0 \end{aligned} \tag{7}$$

one can notice that from $\dot{x}^r = \dot{y}^r \tan(\psi^r)$ we can write $\dot{\psi}^r = 0$ then from (4) and (5) we have

$$\begin{aligned} \dot{z}_1^r &= u^r \\ \dot{z}_2^r &= 0 \end{aligned} \tag{8}$$

we incorporate the errors in z_1 and z_2, with $e_{z_1} = z_1 - z_1^r$ and $e_{z_2} = z_2 - z_2^r$. The time derivative of these errors are as

$$\begin{aligned} \dot{e}_{z_1} =& e_u - e_{z_2}\dot{e}_\psi - z_2^r\dot{e}_\psi \\ \dot{e}_{z_2} =& e_{z_1}\dot{e}_\psi + z_1^r\dot{e}_\psi \end{aligned} \tag{9}$$

where $e_u = u - u^r$ and $e_\psi = \psi - \psi^r$.
The tracking control problem is reduced to the following system

$$\begin{aligned} \dot{e}_x - e_\psi \dot{e}_y =& \dot{y}^r e_\psi + \sin(\psi^r)e_{z_1} - \cos(\psi^r)e_{z_2} \\ \dot{e}_{z_1} =& e_u - z_2\dot{e}_\psi \\ e_\psi \dot{e_x} + \dot{e}_y =& -\dot{x}^r e_\psi + \cos(\psi^r)e_{z_1} + \sin(\psi^r)e_{z_2} \\ \dot{e}_{z_2} =& z_1\dot{e}_\psi \end{aligned} \tag{10}$$

Without loss of generality, let $\dot{\tilde{e}}_x \triangleq \dot{e}_x - e_\psi \dot{e}_y$ and $\dot{\tilde{e}}_y \triangleq e_\psi \dot{e_x} + \dot{e}_y$, meaning that this global regular transformation

$$\begin{pmatrix} \dot{\tilde{e}}_x \\ \dot{\tilde{e}}_y \end{pmatrix} = \begin{pmatrix} 1 & -e_\psi \\ e_\psi & 1 \end{pmatrix} \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \tag{11}$$

Then the system Eq. (10) becomes

$$\begin{aligned} \dot{\tilde{e}}_x =& \dot{y}^r e_\psi + \sin(\psi^r)e_{z_1} - \cos(\psi^r)e_{z_2} \\ \dot{e}_{z_1} =& e_u - e_{z_2}\dot{e}_\psi - z_2^r\dot{e}_\psi \\ \dot{\tilde{e}}_y =& -\dot{x}^r e_\psi + \cos(\psi^r)e_{z_1} + \sin(\psi^r)e_{z_2} \\ \dot{e}_{z_2} =& e_{z_1}\dot{e}_\psi + z_1^r\dot{e}_\psi \end{aligned} \tag{12}$$

as one reasons on the system of errors, we assume that ψ^r is in the neighborhood of zero, then $\cos(\psi^r) \simeq 1$ and $\sin(\psi^r) \simeq \psi^r$. Further the quadratic terms can be ignored. The system of errors becomes

$$\begin{aligned} \dot{\tilde{e}}_x =& \dot{y}^r e_\psi + \psi^r e_{z_1} - e_{z_2} \\ \dot{e}_{z_1} =& e_u - z_2^r\dot{e}_\psi \\ \dot{\tilde{e}}_y =& -\dot{x}^r e_\psi + e_{z_1} + \psi^r e_{z_2} \\ \dot{e}_{z_2} =& z_1^r\dot{e}_\psi \end{aligned} \tag{13}$$

which can be divided in two sub-systems. Then, we obtain

$$\begin{aligned} \dot{\tilde{e}}_x =& \psi^r e_{z_1} + \dot{y}^r e_\psi - e_{z_2} \\ \dot{e}_{z_1} =& e_u - z_2^r\dot{e}_\psi \\ \dot{\tilde{e}}_y =& \psi^r e_{z_2} - \dot{x}^r e_\psi + e_{z_1} \\ \dot{e}_{z_2} =& z_1^r\dot{e}_\psi \end{aligned} \tag{14}$$

The writing Eq. (14) is considered as a perturbed system. The perturbation term results from e_{z_1} and e_{z_2}. We think of Eq. (14) as a perturbation of the nominal system

$$\begin{aligned} \dot{\tilde{e}}_x =& \psi^r e_{z_1} + \dot{y}^r e_\psi \\ \dot{e}_{z_1} =& e_u - z_2^r\dot{e}_\psi \\ \dot{\tilde{e}}_y =& \psi^r e_{z_2} - \dot{x}^r e_\psi \\ \dot{e}_{z_2} =& z_1^r\dot{e}_\psi \end{aligned} \tag{15}$$

One divides Eq. (15) in two disconnected nominal sub-systems. The first one is given by

$$\begin{aligned} \dot{\tilde{e}}_y =& \psi^r e_{z_2} - \dot{x}^r e_\psi \\ \dot{e}_{z_2} =& z_1^r\dot{e}_\psi \end{aligned} \tag{16}$$

and the second is as

$$\begin{aligned} \dot{\tilde{e}}_x &= \psi^r e_{z_1} + \dot{y}^r e_\psi \\ \dot{e}_{z_1} &= e_u - z_2^r \dot{e}_\psi \end{aligned} \tag{17}$$

The stability study of Eq. (16) is as follows: from sliding mode control theory, we constrain the error in yaw to the manifold $s = \dot{e}_\psi + \lambda e_\psi$ $(\lambda > 0)$. The variable s should tends to zero as time tends to infinity guarantees that $(e_\psi, \dot{e}_\psi)$ tends to zero at infinity and the rate of convergence is fixed by λ. In order to determine s, let us introduce this variable like a new controller into Eq. (16) where we suppose $e_\psi = 0$ that should be verified later.

$$\begin{aligned} \dot{\tilde{e}}_y &= \psi^r e_{z_2} \\ \dot{e}_{z_2} &= z_1^r s \end{aligned} \tag{18}$$

We have the following lemma.

Lemma 1 *The variable input $s = -\psi^r z_1^r(\tilde{e}_y + \psi^r e_{z_2})$ ensures the exponential stability of the nominal sub-system Eq. (18).*

Proof. Introduce s given by Lemma1 into Eq. (18) leads to $\ddot{\tilde{e}}_y + (\psi^r)^2(z_1^r)^2\dot{\tilde{e}}_y + (\psi^r)^2(z_1^r)^2\tilde{e}_y = 0$. The last equality means that $(\tilde{e}_y, \dot{\tilde{e}}_y)$ tends to zero as time tends to infinity. As a result e_{z_2} tends to zero, consequently $s \to 0$, meaning that $(e_\psi, \dot{e}_\psi) \to 0$. Then the nominal sub-system Eq. (16) is exponentially stable. This ends the proof.

One returns to the nominal sub-system Eq. (17), from results of Lemma1, we can write the following

$$\begin{aligned} \dot{\tilde{e}}_x &= \psi^r e_{z_1} \\ \dot{e}_{z_1} &= e_u \end{aligned} \tag{19}$$

Lemma 2 *For $e_u = -\psi^r(\tilde{e}_x + \dot{\tilde{e}}_x)$, the nominal sub-system Eq. (19) is exponentially stable.*

Proof. Introduce $e_u = -\psi^r(\tilde{e}_x + \dot{\tilde{e}}_x)$ in Eq. (19), we have $\ddot{\tilde{e}}_x + (\psi^r)^2\dot{\tilde{e}}_x + (\psi^r)^2\tilde{e}_x = 0$ since the roots of the polynomial characteristic have strictly negative a reality part then $\tilde{e}_x$ and $\dot{\tilde{e}}_x$ converge to zero.

Theorem 1 *Consider the perturbed system Eq. (14), Let $g(t,x) = (e_{z_1} \;\; 0 \;\; e_{z_2} \;\; 0)^T$ denotes the vector of perturbation where $x = (\tilde{e}_y \;\; e_{z_2} \;\; \tilde{e}_x \;\; e_{z_1})^T$ is the state of Eq. (14). Then the perturbed system is exponentially stable at the equilibrium.*

Proof. The exponential stabilities of the two nominal sub-systems are given by Lemma1 and Lemma2. Further, we have $g(t,0) = 0$ and $g(t,x)$ is smooth. From the fact that $\dot{g}(t,x)$ tends to zero as time tends to infinity, this guarantees the existence of a small enough γ such that $||g(t,x)|| \leq \gamma||x||$ (vanishing perturbation). One ends the proof from stability results of perturbed systems detailed in (see[16]).

Under the conditions of lemma 1 and lemma 2 we have $u = u^r + e_u$ by consequent u converge to u^r and $\psi = \psi^r + e_\psi$ by consequent ψ converge to ψ^r.

The backstepping approach and the sliding mode technique are combined to solve the tracking control problem. Once e_ψ and e_u are established such that the system above is asymptotically stable, one deduces $u = u^r + e_u$ and $\psi = \psi^r + e_\psi$. The reference inputs u^r and ψ^r are the solutions of the B-UAV reference dynamic. A satisfying tracking examples issue from simulations are depicted in figures 2. To success the tracking behavior constraints in the reference path are added.

III. Conclusions and Futur works

A model based planar behavior of the X4 bidirectionnel flyer vehicle was proposed. We have shown that the displacement of the vehicle can be asserted from the yaw attitude planning. In comparing these results with respect to the conventional X4 flyer, the orientation of the whole vehicle is not needed. A yaw-motion based controller combined with the sliding mode techniques ensure an exponentiel behavior of the tracking objectives. Our analysis for control feedback is interesting in the sens that leads to accelerations and not to velocities for control schemes. The simulation results sketch that the control inputs and yaw attitude references are well followed.

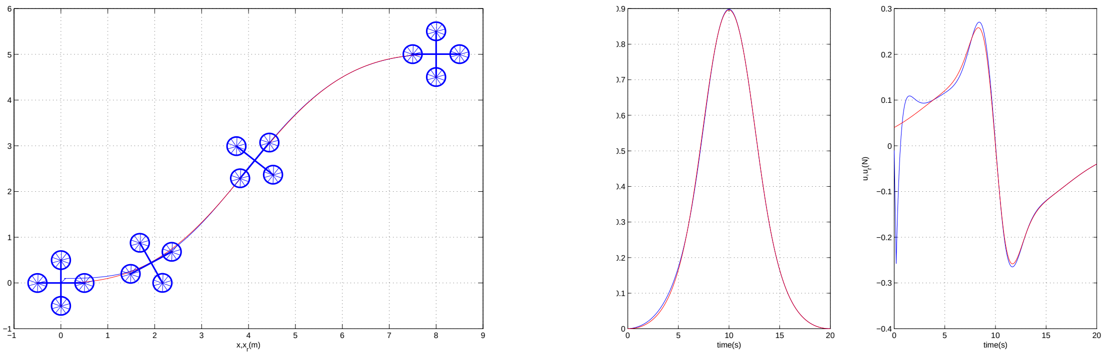

Figure 2. The reference and real trajectories (left) with the real and planned inputs (right).

Acknowledgments

Professor L. BEJI would like to thanks the Tunisian Minister of High Education Scientific Research and Technology for the invitation in within the framework of 2007-SERST program.

References

[1]Chriette, A., *Contribution la Commande et la Modlisation des Hlicoptres : Asservissement Visuel et Commande Adaptative*, Evry University'Thesis, 2001.

[2]Johnson, E.N., Kannan, S.H., *Adaptive Flight Control for an Autonomous Unmanned Helicopter*, AIAA Guidance, Navigation, and Control Conference and Exhibit 5-8 August, Monterey, California, 2002.

[3]Beji, L., Abichou, A., *Streamlined rotors mini rotorcraft: Trajectory generation and tracking*, Int. J. of Control, Automation, and Systems, 3(1), 87–99, 2005.

[4]Guenard, N., Hamel, T., Morceau, V., *Modlisation et laboration de commande de stabilisation de vitesse et de correction d'assiette pour un drone de type X4-Flyer*, CIFA'2004.

[5]Beji, L., Abichou, A., Zemalache, K.-M., *Smooth control of an x4 bidirectional rotors flying robot*, In Proc. of the Workshop RObot MOtion COntrol, Poland, 2005.

[6]Calise, A. J., Leitner, J., Prasad, J.V.R., *Analysis of adaptive neural networks for helicopter flight controls*, in AIAA J.of Guidance, Control, and Dynamics, 20(5), 972–979, 1997.

[7]Corban, J.E., Prasad, J.V.R., Calise, A.J., and Pei, Y., *Adaptive nonlinear controller synthesis and flight test evaluation on an unmanned helicopter*, in IEEE International Conference on Control Applications, 137–143, 1999.

[8]Ostrowski, J., Altug, E., and Mahony, R., *Control of a quadrotor helicopter using visual feedback*, in Proc. of the IEEE Conf. on Robotics and Automation,Washington DC. Virginia,USA, 72–77, 2002.

[9]Hynes, P., Pound, P., Mahony, R., and Roberts, J., *Design of a four rotor aerial robot*, in Proc. of the Australasian Conference on Robotics and Automation, Auckland, 145–150, 2002.

[10]Lozano, R., Hamel, T., Mahony, R., and Ostrowski, J., *Dynamic modelling and configuration stabilization for an x4-flyer*, in IFAC 15th World Congress on Automatic Control, Barcelona, Spain, 2002.

[11]Dzul A., Castillo, P., and Lozano, R., *Real-time stabilization and tracking of a four rotor mini-rotorcraft*, in IEEE Transactions on Control Systems Technology, 510–516, 2004.

[12]Sastry S., Hauser J., and Meyer, G., *Nonlinear control design for slightly nonminimum phase systems:application to v/stol aircraft*, in Journal Automatica, 28(4), 665–679, 1992.

[13]Martin, P., Fliess, M., Levine, J., and Rouchon, P., *Flatness and defect of nonlinear systems:introductory theory and examples*, in Int. Journal of Control, 61, 1327–1361, 1995.

[14]S.Devasia, P.Martin and B.Paden, *A different look at output tracking: Control of vtol aircraft*, in Journal Automatica, 61(1), 101–107, 1996.

[15]Murray, R.M. , Martin, P., and Rouchon, P., *Flat systems,equivalence and trajectory generation*, Technical report, Ecole des Mines de Paris, France, April 2003.

[16]Khalil-Hassen, K., *Nonlinear Systems*, Third Edition, Prentice-Hall, 2002.