# A Predictive Wave-Based Approach for Time Delayed Virtual Environment Haptics Systems

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## 1 Abstract

This paper extends earlier results of wave based approach for stable force re°ecting systems in the presence of the transmission delay (constant or variable). Theoretically, stability of the whole system is kept for any time delay, but degradation performance increases proportionally with a large delays. In this paper, we propose a new control scheme that is also based on wave variables, but minimizes the degradation performance due to large constant time delay, the time varying delay case is also discussed. The proposed method uses a modi¯ed Smith prediction is simple and easy to implement.

## 2 Introduction

In this paper we examine the performance deterioration of the control of force re°ecting bilateral teleoperators with time delay using wave variables approaches. The standard approaches to control bilateral teleoperators with force feedback, based either on scattering theory [2] and [3], or the equivalent wave variable formulation [13] and [12], preserve passivity only in the case of constant transmission time delay. For teleoperation over the Internet the delay varies with such factors as congestion, bandwidth, or distance, and these varying delays or large delays may severely degrade performance or even result in an unstable system.
In this paper we present a simple modi¯cation to the wave based transformation proposed in [11], that inserts modi¯ed Smith predictor based on the haptic interface model (the master model in the teleoperation case) into the communication block (in the wave domain) which keep passivity and improve performance for arbitrary time constant delays. Many method have been proposed focusing, how to preserve passivity, however, is not su±cient to guarantee acceptable performance [10], [16] and [9]. We therefore, use an additional control in wave domain to reduce considerably degradation of performance. Recently, The principle of Smith prediction have been considered in teleoperation under some conditions on estimation parameters [4] and [5]. The combination of this tow approach gives a good result about quality of performance. Simulation results are presented showing the performance of the overall compensation scheme.
In the ¯rst section, we recall some de¯nition concerning wave variables approach. Second section gives some recent results about model based approach using smith prediction principle. After, we propose how we shall combine this two approaches, in other words, how to predict in the wave domain. Section 4, we demonstrate this complete system via simulation result. Finally, some concluding remarks are given.

## 3 Wave Based Approach in Bilateral Control

### 3.1 Encountered problems due to time delay

It is well known that even small communication delay may destabilize teleoperators systems or haptic interaction with conventional bilateral control methods, such as symmetric position servo and force re°ecting servo [2]. Although the master arm behaves stably without delay. Instability of the force re°ecting system can compromise operator safety and can damage the master display. All existing methods to deal with transmission delay problem are based on passive approaches, therefore, performances are not, all the time, acceptable.

### 3.2 Definition of wave variables

The entire concept of wave variables is based on a more general framework of passivity and was in particular motivated by scattering operators, see Desoer and Vidyasagar [8]. So not surprisingly, the basic definition of wave variables are also derived from Scattering [2]. Let us recall the essential formulation of wave variables approach. The power flow as:

$$P_{in} = \dot{x}^T F = \frac{1}{2}u^T u - \frac{1}{2}v^T v \qquad (1)$$

where: $F$ and $\dot{x}$ are the power parameters respectively force and velocity. $u$ and $v$ are the wave variables respectively incidental and reflected one, figure (1).

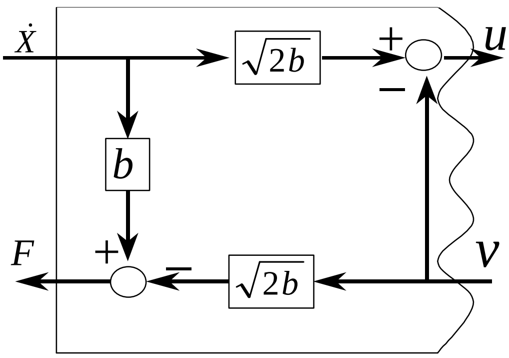


Figure 1: Transformation from power parameters into wave variables (Niemeyer [12]).

The wave variables (u,v) can be computed from the available power parameters ($\dot{x}$,F) by the following bijective transformation:

$$\begin{cases} u = \dfrac{b\dot{x} + F}{\sqrt{2b}} \\ v = \dfrac{b\dot{x} - F}{\sqrt{2b}} \end{cases} \qquad (2)$$

where $b$ in an arbitrary positive constant that determines the properties of the communication line.

In bilateral control of force reflected systems, the transmission process is expressed as follow:

$$\begin{cases} u_s(t) = u_m(t - \tau) \\ v_m(t) = v_s(t - \tau) \end{cases} \qquad (3)$$

where m and s represent side of the waves (master side or slave side) and $T$ is constant time delay.

Note that the inherent combination of velocity and force data makes the system well suited for interaction with unknown environments. Indeed it behaves like a force controller when rigid contact and like a position controller when free motion.

### 3.3 Passivity

The main characteristic of this transformation of power parameters into to wave variables is its effect on passivity property. The power inflow into the communication block at any is given by the equation 1, by substituting equation 3 into equation 1 and assuming that the initial energy is zero, it is easily computed that the total energy stored in the communications during the signal transmission between master and slave is given by:

$$\begin{aligned} E &= \int_0^t P_{in} d\tau = \int_0^t (\dot{x}_{md} F_m - \dot{x}_{sd} F_s) d\tau \\ &= \frac{1}{2}\int_0^t (u_m^T u_m - v_m^T v_m + v_s^T v_s - u_s^T u_s) d\tau \\ &= \frac{1}{2}\int_{t-T}^t \left( u_m^T u_m + v_s^T v_s \right) d\tau \geq 0 \end{aligned}$$

where, $x_{md}$ and $x_{sd}$ are respectively the desired velocity of the master and the slave. Therefore, the system is passive independently of the size of the delay $T$, in other words, this new formulation makes wave variables robust to constant time delays.

# 4 Existing Model-Based Control Scheme

Some model-based control scheme has been proposed, in first time, we have used a some-how prediction (based on Smith prediction principle) of the master part within the remote part [4]. Hence, the developed equations lead to a scheme where only the master model appears and also the estimation of the time delay is necessary. The term "somehow prediction" is used to signify that in fact the proposed solution is not really a prediction since only the master model is required, which means that no prediction of operator behavior or trajectory is needed. However, the upwards and forwards time delays must be known.

Then, we have extend the proposed controller based on Smith prediction principle. We notice, that in fact, the desired remote position is corrupted by a local closed loop on the obtained contact force. Indeed, the actual position of the virtual object $x_e$ is not directly the desired master position $x_m$, but the delayed $x_m$ minus the outputs of the local closed controller based on the master model, the delayed master model and the obtained contact force (look with[6] for more details).

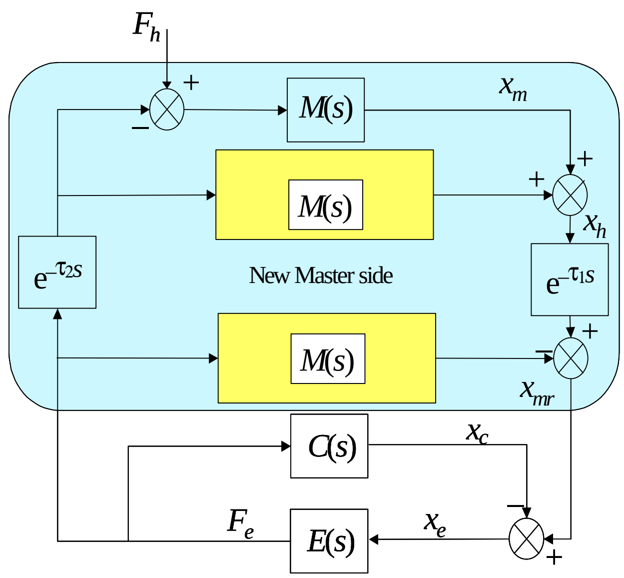


Figure 2: A practical implementation of the controller.

where $M(s)$ is the haptic device transfer function, $s$ is the Laplace transform variable, $E(s)$ is the VE transfer function (assumed continuous due to a high sampling frequency), $x_m; x_c$ and $x_e$ are respectively master, virtual coupling and VE positions, $F_e$ is the VE computed force, $F_h$ is the operator applied force on the device, $C(s)$ is the commonly used virtual coupling [1] and [7].

The new implementation of the controller makes possible an interesting extension which:

- avoids the estimation of time-delay, and
- makes a straightforward extension to time-varying time delay.
- unburden the VE from the buffering in order to compute the controller by sharing the computation on both sites.

This is obtained simply as depicted in figure 2. Nevertheless, this new implementation highlights that it is no more necessary to estimate time delay, and more importantly: the behavior of time delay may have no effect on the stability of the system.

## 4.1 Passivity of model based approach

The dynamic model of an haptic display can be approximated in a linear form[1], considering an apparent mass $M$ and friction:

$$F_h(t) - F_e(t - \tau_2(t)) = M\ddot{x}_m + B\dot{x}_m \quad (4)$$

where $x_m$, $\dot{x}_m$ and $\ddot{x}_m$ are respectively the Cartesian space position, speed and acceleration, $F_h$ and $F_e$ denote respectively human and VE forces applied to the haptic device. Time delays uses for the proof of the stability with the proposed control scheme are variables. The fundamental idea is to emulate a passive behavior of the haptic device and the transmission channel. In the second comparator of figure 2 we have $x_h = x_m + x_c$, using the equality $F_e(t - \tau_2(t)) = \hat{M}\ddot{x}_c + \hat{B}\dot{x}_c$ are obtains:

$$F_e(t - \tau_2(t)) = \hat{M}[\ddot{x}_h - \ddot{x}_m] + \hat{B}[\dot{x}_h - \dot{x}_m] \quad (5)$$

Where $\hat{M}$ is the estimated apparent mass, $\hat{B}$ is the friction estimate. Equation 5 is used to cancel the effect the delayed control $F_e(t - \tau_2(t))$ from the haptic device (master) position $x_m$ to be sent to the slave site.

Just after the transmission channel $\tau_1(t)$, at the third comparator we have:

$$F_e(t) = \hat{M}[\ddot{x}_h(t - \tau_1(t)) - \ddot{x}_{mr}] + \hat{B}[\dot{x}_h(t - \tau_1(t)) - \dot{x}_{mr}] \quad (6)$$

where $x_{mr}$ is the after-master position transmitted to the slave site. Equation 5 delayed by $\tau_1(t)$ substrates into equation 6 leads to:

$$F_e(t - \tau_1(t - \tau_2(t))) - F_e(t) = \hat{M}[\ddot{x}_{mr} - \ddot{x}_m(t - \tau_1(t))] + \hat{B}[\dot{x}_{mr} - \dot{x}_m(t - \tau_1(t))] \quad (7)$$

Finally, equation 4 is also delayed by $\tau_1(t)$, and the obtained force $F_e(t - \tau_1(t - \tau_2(t)))$ substituted in equation 7 leads to:

$$F_h(t - \tau_1(t)) - F_e(t) = \hat{M}\ddot{x}_{mr} + \hat{B}\dot{x}_{mr} + (M - \hat{M})\ddot{x}_m(t - \tau_1(t)) + (B - \hat{B})\dot{x}_m(t - \tau_1(t)) \quad (8)$$

[1] The developped proof holds for the non-linear haptic model described by the classical dynamic equation: $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)$. The demonstration is also trivial, we choose the linear case for the clarity of the presentation.

If we assume that the estimation error of the apparent mass and friction is zero,then equation 8 takes the following form:

$$F_h(t - \tau_1(t)) - F_e = M\ddot{x}_{mr} + B\dot{x}_{mr} \qquad (9)$$

This last equation exhibits a passive behavior of the equivalent new master side. Assuming that the virtual environment is passive, a fundamental property is that the feedback interconnection of passive systems is again passive [15], it ensues from it, that the proposed haptic interaction scheme is stable.

# 5 Wave-Model based control

The combination of both previous methods can be implemented in two manners, the first one, we place the prediction using the master model inside the waves domain (figure 3), such idea was evoked in the works of Saghir [14], but the prediction used concerns the behavior of the slave part or the virtual environment. where G(s) is the transfer function of the entire left

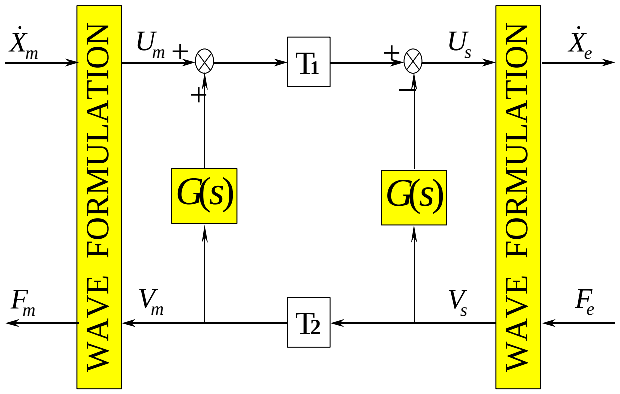


Figure 3: Prediction inside the wave domain

hand plant until before transmission channel, giving by:

$$G(s) = \frac{U_m(s)}{V_m(s)} = \frac{b - M(s)}{b + M(s)}$$

The second case, is to make more robust master-model-based method by imitating a passive behavior of the channel of transmission using the waves variables formulation (figure 4). Both implementations is passive (under some conditions or where the errors estimation of the master model are zero), it should be stated, that passivity property of the equivalent system is not compromised.

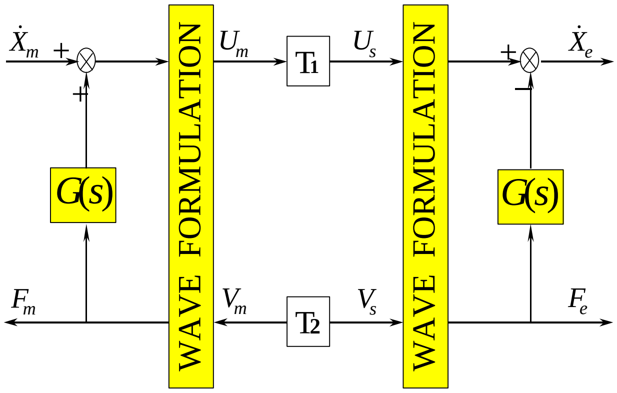


Figure 4: An other implementation (make more robust model based method)

In this paper, we shall present with more details the first proposed scheme, focusing on the improvement of the position error while contact is occur.

## 5.1 Static and dynamic position error

The expression of the position error relation $X_m$ and $X_e$ is giving by the following equation:

$$\Delta x(t) = x_m(t) - x_e(t) = \int_0^t \dot{x}_m(\tau) - \dot{x}_e(\tau) d\tau \qquad (10)$$

in the wave domain the previous equation can be rewritten as:

$$\Delta x(t) = x_m(t) - x_e(t) = \frac{1}{\sqrt{2b}} \int_0^t (u_m + v_m - u_s - v_s) d\tau \qquad (11)$$

Using Laplace transform and the prediction terms, where:

$$\begin{cases} U_s(s) = U_m e^{-sT_1} + V_s G_m(s)(e^{-s(T_1+T_2)} - 1) \\ V_m = V_s e^{-sT_2} \end{cases} \qquad (12)$$

substituting equation (12) into equation (11) gives:

$$\begin{aligned} \Delta X(s) &= X_m(s) - X_e(s) = \Delta X_0(s) \\ &\quad - \frac{1}{\sqrt{2b}s} \left[ V_s(s) G_m(s)(1 - e^{-s(T_1+T_2)}) \right] \end{aligned}$$

where:

$$\Delta X_0(s) = \frac{1}{\sqrt{2b}s} \left\{ U_m(s)(1 - e^{-sT_1}) - V_s(s)(1 - e^{-sT_1}) \right\}$$

is the error without prediction term. Recalling that:

$$v_s = \frac{bX_e - F_e}{\sqrt{2b}}$$

and $F_e = E(s)X_e(s)$, equation (13) becomes:

$$\Delta X(s) = \Delta X_0(s) - \frac{G_m(s)}{s}\frac{(E(s) - b)}{2bE(s)}F_s(1 - e^{-s(T_1+T_2)}) \quad (13)$$

At steady state when the transients have died out, the wave signals decay to zero, at which time require that the position error also be zero. In other words. form the equation (13), if we have:

$$\lim_{t\to\infty} u_m(t) = 0; \quad \lim_{t\to\infty} v_s(t) = 0$$

we have

$$\lim_{t\to\infty} \Delta x(t) = \lim_{s\to 0} s\Delta X(s) = \lim_{s\to 0} s\Delta X_0(s) = 0$$

For the dynamical error while the contact is occurred, in other words $F_s \neq 0$, last term

$$\frac{G_m(s)}{s}\frac{(E(s) - b)}{2bE(s)}F_s(1 - e^{-s(T_1+T_2)})$$

of equation (13) is positive (for more detail details, we give an example in the next section concerning simulation results).

## 6 Simulation results

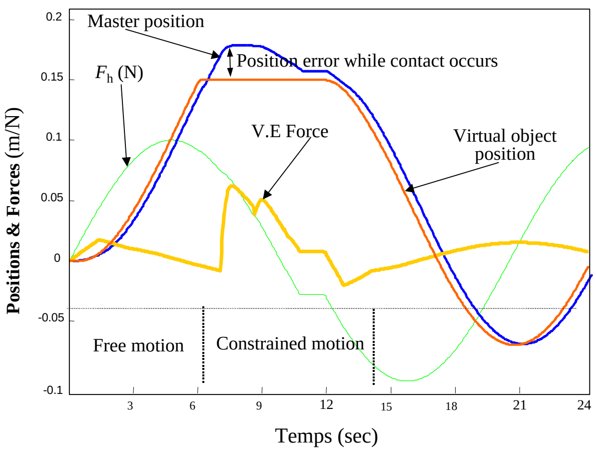


Figure 5: Simulation with wave variables approach

This section presents simulation results of the developed controller. The haptic display is a one DOF actuated arm with apparent mass $m = 0.2$kg and friction of about $b = 3$Ns/m. The contact will be performed between the rigid virtual pen and a virtual walls of high stiffness $K_e$(which representing by $E(s)$). In this first simulation, time delays are taken constant but different, indeed $\tau_1 = 1$sec and $\tau_2 = 0.5$sec. Figure 5 shows the tracking and force feedback behavior when the operator interacts with a VE of stiffness $K_e$ using wave variables approach without any adaptation and the line characteristic $b = 3$. It is assumed that collision detection and force computation are performed simply and do not cost additional time delay.

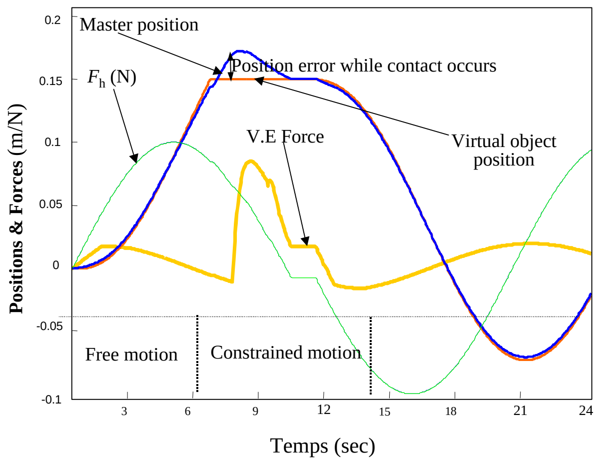


Figure 6: Simulation with wave-model based approach.

Figure 6 shows the result obtained of a simulating virtual contact using the new control scheme proposed in section 5. The position discrepancy, when the contact is made, is unavoidable whatever is the controller or the approach (unless a very prediction is made in the master side), this is due to the undergo physical time-delay $T_2$. Nevertheless, the virtual probe position $x_e$ is stably maintained by the operator during the whole contact time. Several simulations are conducted with multiple hard and viscous contacts, they show that the behavior of the force feedback interaction is stable whatever the time delay. Comparing with the previous result, this new implementation minimizes considerably the time delay between both positions $x_m$ and $x_e$, furthermore, the error of position while contact is occurs is more reduces than that of the first case and depends strongly on $F_e$ value, like we state this in equation (13).

# 7 Conclusion

This paper presents a new control scheme using wave variables approach using a master-model based prediction to stabilize delayed force feedback systems and to improve performance. The proposed method requires only the haptic (mater model in teleoperation case) device model and do not necessitate the estimation of both (upwards and downwards) delays, which become robust to time varying delay as proved in [6]. The simulation results con¯rm a stable force re°ection from the VE in presence of constant time delays. A robustness analysis is not necessary in this case due to prediction of the behavior of the master-model, but the wave variables approach is initially robust. The position error is considerably reduced and the time delay between the haptic device position and the virtual probe.

# References


[1] R. J. Adams and B. Hannaford, \Stable Haptic Interaction with Virtual Environments", IEEE Trans. on Robotics and Automation, pp. 465-474, Vol. 15, No. 3, June 1999.

[2] R.J. Anderson and M.W. Spong, \Bilateral control of teleoperators with time delay", IEEE Transactions On Automatic Control, pp. 494-501, Vol. 34, No. 5, May 1989.

[3] R.J. Anderson and M.W. Spong, \Asymptotic Stability for Force Re°ecting Teleoperators with Time Delay", Int. Journal of Robotics Research, pp. 135-149, Vol. 11, No. 2, April 1992.

[4] H. Arioui, S. Mammar and T. Hamel, \A Smith-Prediction Based Haptic Feedback Controller for Time Delayed Virtual Environments Systems", American Control Conference, pp. 4303-4308, May 2002.

[5] H. Arioui, A. Kheddar and S. Mammar, \Master-Model Based Time-Delayed Force Feedback Interaction: Experimental Results", To appear in Proc. of the 17th IEEE International Symposium on Intelligent Control, October 2002.

[6] H. Arioui, A. Kheddar and S. Mammar, \Stable Shared Virtual Environment Haptic Interaction Under Time-Varying Delay", To appear in Proc. of the 8th IEEE Intern. Conf Methods and Models in Automation and Robotics, September 2002.

[7] L. E. Colgate, M. C. Stanley and J. M. Brown, \Issues in the Haptic Display of Tool Use", IEEE/RSJ Int. Conf. on Intelligent Robotics and Systems, pp. 140-145, November 1995.

[8] C. A. Desoer and M. Vidyasagar, \Feedback Systems: Input-Output Properties", Academic Press, 1975.

[9] B. Hannaford and J. H. Ryu, \Time Domain Passivity Control of Haptic Interfaces", IEEE Trans. on Robotics and Automation, pp. 1-10, Vol. 18, No. 1, February 2002.

[10] K. Kosuge and Al., \Bilateral Feedback Control of Telemanipulators Via Computer Network", IEEE/RSJ Int. Conf. on Intelligent Robotics and Systems, pp. 1380-1385, November 1996.

[11] G. Niemeyer and J. J. Slotine, \Stable Adaptive Teleoperation", IEEE Journal of Oceanic Engineering, pp. 152-162, Vol. 16, 1991.

[12] G. Niemeyer and J. J. Slotine, \Using Wave Variables for system Analysis and Robot Control", IEEE International Conference on Robotics and Automation, pp. 1619-1625, April 1997.

[13] G. Niemeyer and J. J. Slotine, \Designing Force Re°ecting Teleoperators with Time Delays to Appears as Virtual Tools", IEEE International Conference on Robotics and Automation, pp. 2212-2218, April 1997.

[14] M. Saghir And W. J. Book, \Wave-Based Teleoperation with Prediction", American Control Conference, pp. 4605-4611, June 2001.

[15] Arjan Van der Schaft, \$L_2$-Gain an Passivity Techniques in Nonlinear Control", Springer, 1996.

[16] Y. Yokokojhi and T. Imaida and T. Yoshikawa, \Bilateral Control with Energy Balance Monitoring Under Time-Varying Communication Delay", IEEE International Conference on Robotics and Automation, pp. 2434-2439, April 2000.