Yaw-based Control of a X4-bidirectional Flyer Planar Motion

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Abstract: In this paper we treat the problem of the planar tracking motion of a particular structure of a four rotors mini-flying robot where two rotors are directional (bidirectional). Thus, the yaw angle is used to control the planar movement instead off inclining the engine. This last technique is usually adopted in the literature for this kind of aerial vehicle. An explicit time-varying feedback law is proposed for the bidirectional system that fails the Brockett'conditions. In order to solve the tracking problem, we propose a point-to-point path steering considering the system flat output. The equations of motion are dynamically represented and partially controlled by the yaw angle. Theoretical results are supported by an important part of analysis in simulation.
Key-Words: - Planar motion, Stabilization, Tracking, flatness

1. INTRODUCTION

Unmanned Aerial Vehicles (UAV) terrain control is a matter of both interest for scientific research and military control constraint consist in planning and following predefined trajectories. Examples range from unmanned and remotely piloted airplanes and submarines performing surveillance and inspection, mobile robots moving on factory floors and multi-fingered robot hands performing inspection and manipulation inside the human body under a surgery control. All these systems are highly nonlinear and require accurate performance. The Modeling and control of aerial vehicles were developed for blimps [1] and mini rotor-crafts (X4-flyer) [2, 3]. The industrial technical characteristics of the mini-UAV presented in this paper should respect 2 kg in mass, a wingspan of 50 cm and with a 30 mn flying-time. It is an autonomous hovering system, capable of vertical take-off, landing, lateral motion and quasi-stationary (hover or near hover) flight conditions. Compared to helicopters, named quad-rotor,[4, 5, 6, 7] the four-rotor rotorcraft has some advantages[8]: given that two motors rotate counter clockwise while the other two rotate clockwise, gyroscopic effects and aerodynamic torques tend, in trimmed flight, to be canceled. A model for the dynamic and configuration stabilization of quasi-stationary fight conditions of a four rotor vertical take-off and landing (VTOL) was studied by [8] where the dynamic motor effects are incorporating and a bound of perturbing errors was obtained for the coupled system. The stabilization problem of a four rotor rotorcraft is also presented in [9] where the nested saturation algorithm is considered. With the intent to stabilize aircrafts that are able to take-off vertically as helicopters, the control problem was solved for the planar vertical take-off and landing (PVTOL) with the input/output linearization procedure [10] and theory of flat systems [11, 12, 13]. An X4 bidirectional rotors mini-flyer operates as a omnidirectional UAV. Vertical motion is controlled by collectively increasing or decreasing the power for all motors. Lateral motion, in x-direction or in y-direction, is not achieved by differentially controlling the motors generating a pitching/rolling motion of the airframe that inclines the collective thrust (producing horizontal forces) and leads to lateral accelerations (case of the X4-flyer). But, two rotors are directional introduce two internal degrees of freedom used to permute between the x/y motion. This work completes the paper presented in [3] where, to overcome singularities, we presented a continuous variable structure controller including path planning. The x/y motion considered as a planar movement needs to stabilize the aerial vehicle in pitch and roll. Hence, two independent controllers can be easily established as the dynamics in pitch and roll are in the form of a decoupled second order differential equations: $\tau_\theta = \ddot{\theta}$ and

$\tau_\phi = \ddot{\phi}$ [2]. We consider the planar movement and only the yaw based control will steer the system between the two directions. We show that path planning for the yaw motion steers the system along the reference path.
The paper is organized as following. Section 2 deals the planar model and the necessary considered variables. The *STLC* is investigated in section 3 where we prove that the system fails Brockett'conditions, thus, the necessity of the time varying controller. In order to stabilize the equilibrium, in section 4, we propose a control algorithm based on the augmented system. Section 5 details the problem of the tracking with path planning based on the system flat output. Finally, we conclude the paper with simulations and comments.

2. EQUATION OF MOTION

The X4-bidirectional aerial vehicle is minimum in size consisting of four individual electrical fans attached to a rigid bar. Two of them can be oriented by an electric servo-mechanism. This makes the system different of a conventional X4-flyer.
We consider a local reference airframe $\Re_G = \{G, E_1^g, E_2^g, E_3^g\}$ attached to the center of mass G of the vehicle. The center of mass is located at the intersection of the two rigid bars, each of which supports two motors. Equipment (controller cartes, sensors, etc.) onboard are placed not far from G. The inertial frame is denoted by $\Re_o = \{O, E_x, E_y, E_z\}$ such that the vertical direction E_z is upwards. Let the vector $\xi = (x, y, z)$ denote the position of the center of mass of the airframe in the frame $\Re_o$. While the rotation of the rigid body is determined by a rotation $R : \Re_G \rightarrow \Re_o$, where $R \in SO(3)$ is an orthogonal rotation matrix. This matrix is defined by the three Euler angles, θ(pitch), ϕ(roll) and ψ(yaw) which are regrouped. A sketch of the X4-bidirectional is given in Fig1 and Fig.2. In the following we recall only equations due to translations and the attitude yaw dynamic. The reader can refer to [2] for further details in modeling.

$$
\begin{aligned}
m\ddot{x} &= u\sin(\psi) \\
m\ddot{y} &= u\cos(\psi) \\
m\ddot{z} &= mg - v \\
\ddot{\psi} &= \tau_\psi
\end{aligned}
\tag{1}
$$

where $(x, \dot{x}, y, \dot{y}, z, \dot{z}, \psi, \dot{\psi})^t \in \mathbb{R}^8$ is the state, $(u, v, \tau_\psi)^t \in \mathbb{R}^3$ is the control vector. The lift (collective) force v and the direction input u combine the principal non conservative forces applied to the system including forces generated by the motors and drag terms. Drag forces and gyroscopic due to motors effects are not considered. Finally, τ_ψ denotes the torque input will control the yaw motion.

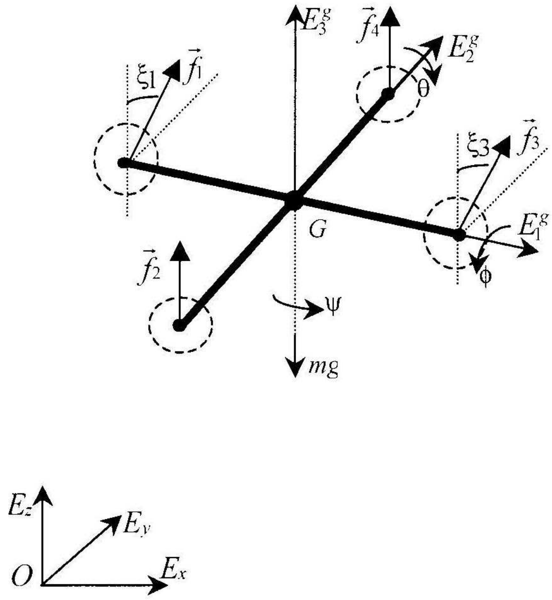

Figure 1: Frames considered in the modeling.

3. LOCAL CONTROLLABILITY INVESTIGATION

In the following we show that the system (1) is *Small Time Locally Controllable* (STLC) in the neighborhood of the equilibrium. First, we introduce the equivalent system

$$
\begin{aligned}
\dot{x} &= x_1 \quad ; \quad \dot{x}_1 = \frac{1}{m} u \sin(\psi) \\
\dot{y} &= y_1 \quad ; \quad \dot{y}_1 = \frac{1}{m} u \cos(\psi) \\
\dot{z} &= z_1 \quad ; \quad \dot{z}_1 = g - \frac{1}{m} v \\
\dot{\psi} &= w \quad ; \quad \dot{w} = \tau_\psi
\end{aligned}
\tag{2}
$$

By adding an integrators, we obtain the augmented system

$$
\begin{aligned}
\dot{x} &= x_1 \quad ; \quad \dot{x}_1 = \frac{1}{m} \alpha \sin(\beta) \\
\dot{y} &= y_1 \quad ; \quad \dot{y}_1 = \frac{1}{m} \alpha \cos(\beta) \\
\dot{\alpha} &= u \quad ; \quad \dot{\beta} = \psi
\end{aligned}
\tag{3}
$$

Since the behavior of z and ψ can be achieved respectively by v and τ_ψ, then, these dynamics are omitted. For the augmented system above u and ψ are considered as new input variables.

Thus, we have the following result
Proposition. The system (3) is STLC near the equilibrium.

Proof. The new system (3) can be rewritten in compact form as

$$\dot{X} = f_0(X) + uf_1(X) + \psi f_2(X) \tag{4}$$

where $X = (x, x_1, y, y_1, \alpha, \beta)^t \in \mathbb{R}^6$ is the vector of states and $U = (u, \psi)^t \in \mathbb{R}^2$ is the control vector.
Further, we have

$$\begin{aligned} f_0(X) =&(x_1, \frac{1}{m}\alpha sin(\beta), y_1, \frac{1}{m}\alpha\cos(\beta), 0, 0)^t \\ f_1(X) =&(0, 0, 0, 0, 1, 0)^t \\ f_2(X) =&(0, 0, 0, 0, 0, 1)^t \end{aligned}$$

It is clear that vector fields $f_0(X)$, $f_1(X)$ and $f_2(X)$ are real-analytic.
Let's compute the different Lie Brackets, we have

$$\begin{aligned} [f_1, f_0] =&(0, \frac{1}{m}\sin(\beta), 0, \frac{1}{m}\cos(\beta), 0, 0)^t \\ [f_2, [f_1, f_0]] =&(0, \frac{1}{m}\cos(\beta), 0, -\frac{1}{m}\sin(\beta), 0, 0)^t \\ [[f_1, f_0], f_0] =&(\frac{1}{m}\sin(\beta), 0, \frac{1}{m}\cos(\beta), 0, 0, 0)^t \\ [f_2, [[f_1, f_0], f_0]] =&(\frac{1}{m}\cos(\beta), 0, -\frac{1}{m}\sin(\beta), 0, 0, 0)^t \end{aligned}$$

Then, after computing the different Lie brackets, we obtain

$$\begin{aligned} B(X) =&span\{f_1, f_2, [f_1, f_0], [f_2, [f_1, f_0]], [[f_1, f_0], f_0], \\ &[f_2, [[f_1, f_0], f_0]]\}(X) \end{aligned}$$

which has dimension 6 for every $X \in \mathbb{R}^6$. Thus, the strong accessibility rank condition is satisfied and consequently, the system (S) is locally strongly accessible for all $X \in \mathbb{R}^6$. Furthermore, it is straightforward to prove that system (4) is small time locally controllable in neighborhood of the equilibrium within the meaning of Sussmann [14].

4. STABILIZATION PROBLEM

The Brockett necessary condition related to static stabilization is addressed in the follows (more explanations are in [15]).

Proposition 1. *The system (3) cannot be stabilized by a static smooth feedback law.*
Proof. Based on the Brockett's result, we prove that we cannot stabilize the model with a stationary continuous control. In fact, for any point under the form $X_c = (0, 0, 0, 0, 0, \varepsilon)$ in the neighborhood of $0_{\mathbb{R}^n}$, where $\varepsilon \neq 0$ is not part of the image set.
In the following proposition we develop a continuous time-varying feedback law (more explanations are given in [16]).

Proposition 2. *Consider the following time-varying feedback law*

$$\begin{aligned} \alpha_d =&2m\rho(x)\sin(\frac{t}{\varepsilon}) - 2m(y + y_1) \\ \beta_d =& - 2\frac{\sin(\frac{t}{\varepsilon})}{\rho(x)}(x + x_1) \\ u =& - k_1(\alpha - \alpha_d) + \dot{\alpha}_d \\ \psi =& - k_2(\beta - \beta_d) + \dot{\beta}_d \end{aligned} \tag{5}$$

with $\rho(x) = (x^2 + x_1^2 + y^2 + y_1^2)^{\frac{1}{2}}$. Then for a suitable choice of positive parameters (k_1, k_2), there exists ε_0 such that for any $\varepsilon \in (0, \varepsilon_0)$ and large enough (k_1, k_2) the feedback defined above stabilizes locally-exponentially the system (3). ε is a parameter that we need to adjust.

Proof. The initial system can be rewritten in compact form as following

$$\dot{X} = f(X, t)$$

where

$$X = (x, x_1, y, y_1, \alpha, \beta)^t \in \mathbb{R}^6$$

and

$$f(X, t) = (x_1, \frac{1}{m}\alpha\sin(\beta), y_1, \frac{1}{m}\alpha\cos(\beta), u, \psi)^t \in \mathbb{R}^6$$

The associated linearized model is obtained

$$\begin{aligned} \dot{x} =&x_1 \quad ; \dot{x_1} = \frac{1}{m}\alpha\beta \\ \dot{y} =&y_1 \quad ; \dot{y_1} = \frac{1}{m}\alpha \\ \dot{\alpha} =&u \quad ; \dot{\beta} = \psi \end{aligned} \tag{6}$$

The analysis consists to take part of $u = -k_1(\alpha - \alpha_d) + \dot{\alpha}_d$ and $\psi = -k_2(\beta - \beta_d) + \dot{\beta}_d$ which ensure that $\alpha \to \alpha_d$ and $\beta \to \beta_d$ as time go to ∞. Therefore, in closed loop

$$\begin{aligned} \dot{x} =&x_1 \quad ; \dot{x_1} = \frac{1}{m}\alpha_d\beta_d \\ \dot{y} =&y_1 \quad ; \dot{y_1} = \frac{1}{m}\alpha_d \end{aligned} \tag{7}$$

Due the periodic time-invariant control, the resulting system is also a periodic time-invariant system which can be written in the form:

$$\dot{X} = h(X, \frac{t}{\varepsilon}) = h_0(X) + g_1(\frac{t}{\varepsilon})h_1(X) + g_2(\frac{t}{\varepsilon})h_2(X)$$

where $h(X, t) = (x_1, \frac{1}{m}\alpha_d\beta_d, y_1, \frac{1}{m}\alpha_d)$

is homogeneous of degree zero with respect to the dilation

$$\delta_\lambda(X,t) = (\lambda x, \lambda x_1, \lambda y, \lambda y_1, t)$$

further

$$\begin{aligned} h_0(X) =& (x_1, -2(x+x_1), y_1, -2(y+y_1)) \\ h_1(X) =& (0, \frac{4}{\rho(X)}(x+x_1)(y+y_1), 0, \frac{2}{\rho(X)}) \\ h_2(X) =& (0, -4(x+x_1), 0, 0) \\ g_1(t) =& \sin(t) \\ g_2(t) =& \sin^2(t) - \frac{1}{2} \end{aligned} \tag{8}$$

are continuous 2π-periodic functions such that $\int_0^{2\pi} g_i(\tau)d\tau = 0$. We approximate this system by an averaged system which is autonomous. The averaged system is defined as

$$\begin{aligned} \dot{x} =& x_1 \\ \dot{x}_1 =& -2(x+x_1) \\ \dot{y} =& y_1 \\ \dot{y}_1 =& -2(y+y_1) \end{aligned} \tag{9}$$

Now it is straightforward to prove the exponential stability of the averaged system origin. Consequently, the origin of the initial system is asymptotically stable (more theoretical analysis is in [**?**]).

5. TRACKING WITH POINT-TO-POINT STEERING

In this section we solve the tracking problem using the flat output of the system. More explanation about flatness theories can be found in [11, 12, 13].
For a given smooth reference trajectory (x^r, y^r), the model (10) takes this form, where the subscript r denotes the reference.

$$\begin{aligned} m\ddot{x}^r =& u^r \sin(\psi^r) \\ m\ddot{y}^r =& u^r \cos(\psi^r) \\ \ddot{\psi}^r =& \tau_\psi^r \end{aligned} \tag{10}$$

where the dynamic of z is omitted. Recall that the objective is to design the controllers u and ψ that ensure the reference trajectory tracking. We introduce $u = u^r + \Delta u$ and $\psi = \psi^r + \Delta\psi$.
In the neighborhood of the reference trajectory and their derivatives (considered smooth), the system (10) is transformed to the following matrix form

$$m \begin{pmatrix} \ddot{\widehat{\Delta x}} \\ \ddot{\widehat{\Delta y}} \end{pmatrix} = \begin{pmatrix} cos(\psi^r) & -u^r sin(\psi^r) \\ sin(\psi^r) & u^r cos(\psi^r) \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta\psi \end{pmatrix} \tag{11}$$

The matrix $(2x2)$ in (11) is invertible if $u^r \neq 0$. This imposes a constraint in the reference trajectory which should be taken into account. In the following we consider $u^r \neq 0$, hence from (11) we propose the feedback linearized control law

$$\begin{pmatrix} \Delta u \\ \Delta\psi \end{pmatrix} = \frac{1}{u^r} \begin{pmatrix} u^r cos(\psi^r) & u^r sin(\psi^r) \\ -sin(\psi^r) & cos(\psi^r) \end{pmatrix} \begin{pmatrix} \nu_x \\ \nu_y \end{pmatrix} \tag{12}$$

where ν_x and ν_y are the new inputs for the new system

$$\begin{aligned} \ddot{\widehat{\Delta x}} &= \nu_x \\ \ddot{\widehat{\Delta y}} &= \nu_y \end{aligned} \tag{13}$$

For the appropriate chose of the gain parameters $(k_i^x, k_i^y)_{i=1,2}$, the following inputs ensure that Δx and Δy tend to zero asymptotically.

$$\begin{aligned} \nu_x &= -k_1^x \dot{\widehat{\Delta x}} - k_2^x \Delta x \\ \nu_y &= -k_1^y \dot{\widehat{\Delta y}} - k_2^y \Delta y \end{aligned} \tag{14}$$

Proposition. *The following system*

$$\begin{aligned} m\ddot{x} =& u\sin(\psi) \\ m\ddot{y} =& u\cos(\psi) \end{aligned} \tag{15}$$

is locally asymptotically stable under the controllers

$$\begin{aligned} u =& u^r + \Delta u \\ \psi =& \psi^r + \Delta\psi \end{aligned} \tag{16}$$

where $u^r = \pm\sqrt{(\ddot{x}^r)^2 + (\ddot{y}^r)^2}$ *and* $\psi^r = \arctan(\ddot{x}^r/\ddot{y}^r)$. *The approximated errors* Δu *and* $\Delta\psi$ *are in (14) through out (12).*

Remark. In order to circumvent the singularity due to u^r who is present in $\Delta\psi = \frac{1}{u^r}(-sin(\psi^r)\nu_x + cos(\psi^r)\nu_y)$, we suggest to take in consideration $\tau_\psi = \tau_\psi^r - k_1^\psi \dot{\widehat{\Delta\psi}} - k_2^\psi \Delta\psi$. In closed loop, this leads to $\ddot{\widehat{\Delta\psi}} + k_1^\psi \dot{\widehat{\Delta\psi}} + k_2^\psi \Delta\psi = 0$ with $k_1^\psi, k_2^\psi > 0$. The residue in $\Delta\psi$ can be solved from there. Recall that from the reference path $\tau_\psi^r = \ddot{\psi}^r$.

Figure 2: Conceptual form of the X4 Super-Flyer.

6. PATH PLANNING AND SIMULATION RESULTS

In the case of the point to point steering, the following reference trajectory is introduced.

$$x^r(t) = y^r(t) = p_d \frac{t^2}{t^2 + (T_f - t)^2} \quad (17)$$

where p_d is the desired position and T_f is the necessary final time to reach the point. Constraints to perform each trajectory

$$x^r(0) = y^r(0) = 0; \;\; x^r(T_f) = y^r(T_f) = p_d$$
$$\dot{x}^r(0) = \dot{y}^r(0) = 0; \;\; \dot{x}^r(T_f) = \dot{y}^r(T_f) = 0$$
$$\ddot{x}^r(0) = \ddot{y}^r(0) = 0; \;\; \ddot{x}^r(T_f) = \ddot{y}^r(T_f) = 0$$

The X4-bidirectional flyer robot development and equipments are in progress in our laboratory. The tests are envisaged in the future. The total mass of the drone is $m = 2kg$. The technical characteristics of the flying vehicle were presented in [2] (see Fig.2). Fig.3 shows the stabilization of the origin with the time varying controller. The X4-flyer seeks to stabilize at the origin. Note that the initial configuration influences the response behavior as well as the energy consumption. An example is given in Fig.4 where the behavior of the controller is very oscillating. The importance of the stabilizing problem with the point to point steering (tracking) is that it permits to control the amplitude of the controller as well as the orientation of the aerial vehicle. Therefore we have considered a regular reference trajectory respecting the physical limits of the system. Results are shown in Fig.5,6. The acceleration at the beginning and deceleration when the X4-flyer reaches the objective are considered in the path planning.

7. CONCLUSION

The aim of this work was to steer the X4-bidirectional flying robot using the yaw attitude and the two inclined rotor forces both in the $x - y$ directions or simultaneously. The model describing the dynamic of the aerial

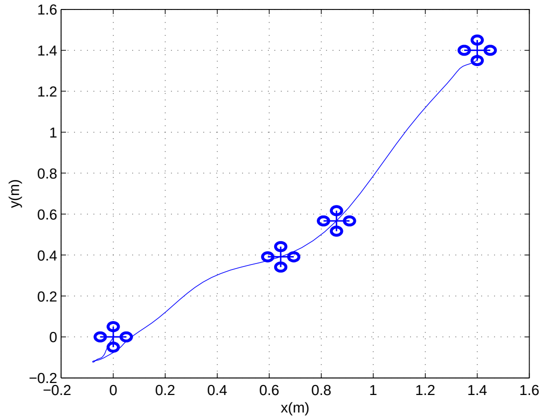

Figure 3: Time varying stabilization of the origin

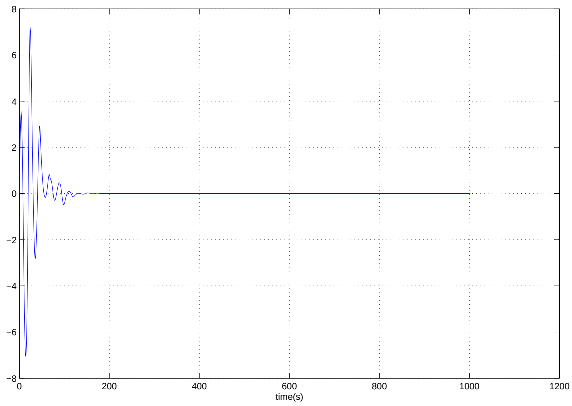

Figure 4: The necessary non stationary input to stabilize the origin

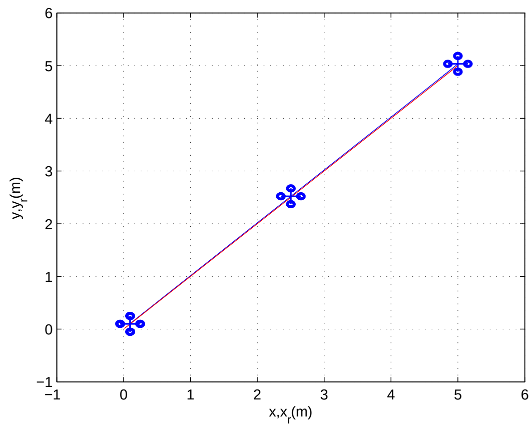

Figure 5: Tracking with motion planning in the x-y plan

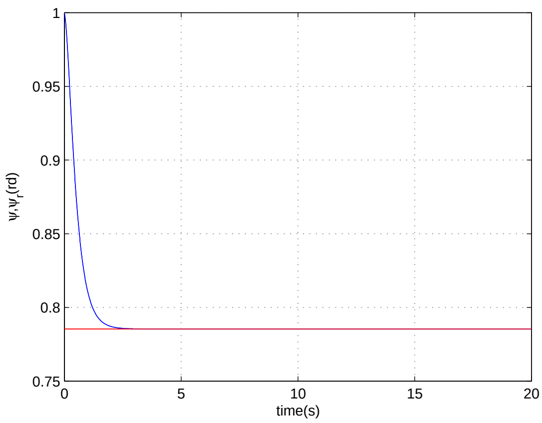

Figure 6: The yaw angle and the yaw reference.

vehicle considers the roll and pitch behaviors stabilized. Control based time-varying and averaging are proposed which permits to stabilize the system origin. We have proved that the planar control of the vehicle using the yaw angle is possible with a constrained planning path. The system presents a flat output useful to calculate states and inputs as function of the reference trajectory. The later helped us in tracking based point-to-point steering. Complex paths will be studied in the future for other missions.

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