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# Finite-time stabilization of interconnected nonlinear systems

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*Abstract*— **This paper deals with the finite-time stability and stabilization problems of interconnected nonlinear systems. We consider that there exist a finite-time stable controller for each isolated system for which is added a control part that preserves the systems finite-time stability. Sufficient conditions for finite-time stability are achieved, and permit to extend the asymptotic stability results presented in the literature. The procedure can be applied for a large variety of autonomous nonlinear multi-system with and without drift terms. The finite-time stabilizing-tracking control of kinematically nonholonomic model of multiple wheeled mobile robots is presented and illustrated.**

## I. INTRODUCTION

Many researchers tired to concentrate their works on asymptotic or exponential stability of interconnected systems, which contributes to provide information about the stability analysis of interconnected systems. For example, the string stability in [1] was described with linear interconnection. Further, necessary and sufficient conditions for stability of linear interconnected systems based on graph Laplacian matrix is presented in [2]. More general form of interconnected nonlinear systems [3] [4] are used instead of stability concepts because of the complexity of system models.

The finite-time stability of dynamical systems implies that trajectories converge to an equilibrium state in finite-time. With respect to the classical control theory, the finite-time stability theory is a more practical concept. Haimo [5] studied autonomous scalar systems and gives necessary and sufficient conditions for finite-time stability of the system's origin. Further, the stability problem in finite-time of nonautonomous systems was treated by several authors such as Orlov [7] for switched systems and Moulay in [6] proposed sufficient conditions using Lyapunov function. Haddad [11] provides Lyapunov and converse Lyapunov conditions for finite-time stability.The principal result of finite-time stability for homogeneous nonautonomous systems was obtained by Bhat in [10]. Sufficient conditions for finite-time stability of homogeneous and T-periodic systems are presented in [12],and where the averaging method has lead to a perturbed average system.

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The analysis of multi-system in group has witnessed a large and growing literature concerned with the coordination of multi-mobile autonomous agents including flocking and formation [13] [14] [16] [15]. In this area, the $i^{th}$ agent model is considered as a driftless subsystem, and taken kinematically as a first order ($\dot{x}_i = u_i$, $u_i$ is the input) or dynamically as a second order ($\ddot{x}_i = u_i$) leading to asymptotic or finite-time stability results.

As a purpose, we will propose to solve the finite-time stabilization of interconnected system where each individual kinematic/dynamic isolated system is nonlinear. Each isolated system is considered as finite-time stable with the associated Lyapunov function. Consequently, each nonlinear control law is constructed taking each system properties and should enables the formation's stability in finite-time. As a result, the stabilizing control procedure must be applied to wide nonlinear form of applications with/without drift terms. As an application, the finite-stability of nonholonomic wheeled multi-mobile robots is presented.

The paper is organized as follows: the second section is devoted to some preliminary mathematical results of finite-time stability and graph theory. In the third section, We present a class of interconnected nonlinear systems, and sufficient conditions for stability. Driftless interconnected systems and their stabilization in finite-time are presented in section IV. Section V deals with the tracking control problem of multiple kinematically non-holonomic wheeled mobile robots. To illustrate our results, numerical examples are simulated in section VI.

## II. MATHEMATICAL PRELIMINARIES

In this section, we introduce notations, definitions and present some results needed for the development of our main approach.

### A. Finite-time stability

Consider the system of differential equations

$$\dot{x}(t) = f(x(t)) \tag{1}$$

where $f : \mathcal{D} \to \mathbb{R}^n$ is continuous on an open neighborhood $\mathcal{D} \subseteq \mathbb{R}^n$ of the origin and $f(0) = 0$. We denote by $\psi^x(.)$ a solution of (1) satisfying $\psi^x(0) = x$.

*Definition 2.1:* [8]: The origin is said to be a finite-time stable equilibrium of (1) if there exists an open neighborhood $\mathcal{N} \subseteq \mathcal{D}$ of the origin and a function $T : \mathcal{N}\backslash\{0\} \to (0, \infty)$, called the settling-time function, such that the following statements hold:

(i) Finite-time convergence: For every $x \in \mathcal{N}\backslash\{0\}$, $\psi^x$ is defined on $[0, T(x))$, $\psi^x(t) \in \mathcal{N}\backslash\{0\}$ for all $t \in [0, T(x))$, and $\lim_{t \to T(x)} \psi^x(t) = 0$.

(ii) Lyapunov stability: For every open neighborhood $\mathcal{U}_\varepsilon$ of 0 there exists an open subset $\mathcal{U}_\delta$ of $\mathcal{N}$ containing 0 such that, for every $x \in \mathcal{U}_\delta\backslash\{0\}$, $\psi^x(t) \in \mathcal{U}_\varepsilon$ for all $t \in [0, T(x))$.

The origin is said to be a globally finite-time stable equilibrium if it is a finite-time stable equilibrium with $\mathcal{D} = \mathcal{N} = \mathbb{R}^n$.

Note that if the equilibrium of (1) is finite-time stable, then it is asymptotically stable,therefor finite-time stability is a stronger notion than asymptotic stability.

*Theorem 2.2:* [9]: Suppose there exists a continuous function $V : \mathcal{D} \to \mathbb{R}$ such that the following conditions hold:

(i) $V$ is positive definite.

(ii) $\dot{V}$ is continuous and negative on $\mathcal{D}\backslash\{0\}$.

(iii) There exist real numbers $k > 0$ and $\alpha \in (0, 1)$, and a neighborhood $\mathcal{V} \subset \mathcal{D}$ of the origin such that $\dot{V} + kV^\alpha \le 0$ on $\mathcal{V} \subset \mathcal{D}$.

Then, the origin is finite-time stable equilibrium of (1). Moreover, if $\mathcal{N}$ is as in definition 2.1, then $T(x) \le \frac{1}{k(1-\alpha)} V(x)^{1-\alpha}$ for all $x \in \mathcal{N}$.

### B. Graph theory

In this subsection, we introduce some basic concepts in algebraic graph theory for multi-agent networks. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a directed graph, where $\mathcal{V} = \{1, 2, ..., n\}$ is the set of nodes, node $i$ represents the $i$th agent, $\mathcal{E}$ is the set of edges, and an edge in $\mathcal{G}$ is denoted by an ordered pair $(i, j)$. $(i, j) \in \mathcal{E}$ if and only if the $i$th agent can send information to the $j$th agent directly.

$A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is called the weighted adjacency matrix of $\mathcal{G}$ with nonnegative elements, where $a_{ij} > 0$ if there is an edge between the $i$th agent and $j$th agent and $a_{ij} = 0$ otherwise. In this paper, we will refer to graphs whose weights take values in the set$\{0, 1\}$ as binary and those graphs whose adjacency matrices are symmetric as symmetric. Let $D = diag\{d_1, ..., d_n\} \in \mathbb{R}^{n \times n}$ be a diagonal matrix, where $d_i = \sum_{j=0}^{n} a_{ij}$ for $i = 1, ..., n$. Hence, we define the Laplacian of the weighted graph

$$L = D - A \in \mathbb{R}^{n \times n}$$

*Theorem 2.3:* [18] The Laplacian matrix $L$ of a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ has at least one zero eigenvalue and all of the nonzero eigenvalues are in the open right-half plane. In addition, $L$ has exactly one zero eigenvalue if and only if $\mathcal{G}$ has a directed spanning tree. Furthermore, Rank(L)=n if only if $L$ has a simple zero eigenvalue.

## III. Interconnected systems integrating drift terms: finite-time stabilization

Based on properties of each system and those of interconnections, sufficient conditions are given for finite-time stabilization of interconnected nonlinear systems. Let consider the dynamic of $N$ systems indexed by the set $\mathcal{I} = \{1, ..., N\}$, in algebraic form

$$\dot{x}_i = f_i(x_i) + \sum_{j=1}^{m} g_{i,j}(x_i) u_{i,j} \quad \forall i \in \mathcal{I} \tag{2}$$

where $x_i \in \mathbb{R}^n$ and the continuous maps $f_i : \mathbb{R}^n \to \mathbb{R}^n$ and $\forall 1 \le j \le m \; g_{i,j} : \mathbb{R}^n \to \mathbb{R}^m$. In matrix form, the $i^{th}$ system (2) leads to,

$$\dot{x}_i = f_i(x_i) + B(x_i) u_i \tag{3}$$

where $B(x_i) \in \mathbb{R}^{n \times m}$ and $u_i \in \mathbb{R}^m$.

In the following, we stay the main result of the paper, which gives sufficient conditions for finite-time stability of interconnected systems with drift.

The all system is defined by

$$\dot{x} = f(x) + (I_N \otimes B(x)) u \tag{4}$$

where $I_N$ is the identity matrix, $x \in \mathbb{R}^{Nn}$, $u \in \mathbb{R}^{Nm}$ and $f(x) = (f_1(x_1), ..., f_N(x_N))^T$

Interconnection in system (4) is the subject to design the control-input $u$ tacking the Laplacian $L$ related to a proposed graph $\mathcal{G}$.

*Proposition 3.1:* Let the interconnection control input given by

$$u(x) = -[L \otimes I_m][I_N \otimes C]x \tag{5}$$

where the matrix $C$ is in $\mathbb{R}^{m \times n}$. Under the control input (5), the origin of the interconnected system (4)is finite-time stable if the following assumptions hold:

(1) there exist $k \ge 0$ such that $x^T f(x) \le -k(x^T x)^\beta$ where $\beta \in ]0, 1[ \; \forall x \ne 0$.

(2) $x^T [L \otimes B(x_i) C] x \ge 0$.

□

**Proof.** Using the given feedback and the Kronecker product properties, in closed loop the dynamic of the interconnected systems is given by

$$\begin{aligned} \dot{x} &= f(x) + (I_N \otimes B)u \\ &= f(x) - (I_N \otimes B)[L \otimes I_m](I_N \otimes C)x \\ &= f(x) - [L \otimes BC]x \end{aligned} \tag{6}$$

Using the quadratic Lyapunov function $V(x) = \frac{1}{2} x^T x$ and evaluating the time derivative of $V$ along (6), we have

$$\begin{aligned} \dot{V} &= x^T f(x) - x^T [L \otimes BC]x \\ &\le -k(x^T x)^\beta - x^T [L \otimes BC]x \end{aligned}$$

Assumption 2 in proposition 3.1 implies that the second term in $\dot{V}$ is negative semi-definite, therefore

$$\begin{aligned} \dot{V} &\le -k(x^T x)^\beta \\ &\le -2^\beta k [V]^\beta \end{aligned}$$

Hence, the origin of the interconnected system (4) is finite-time stable.

■

*Remark 3.2:* Assumption (1) in proposition 3.1 describes the property of the subsystem which implies that each subsystem is finite-time stable with the appropriate quadratic Lyapunov function.

*Remark 3.3:* Assumptions (2) is equivalent to say that the matrix $L\otimes BC$ is positive semi-definite. Therefore, using the Kronecker product properties, $BC$ and $L$ are semi-definite positive matrix (or semi-definite negative matrix). Then, the feedback (5) depends on the choice of $\mathcal{G}$ and $C$.

## IV. Finite-time Stabilization of interconnected driftless systems

In this paragraph, we present an extension of the previous results for the finite-time stabilization of interconnected driftless systems. Hence, the control feedback is decomposed into two spots $u = u^{fts} + u^{inter}$, where $u^{fts}$ performs the finite-time stability of each isolated system and $u^{inter}$ ensures the interconnection objectives. Under the control-input $u$, in closed loop, the stabilization problems is reduced to a system with drift. To do, let consider the dynamic of $N$ systems indexed by the set $\mathcal{I} = \{1, ..., N\}$, in matrix form $\forall i \in \mathcal{I}$

$$\dot{x}_i = B(x_i)u_i \tag{7}$$

where $B(x_i) \in \mathbb{R}^{n\times m}$ and $u_i \in \mathbb{R}^m$.
and the driftless interconnected systems in the state vector $x = (x_1, ..., x_N)^T$ is written as,

$$\dot{x} = (I_N \otimes B(x_i))u \tag{8}$$

with $u \in \mathbb{R}^{Nm}$.

*Proposition 4.1:* Let subdivide the control $u$ into two parts

$$u = u^{fts} + u^{inter} \tag{9}$$

such that

$$x^T(I_N \otimes B(x_i))u^{fts}(x) \leq -k(x^Tx)^\beta, \tag{10}$$

$$and \quad u^{inter}(x) = -[L\otimes I_m][I_N \otimes C]x \tag{11}$$

whith $k \geq 0$, $\beta \in ]0, 1[$ and the matrix $C \in \mathbb{R}^{m\times n}$.
Assume that assumption (2) in Proposition 3.1 holds. Under (10) and (11), the interconnected system (8) origin is finite-time stable.

□

**Proof.** The dynamic of the interconnected driftless systems is given by

$$\begin{aligned}\dot{x} &= (I_N \otimes B)u^{fts}(x) + (I_N \otimes B)u^{inter}(x)\\ &= f(x) - (I_N \otimes B)[L\otimes I_m](I_N \otimes C)x\\ &= f(x) - [L\otimes BC]x\end{aligned} \tag{12}$$

where $f(x) = (I_N \otimes B)u^{fts}(x)$.
Taking $V = \frac{1}{2}x^Tx$ the proof steps are similar to Proposition 3.1.

■

## V. Finite-time tracking control of nonholonomic wheeled mobile robots

The tracking control problem of non-holonomic wheeled mobile robots has received great attention. Different methods are proposed to solve the tracking control problem, such as the work of Samson [20] that proposes a global tracking control results. Also, in [21] the tracking control is developed leading to exponentially stability results of errors. Further, based on perturbed systems theory, Li in [17] proposed a finite-time stability result for a single wheeled mobile robot. Given the quadratic form of a Lyapunov function, our aim is to find adequate control inputs that success a $N$ wheeled mobile robots formation. Hence, our approach is different from Li [17], but is restricted to the quadratic form of Lyapunov's stability. The asymptotic result proposed by Morin [22] initiates our finite-time approach for the tracking control. In the subsequent section, we develop our preliminary result for a single unicycle that will be conducted to the multiple unicycle system.

### A. Finite-time tracking of one unicycle

The tracking trajectory problem is presented as follows: given an eligible trajectory $q_r(t)$ for $t \geq 0$ and define the error $q_e = q - q_r$, the goal is to find a state feedback control $u(q, q_r, t)$ such that the origin of $\dot{q}_e = f(q, u(q, q_r, t)) - f(q_r, u^r)$ is finite-time stable.

Recall that the kinematically nonholonomic model of the unicycle is given by :

$$\begin{aligned}\dot{x} &= u_1\cos(\theta)\\ \dot{y} &= u_1\sin(\theta)\\ \dot{\theta} &= u_2\end{aligned} \tag{13}$$

where $(x, y)$ denotes the center of mass coordinates, $\theta$ is the angle between the heading direction and the $x$ axis, and inputs $u_1$ and $u_2$ are the linear and angular velocities, respectively. For the stabilizing problem meaningful, we must first characterize the achievable trajectories, one way to do this is to consider the model. The $(x_r(t), y_r(t))$ should verifies

$$\begin{aligned}\dot{x}_r &= u_{1,r}\cos(\theta_r)\\ \dot{y}_r &= u_{1,r}\sin(\theta_r)\\ \dot{\theta}_r &= u_{2,r}\end{aligned} \tag{14}$$

Let us define the trajectory tracking errors $e = (x - x_r, y - y_r, \theta - \theta_r)$. The time derivative of $e$ is given by:

$$\begin{aligned}\dot{e}_{1,r} &= u_{2,r}e_{2,r} + u_1\cos(e_{3,r}) - u_{1,r}\\ \dot{e}_{2,r} &= -u_{2,r}e_{1,r} + u_1\sin(e_{3,r})\\ \dot{e}_{3,r} &= u_2 - u_{2,r}\end{aligned} \tag{15}$$

where $e_{3,r} \in ]-\pi/2, \pi/2[$. A change of coordinates and control variables lead to

$$\begin{aligned} z_1 &= e_{1,r} \\ z_2 &= e_{2,r} \\ z_3 &= \tan e_{3,r} \\ w_1 &= u_1 \cos(e_{3,r}) - u_{1,r} \\ w_2 &= \frac{u_2 - u_{2,r}}{\cos^2(e_{3,r})} \end{aligned} \quad (16)$$

Taking $w_1$ and $w_2$ as new inputs, from (15), we obtain :

$$\begin{aligned} \dot{z}_1 &= u_{2,r} z_2 + w_1 \\ \dot{z}_2 &= u_{2,r} z_1 + u_{1,r} z_3 + w_1 z_3 \\ \dot{z}_3 &= w_2 \end{aligned} \quad (17)$$

*Proposition 5.1:* Under the following tracking control laws

$$w_1 = -|u_{1,r}|(sign(z_1)|z_1|^\alpha + z_2 z_3) \quad (18)$$

$$\begin{aligned} w_2 = &-|u_{1,r}|[z_2 - sign(z_1)|z_1|^\alpha + (z_1 z_2 + sign(z_1)|z_1|^\alpha z_2 \\ &+ z_2^2 z_3 - \frac{1}{arctan(\varepsilon)} sign(z_3)|z_2|^{\alpha+1})] \end{aligned} \quad (19)$$

the origin of (17) is finite-time stable, consequently, the system (13) tracks the reference (14) in finite-time whith $\varepsilon > 0$, $e_{3,r} > \varepsilon$ and $\alpha \in ]0, 1[$.

**Proof**. Let us take the quadratic Lyapunov function

$$V(z_1, z_2, z_3) = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2)$$

The time derivative of $V$ through system (17) leads to

$$\begin{aligned} \dot{V} &= z_1 \dot{z_1} + z_2 \dot{z_2} + z_3 \dot{z_3} \\ &= z_1 w_1 + z_2 z_3 w_1 + z_3 w_2 \\ &= -|u_{1,r}|((|z_1|^{\alpha+1} + |z_2|^{\alpha+1} + |z_2|^{\alpha+1}) \\ &= -|u_{1,r}| \sum_{i=1}^{3} |z_i|^{\alpha+1} \end{aligned} \quad (20)$$

View the convexity of the function $z_i \mapsto z_i^{\alpha+1}$, we have

$$(\sum_{i=1}^{3} \frac{z_i}{3})^{\alpha+1} \leq \frac{1}{3} \sum_{i=1}^{3} z_i^{\alpha+1}$$

Also this inequality holds

$$(\sum_{i=1}^{3} z_i)^{\alpha+1} \geq (\sum_{i=1}^{3} z_i^2)^{\frac{\alpha+1}{2}}$$

The above inequalities with (20) permit to write

$$\dot{V} \leq -\frac{|u_{1,r}|}{3^\alpha} 2^{\frac{\alpha+1}{2}} V^{\frac{\alpha+1}{2}}$$

As the tracking problem is reduced to a stabilizing one, then from the inequality above we can conclude that systems (17) origin is finite-time stable. This ends the proof.

### B. Finite-time tracking of multi-unicycle

We now present a direct application of Proposition 4.1. Consider a set of $N$ unicycles where for $i \in \{1, ..., N\}$ the $i^{th}$ unicycle model is given by (14):

$$\dot{q}_i = B(q_i) u_i \quad (21)$$

with $\forall i \in \{1, ..., N\}$, $q_i = (x_i, y_i, \theta_i)^T$, $u_i = (u_{1,i}, u_{2,i})^T$ and

$$B(q_i) = \begin{pmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{pmatrix}.$$

Each unicycle's reference model is shown by (14), hence the $i^{th}$ system of errors is deduced from (17),

$$\begin{aligned} \dot{z}_{1,i} &= u_{2,r} z_{2,i} + w_{1,i} \\ \dot{z}_{2,i} &= u_{2,r} z_{1,i} + u_{1,r} z_{3,i} + w_{1,i} z_{3,i} \\ \dot{z}_{3,i} &= w_{2,i} \end{aligned} \quad (22)$$

Let consider a Laplacian matrix $L$ that describes the graph connection between all unicycles, and $C$ is in $\mathbb{R}^{2\times3}$ such that proposition 3.1 assumption 2 is verified.

*Proposition 5.2:* consider the following

$$w_{1,i}^{fts} = -|u_{1,r}|(sign(z_{1,i})|z_{1,i}|^\alpha + z_{2,i} z_{3,i}) \quad (23)$$

$$\begin{aligned} w_{2,i}^{fts} = &-|u_{1,r}|[z_{2,i} - sign(z_{1,i})|z_{1,i}|^\alpha \\ &+ (z_{1,i} z_{2,i} + sign(z_{1,i})|z_{1,i}|^\alpha z_{2,i} \\ &+ z_{2,i}^2 z_{3,i} - \frac{1}{\arctan(\varepsilon)} sign(z_{3,i})|z_{2,i}|^{\alpha+1})] \end{aligned} \quad (24)$$

such that $w_i^{fts} = (w_{1,i}^{fts}, w_{2,i}^{fts})^T$ and for $z_i = (z_{1,i}, z_{2,i}, z_{3,i})^T$ let $w_i^{inter} = -\sum_{j=1}^{N} a_{ij} C(z_i - z_j)$ under the control vector $w_i = w_i^{fts} + w_i^{inter}$, the system (22) origin is finite-time stable. $a_{ij}$ is the adjacency matrix element.

□

**Proof**. With respect to the first part of $w_i^{fts}$, system (22) is written in the form given by (3):

$$z_i = f_i(z_i) + B(z_i) w_i^{inter}$$

with $z_i = (z_{1,i}, z_{2,i}, z_{3,i})^T$, $B(z_i) = \begin{pmatrix} 1 & 0 \\ z_{3,i} & 0 \\ 0 & 1 \end{pmatrix}$ and $f_i(z_i) = (u_{2,r} z_{2,i}, u_{2,r} z_{1,i} + u_{1,r} z_{3,i}, 0)^T + B(z_i) w_i^{fts}$. Based on Proposition 5.1 proof, assumption (1) in Proposition 3.1 is verified with $k = \frac{|u_{1,r}|}{3^\alpha} 2^{\frac{\alpha+1}{2}}$ and $\beta = \frac{\alpha+1}{2}$.

### C. Simulation results

A set of $N = 4$ unicycles is considered. Following to the graph $\mathcal{G}$ in Fig.1, for one unicycle the matrix $C$ and the Laplacian matrix $L$ are given by :

$$C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad L = \begin{pmatrix} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 3 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

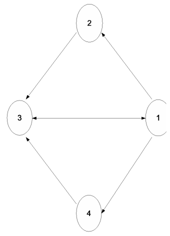


Fig. 1. $\mathcal{G}$ for a system with 4 unicycles.

The control parameter is taken $\alpha = 0.5$. Each unicycle initial positions is given by (meters):

$$(x_1, x_2, x_3, x_4)(t = 0) = (-3, -2, -1, -4)$$

$$(y_1, y_2, y_3, y_4)(t = 0) = (0, 0, 0, 0)$$

and the initial heading angles are such that (in radian)

$$(\theta_1, \theta_2, \theta_3, \theta_4)(t = 0) = (\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{3\pi}{4}, \frac{3\pi}{4})$$

The initial position of the reference is $(x_r, y_r, \theta_r)(t = 0) = (0, 0, 0)$. The reference velocities are as

$$u_{1,r} = 1m.s^{-1}, \quad u_{2,r} = \cos(t) - \frac{1}{4}rad.s^{-1}.$$

Figures Fig.2-4 sketch errors in term of $z_i$ variables. These also images of the multi-unicycle position errors, hence this confirm the stability results. Consequently, the controller $w_{1,i}$ (Fig.7) and $w_{2,i}$ (Fig.8) realize the objectives and tend to zero in finite-time. Further, behaviors in tracking of the multi-unicycle system is presented by figure Fig.9, and the corresponding velocity controllers are shown by figures Fig.5-6. Consequently, the velocity references are reached in finite time.

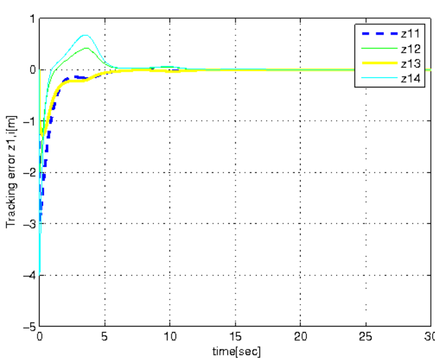


Fig. 2. Tracking errors $z_{1,i}$

## VI. Conclusion

The finite-time stability problem of interconnected nonlinear systems was solved using Lyapunov quadratic functions and the Graph theory. We have provided sufficient conditions for finite-time stability of some nonlinear classes and may be structurally different systems including interconnections. Interconnections were suggested to be as an additional term

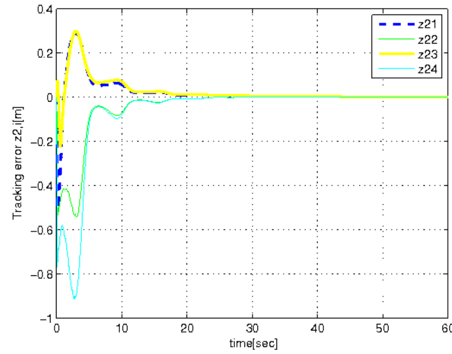


Fig. 3. Tracking errors $z_{2,i}$

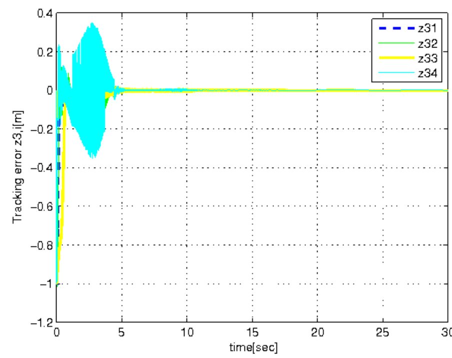


Fig. 4. Tracking errors $z_{3,i}$

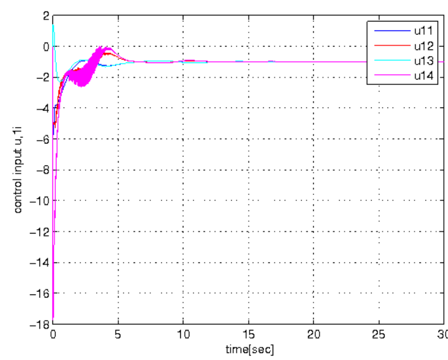


Fig. 5. control input $u_{1,i}$

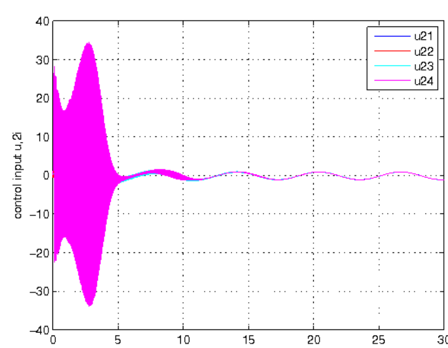


Fig. 6. control input $u_{2,i}$

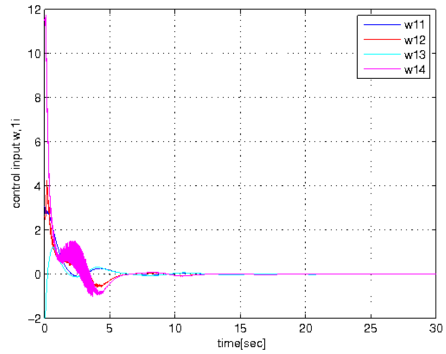


Fig. 7. control input $w_{1,i}$

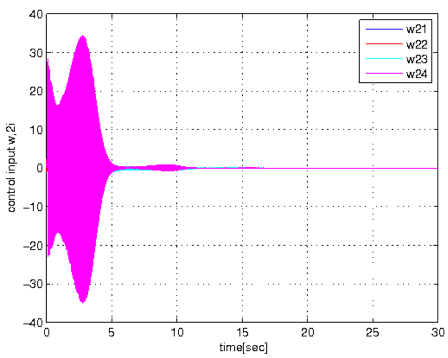


Fig. 8. control input $w_{2,i}$

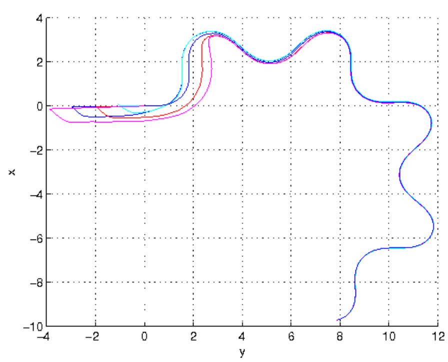


Fig. 9. The phase plot on plane

in the controller and had showed to preserve the system finite-time stability and lead to behaviors like-consensus. Finite-time stability including finite-time consensus of heterogeneous systems in formation is a perspective of this work.

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