Dynamic Analysis of Airships with small deformations

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ABSTRACT

The theory of modelling and control of airships was mostly imported from the fields of science and engineering of underwater vehicles. However the main hypothesis of rigidity of the structure may be improved to take into account the flexibility of the hull of an airship. In this paper we present a new approach for analysing dynamic elastic deformable airships in an ideal fluid. Our model uses the Updated Lagrangian Method in combination with a large displacement and small deformation (Green) strain tensor formulation. Analogy is made with classical structural dynamics. The description of the behaviour of the airship consists of a continuous differential equation that is solved using a Rayleigh-Ritz technique combined to the Finite Element Method. The method proposes also to take into account the coupling between the rigid body motion and the deformation as well as the interaction and coupling with the fluid surrounding.

Keywords : *Flexible airships, strain tensor, interaction fluid-structure, reference configuration.*

1. Introduction

Automated dirigible are low speed platforms. Such vehicles must be able to hover above an area and must have extended airborne capabilities for long duration studies. In order for unmanned aerial vehicles to perform complex missions, significant advances in modeling, motion planning and control must be made. The system under study is a non rigid airship. They are the most common form nowadays. They are basically large gas balloon. Their shape is maintained by their internal overpressure. The only solid parts are the car and the tail fins. Airships are governed also by the aerodynamic forces that have to be controlled. Since the airship is a buoyant vehicle, many concepts relating to its modeling were imported from the well established fields of science and engineering of underwater vehicles [FOS96, MUN24]. One of the earliest texts to discuss the modeling and stability of the airship from a mathematical point of view is the formative text on applied aerodynamics by Bairstow [BAI20], in which a significant part is devoted to the subject. The basis for analyzing the motion of a rigid vehicle in a perfect fluid was established in the 19^{th} century and is described by Lamb[LAM45]. In his work, Lamb initially considers the case of a single solid moving through an infinite mass of liquid and where the motion of the fluid is entirely due to that of the solid, the motion is irrotational and acyclic. When Lamb considers the impulsive wrench (i.e. the combined force and moment) required to be applied to the solid to generate the motion instantaneously from rest, he shows that it varies in exactly the same way as the momentum of finite dynamical system, even though the total momentum of the vehicle and fluid is indeterminate. As part of the analysis, Lamb shows that the kinetic energy of the fluid can be expressed as a quadratic form involving the 3 translational and 3 rotational velocities of the vehicle. The derivations given by Lamb were used as the basis of the equations of motion of airships, in a steady uniform atmosphere. The acceleration dependent terms or added masses arise from the work done in accelerating the perfect fluid. In a real fluid, additional acceleration effects such as the increase in vorticity and its convection past tails come into play. It is assumed that all such effects are added into the corresponding perfect fluid added mass terms. This is a reasonable assumption for streamlined vehicles supported by buoyancy such as airships. It follows from Green analysis that when an ellipsoidal body moves in a infinite incompressible inviscid fluid in such a way that the flow is everywhere of the irrotational, continuous eulerian type, the kinetic energy of the fluid produces an apparent increase in the mass and moments of inertia of the body [THO00, KHO99, BES00].

Actuators provide the means for maneuvering the airship along its course. The propulsion vector contains the terms associated with the propulsive forces and moments and it is a function of the geometrical arrangement of the propulsive units

around the body axes. The dynamics and stability of a rigid vehicle immersed in an irrotational fluid have been studied and interpreted in a general Hamiltonian setting by Marsden and al [MAR99].
In general, the airship is modeled as a rigid body. This rigidity assumption is rather strong considering that many airships show some degree of flexibility and internal motion. These non rigidity effects may sometimes be modeled as disturbances. In this paper, we introduce the effect of the non rigidity as additional unactuated degrees of freedom. The flexibility of the airship is not considered as a sample disturbance issued from the inertia and the elasticity of the body, but considered as acting on the overall motion when taking into account the coupling between the rigid body motion and the deformation as well as the interaction and coupling with the fluid surrounding.

2. Dynamical model of the airship

The dynamics of the airships with small deformations are highly non-linear. The non-linearities being essentially due to the large rotations of the bodies and the interaction between the overall rigid body motion and the deformation. In the literature, many approaches were proposed to treat the problems of flexible bodies. These methods could be classified into two groups. The first one uses the Newton-Euler description [VER91], which is an interesting method in regard of the time computation, but it is not easy to use in the case of bodies with complex shapes because of the relative instability of the resolution. The second group uses the lagrangian formulation [BAT75]. This formulation induces very complex equations of motion in this case because of the interaction fluid-structure between the airships and their environment.
In our formulation we use an Updated Lagrangian Method (U.L.M.). The description of the motion of the airships is not made relatively to the initial configuration but made on a reference configuration which change in the time similarly to the actual configuration of the airship. This method is chosen because it takes into account the buckling between the rigid body motion and the deformation, and allows a relative easiness to describe the interaction fluid-structure. Moreover, many dynamic parameters are kept constant during the simulation. In the case of a small deformation , the real configuration at time t (C^t) is not so different from the configuration (C_r) obtained by a rigid body transformation of the initial configuration. We choose then as a reference configuration (C_{ref}) a "rigid" configuration. We can therefore linearize the motion of the airship around this configuration.
Two reference frames are considered in the derivation of the kinematics and dynamic equations of motion. These are the Earth fixed frame $\mathcal{R}_f$ and the moving frame $\mathcal{R}_m$ fixed to the reference configuration (C_{ref}). The position and orientation of the vehicle should be described relative to the inertial reference frame. The origin C of $\mathcal{R}_m$ coincides with the centre of gravity of the vehicle. Its axes (x_c, y_c, z_c) are the principal axes of symmetry when available. They must form a right handed orthogonal normed frame.

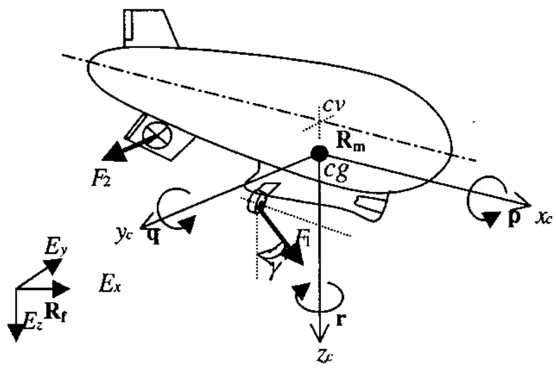

Figure 1. Presentation of the different frames.

2.1 Description of the reference configuration

In this model, we use an incremental scheme (Figure 2). At each time t^n we define a local reference frame on which we study the motion of the body in the step [t^n, $^{tn+1}$].

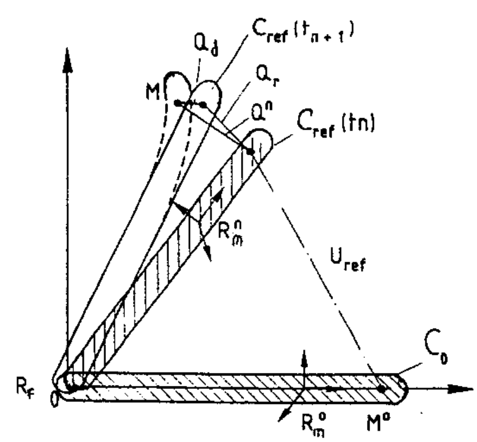

Figure 2. Scheme of a planar motion in one step.

The displacement of each point M is given by:

$$U(M) = U_{ref}(M) + \underbrace{\Delta U_{ref}(M) + U_d(M)}_{Q} \qquad (1)$$

where $\mathbf{U}_d$ is the small displacement corresponding to the deformation, $\Delta \mathbf{U}_{ref}$ is the increment of the rigid body motion. Let us note $\mathbf{V}^L = R^T \mathbf{V}$ the projection of a vector $\mathbf{V}$ in the local reference frame ($\mathcal{R}_m$). R is the rotation matrix between the inertial frame and the local reference frame.
From equation (1), we can obtain the following relation :

$$\ddot{U}^L = R^T \ddot{\bar{U}} = R^T \ddot{\bar{U}}_{ref} + R^T (R\ddot{\tilde{Q}} + 2\dot{R}\dot{\tilde{Q}} + \ddot{R}\tilde{Q}) \qquad (2)$$

$\tilde{Q}$, $\dot{\tilde{Q}}$ and $\ddot{\tilde{Q}}$ represent the displacement, velocity and acceleration of the airship in the local reference frame.

2.2 Description of the strains

We supposed in this part that the airship behaves as a flexible structure. We will demonstrate in the part 4 the validity of this assumption.
To describe the strains, we use the Green-Lagrange tensor $\bar{\bar{\varepsilon}}$ which we refer to the reference configuration (C_{ref}). We denote J the gradient tensor relatively to the reference co-ordinates. We have $J = \nabla.\tilde{Q}$ and the Green tensor $\bar{\bar{\varepsilon}} = (\varepsilon_{ij})$ could be expressed as follows: $\bar{\bar{\varepsilon}} = \frac{1}{2}\left[\left(J^T + J\right) + J^T.J\right]$.

∇ is the nabla differential operator. $\nabla. = \left(\partial/\partial x_1 \;\cdots\; \partial/\partial x_n\right)$

In this paper we limit our study to the small deformations, we have therefore $\|J\| \ll 1$. We can then neglect the non-linear terms in the strain tensor, which becomes: $\bar{\bar{\varepsilon}} \approx \frac{1}{2}\left[J^T + J\right]$. If we use for the more practical strain vector $\bar{\varepsilon}$ defined as : $\bar{\varepsilon}^T = [\varepsilon_{11}\ \varepsilon_{22}\ \varepsilon_{33}\ \varepsilon_{12}\ \varepsilon_{13}\ \varepsilon_{23}]$, this gives $\bar{\varepsilon} = D.\tilde{Q}$, With D a differential operator.

2.3 Description of the stresses

For the description of the stresses applied on the airship, we use the second tensor of Piola-Kirchoff $\bar{\bar{P}}_K$ defined on the reference configuration. This tensor is deduced from the Cauchy tensor $\bar{\bar{\sigma}}$ describing the stresses applied on the real configuration (C^t) by : $\bar{\bar{P}}_K = \frac{\rho_{ref}}{\rho}.J_a^{-1}.\bar{\bar{\sigma}}.J_a^{-T}$.
With $J_a = I_d + J$, I_d is the identity 3x3 tensor, ρ_{ref} is the mass density of the reference configuration equal to its initial value ρ^0 defined on the initial configuration (C^0). In our study the mass density of the real configuration ρ is close to the mass density of the reference configuration $\rho \approx \rho_{ref}$ and we can merge the stress tensor $\bar{\bar{P}}_K$ into $\bar{\bar{\sigma}}$. If we consider the airship with linear elastic behaviour , the stresses could be expressed in function of the strains as : $\bar{P}_K = E.\bar{\varepsilon}$, where E is the matrix of the elastic coefficients.

2. 4 Dynamic equilibrium

Let us consider the equilibrium of the moving airship. Referring to the basis of continuum mechanics [CIA88], the equilibrium is represented by two relations :

$$\begin{cases} \operatorname{div} \bar{\bar{\sigma}} + f_V = \rho \ddot{U} & \text{on } \left(C^t\right) \quad (3.a) \\ \text{with} & \\ \bar{\bar{\sigma}}.n = t_n & \text{on the boundary } \left(\partial C^t\right) \quad (3.b) \end{cases}$$

div is the divergence of a tensor, $\operatorname{div}\bar{\bar{\sigma}} = \left(\sigma_{ij,j}\right)$.
where f_v are the forces acting on the element of volume such as gravitational, buoyancy, and inertia forces, **n** a unit vector directed along the outward normal to the surface at a point M. $\mathbf{t}_n$ is the boundary stress vector. It represents boundary forces such as the pressure p of the fluid on the external surface of the airship.

Let us consider the following assumptions :
the volume $V \approx V_{ref}$, and the surface $S \approx S_{ref}$.

If we use the principle of virtual works and consider a virtual displacement $\delta\tilde{Q}$, the equilibrium equation, in regards of equation (2), becomes [AZO98] :

$$\int_{V^{ref}} \rho.\delta\tilde{Q}^T.(\ddot{\tilde{Q}} + 2R^T\dot{R}\dot{\tilde{Q}} + R^T\ddot{R}\tilde{Q})\,dV + \int_{V^{ref}} \delta\varepsilon_{ij}.(P_K)_{ij}\,dV$$
$$=$$
$$\int_{V^{ref}} \delta\tilde{Q}^T.f_V^L\,dV + \int_{S^{ref}} \delta\tilde{Q}^T.t_n^L\,dS - \int_{V^{ref}} \rho.\delta\tilde{Q}^T.R^T\ddot{\bar{U}}_{ref}\,dV \quad (4)$$

One can note that as a consequence of the choice of the reference configuration, the internal domain of V_{ref} and S_{ref} are equal of those of V^O and S^O defined in the initial configuration (C^0).
Hence, and as a result of our formulation, we can see that the equation (4) is similar to dynamic equations of deformable bodies issued from classical structural dynamics. As a consequence of the use of a moving reference frame, we have two additional terms that appear in the first integral.

3. Resolution of the dynamic equation

To define the exact configuration of the deformable airship, we should use an infinite number of co-ordinates that are required in order to calculate the location of every point of the body. It has been shown that the motion of the deformable airship is governed by a set of partial differential equations that are space and time dependent. Because of the computational problems associated with infinite dimensional spaces, we will use approximate techniques namely Rayleigh-Ritz methods coupled with Finite Elements discretization [ZIE97]. An important effort is made to optimise the computation cost required for the resolution of the dynamic equation.

3.1 Discretization of the airship

The discretization consists in subdividing the airship in a number of sub-domains with simple shapes that could be studied easily.
The displacement of the airship is then described through some interpolations functions and nodal displacements. We must note that the number of elements required for an acceptable approximation of the displacement should be high. However we will show that, when using modal synthesis, this number of elements intervene only in a preliminary computation. Just a few number of "useful" nodes are kept, i.e: nodes defining the position of actuators or characteristic nodes allowing an optimal graphic display.

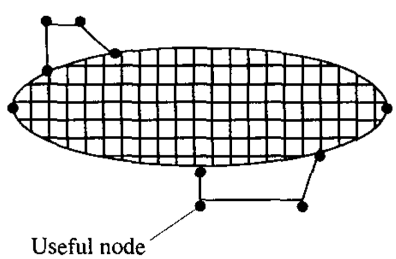

FIGURE 3. Discretization of the airship

3.2 Characteristics of the discretized airship

The displacement $\tilde{Q}^i$ of each element i from the airship is expressed in function of the column matrix $\bar{q}$ representing the displacement, relatively to the local reference frame, of all the nodes of the discretized airship as follows :

$$\tilde{Q}^i = H^i \bar{q} \tag{5}$$

where H^i is an interpolation function.

Let us note M^i and K^i the mass and stiffness matrices of the element i defined as follows :

$$\left.\begin{aligned} M^i &= \int_{V^i} \rho . \left(H^i\right)^T . H^i \, dV \\ K^i &= \int_{V^i} \left(D^i H^i\right)^T . E^i . D^i . H^i \, dV \end{aligned}\right\} \tag{6}$$

V^i and E^i are respectively the volume and the tensor of the elastic properties of the element i.
The matrix of mass and stiffness of the whole airship is built by an adequate assembly of elementary matrices of mass and stiffness.

$$\begin{cases} M = \sum_{i=1}^{ne} M^i \\ K_L = \sum_{i=1}^{ne} K_L^i \end{cases}$$

ne is the number of elements of the airship. This number could be great (thousands).

3.3 Equation of motion of the discretized airship

The dynamic equation of the discretized airship becomes then :

$$M . \ddot{\bar{q}} + B . \dot{\bar{q}} + K_d \bar{q} + K . \bar{q} = \bar{F}_e + \bar{F}_f - \bar{F}_R \tag{7}$$

We define here $\bar{R}$ as an hypermatrix of rotation between the inertial frame and the local reference frame.

$$\bar{R} = \begin{bmatrix} [R] & & & \\ & [R] & & \\ & & \ddots & \\ & & & [R] \end{bmatrix}$$

and $K_d = M \bar{R}^T \ddot{\bar{R}}$ a dynamic stiffness due to the Coriolis effect.

$B = 2 M \bar{R}^T \dot{\bar{R}}$ is a anti-symmetric matrix of gyroscopic terms.

K_d and B are consistent matrices which take into account the inertial coupling between the overall movement and the deformation.

$\bar{F}_R = MR^T \ddot{U}_{ref}$ is a column matrix of residual inertia terms due to the motion of the reference configuration. $\bar{F}_e$ is the column matrix of all external forces and torques applied on the airship

and its boundary including restoring forces. $\tilde{F}_f$ represents the fluid forces.
The dynamic equation (7) has a huge size. If we choose to model the airship with small elements this induces an important number of unknowns, which leads to a very high computational cost and many problems to establish control laws. For this reason, we tried to solve this dynamic equation using a modal synthesis.

3. 4 Modal synthesis

The increment of displacement $\overline{q}$ is divided into a rigid contribution $\overline{q}_r$ and a deformable part $\overline{q}_d$.

$$\overline{q}_r = \sum_{i=1}^{6} Y_{ri} X_{ri} \qquad (8)$$

Y_{ri} is a time dependent variable of the rigid mode i. We have then six variables for the motion of the airship (Y_{trax}, Y_{tray}, Y_{traz}, Y_{rotx}, Y_{roty}, Y_{rotz})

$\mathbf{X}_{ri}$ is a space dependent column matrix of the "rigid" displacement of all the nodes.

We can denote the matrix of all rigid modes as:

$$[X_r] = [[X_{tra}] \vdots [X_{rot}]]$$

$[\mathbf{X}_{tra}]$ are the three modes of the motion of translation.
$[\mathbf{X}_{rot}]$ are the three modes of rotation.

The displacement due to the deformation of the airship is :

$$\overline{q}_d = \sum_{i=1}^{nd} Y_{d_i} X_{d_i} \qquad (9)$$

nd is the number of significant deformable modes kept for the study.

Y_{d_i} is a time dependent variable of the deformable mode i.

$\mathbf{X}_{di}$ is a natural mode of rank i. For the discretized body, it represents a space dependent column matrix of the "deformable displacement" of all the nodes.

The natural modes are assumed to be constant in the local reference frame relative to the rigid reference configuration. The deformable airship could be considered as a low frequency structure. We can model its vibration accurately with the three first modes of lower frequencies.

4. Interaction fluid-airship

Consider a simple dynamical model of the action of a medium on a body. To present this model, we assume that the flow is quasi-steady, i.e. the distribution of the velocities of particles of the medium coincides with the distribution corresponding to the steady motion of the body. Thus the medium responds only to the current motion of the body and forgets its initial conditions. Therefore, within the framework of this hypothesis, the resultant force and torque acting on the body can be represented in the form of a function of the instantaneous distribution of velocities in this body. Thus we arrive at the statement of the problem of the motion of a body in a dragging medium as a problem of classical dynamics.

4.1 Flow representation

To take into account the interaction of the airship with the surrounding fluid medium, a model of the flow is needed. Here, we rely on the potential flow theory corresponding to the following hypothesis :

a. the air can be considered as a perfect fluid with uniform density ρ_{air}, i.e. an incompressible gaz with vanishing viscosity,
b. the flow is irrotational,
c. only the flow outside the airship contributes significantly to the aerodynamics forces.

Denoting by **u** the velocity field in the fluid domain Ω_{air}, the incompressibility and irrotational assumptions leads to :

$$\nabla . u = 0 \quad ; \quad \nabla \wedge u = 0 \qquad (10)$$

$\wedge$ is the vectorial product of two vectors.
and the flow field may be described in terms of a potential F such that :

$$u = \nabla \Phi \quad . \qquad (11)$$

From the incompressibility constraint, it is easy to show that the potential obeys to the homogeneous Laplace equation :

$$\nabla^2 \Phi = 0 \quad \text{in } \Omega_{air} , \qquad (12)$$

with Newman boundary conditions :

$$\nabla \Phi . n = -\dot{q}.n = -([X]\dot{Y}).n \quad \text{on } (\partial C^t) \qquad (13)$$

$[X]$ is the matrix of all the modes (rigid and deformable modes), $\mathbf{Y}$ is the column matrix of all the time dependent variables defined in 3.4., **n** is a unit vector, normal to (∂C^t).

Thus, one of the important characteristic of this representation is that **u** only depends on the current boundary conditions, and not on the history of the flow : the model is quasi-steady. To solve the potential equation, we use the boundary integral representation of the Laplace equation, together with standard boundary element method. It consists in the determination of a piecewise constant distribution of singularities over (∂C^t) (see [KAT91] for details on the numerical treatment).

4.2 Fluid forces

For this assumption, the pressure at any point in the fluid domain (including (∂C^t)) is given by Bernoulli theorem :

$$P+\rho_{air}\left[\frac{1}{2}u.u+\frac{\partial\Phi}{\partial t}\right]=P_\infty+\rho_{air}.\frac{1}{2}u_\infty.u_\infty \qquad (14)$$

where the subscript ∞ denotes the undisturbed conditions far from the airship. This pressure distribution over the airship surface can be integrated to compute the resulting forces and torques which in turn are projected on the modal basis. At the end, and with the linear property of the Laplace equation, the generalised fluid forces vector can be rewritten as :

$$F_f=-M_{ad}\ddot{Y}-B_f\dot{Y} \qquad (15)$$

where M_{ad} is the matrix of the added masses (virtual masses), and Y is the column matrix of all Y_i, B_f is a modal damping due to the flexibility of the hull.

4.3 Modal projection

Taking into account the previous development, the dynamic equation (7) can be projected on the modal basis, this gives :

$$\mathbf{M}.\ddot{Y}+\mathbf{B}.\dot{Y}+\mathbf{K_d}Y+\mathbf{K}.Y=\bar{\mathbf{F}}_e+\bar{\mathbf{F}}_f-\bar{\mathbf{F}}_R \qquad (16)$$

M and **K** are the constant mass and stiffness matrix projected in the modal basis ($\mathbf{M}=X^T.M.X$), and the bold terms are terms projected in the modal basis.
When using equation (15) the dynamic modal equation becomes :

$$\mathbf{M'}.\ddot{Y}+\mathbf{B'}.\dot{Y}+\mathbf{K_d}Y+\mathbf{K}.Y=\bar{\mathbf{F}}_e-\bar{\mathbf{F}}_R \qquad (17)$$

we note $\mathbf{M'}=\mathbf{M}+M_{ad}$, $\mathbf{B'}=\mathbf{B}+B_f$. The effect of the fluid on the structure is then represented mainly by the adjunction of the added masses matrix $\mathbf{M_{ad}}$ to the mass matrix of the structure. For an airship the extra-diagonal terms of $\mathbf{M_{ad}}$ can be neglected (for more details about the constitutive terms of $\mathbf{M_{ad}}$, the reader can see [LAMB45, FOS96]).

Equation (17) is being solved numerically at each step. After modal recombination we retrieve the displacement of each node and the new configuration of the airship. We should note that the dynamic equation (17) has 9 degrees of freedom (d.o.f) in this case. On the other hand the airship has only 3 available inputs: the main and tail thrusters and the tilt angle of the main propeller.
The roll as well as the three vibration modes are totally unactuated. The same input controls both pitch and surge, while yaw and sway are related. The unactuated dynamics imply constraints on the acceleration.

5. Global positioning

5.1 Actualisation of the rotation matrices

The matrix R characterises the rotation between the inertial frame and the local reference frame ($\mathcal{R}_m$). Because of the incremental scheme this matrix should be actualised at each step. We choose to define the increment of rotation in the current reference frame ($\mathcal{R}_m$) at time t^n. We can then compute the rotation matrix at the next step as :

$$R\left(t^{n+1}\right)\equiv R^{n+1}=R^n.\Delta R^{n+1} \qquad (18)$$

Here ΔR^{n+1} is the increment of rotation in the step n+1.
The increment of the rotation motion is defined by the roll, pitch and yaw angles (Δp, Δq, Δr) of the reference configuration, which correspond to (Y_{rotx}, Y_{roty}, Y_{rotz}) issued from the dynamic equation (17).

$$\Delta R^n = \Delta R_{x_c}.\Delta R_{y_c}.\Delta R_{z_c} \qquad (19)$$

The previous expression is commutative. This is due to the fact that we use infinitesimal rotations.
The use of the incremental scheme allows the use of this description of the rotation instead of global description such as Euler angles. The global orientation of the airship is defined uniquely by this description.
The actualisation of the translation motion is made similarly to the rotation as follows :

$$T^{n+1}=T^n+\Delta T^{n+1} \qquad (20)$$

where $\Delta T=\begin{bmatrix}Y_{trax}\\Y_{tray}\\Y_{traz}\end{bmatrix}$ is the increment of the translation motion.
In practice we combine the actualisation of both translation and rotation by using the 4x4 transformation matrices :

$$\tau = \begin{bmatrix} [R] & T \\ 0\,0\,0 & 1 \end{bmatrix} \quad \text{and} \quad \Delta\tau = \begin{bmatrix} [\Delta R] & \Delta T \\ 0\,0\,0 & 1 \end{bmatrix}$$

this gives :

$$\begin{cases} \tau^{n+1} = \tau^{n} . \Delta\tau \\ \dot{\tau}^{n+1} = \tau^{n} . \Delta\dot{\tau} + \dot{\tau}^{n} . \Delta\tau \\ \ddot{\tau}^{n+1} = \tau^{n} . \Delta\ddot{\tau} + 2\,\dot{\tau}^{n} . \Delta\dot{\tau} + \ddot{\tau}^{n} . \Delta\tau \end{cases} \qquad (21)$$

these matrices define the rigid body motion of all the points of the airship.

5.2 Actualisation of the real configuration

The position of each node in the real configuration could be defined by adding the contribution of the deformation. In practice, we actualise just the useful nodes. The displacement due to the deformation of an useful node N_i is :

$$\tilde{U}_d(N_i) = \mu_i . \sum_{j=1}^{nd} Y_{dj} . X_{dj} \qquad (22)$$

with μ_i the boolean matrix, allowing to extract the displacement of N_i from the whole meshing.
In the inertial frame, the deformation motion becomes :

$$\begin{cases} U_d = \tau\,\tilde{U}_d \\ \dot{U}_d = \dot{\tau} . \tilde{U}_d + \tau . \dot{\tilde{U}}_d \\ \ddot{U}_d = \ddot{\tau}\,\tilde{U}_d + 2\,\dot{\tau}\,\dot{\tilde{U}}_d + \tau\,\ddot{\tilde{U}}_d \end{cases} \qquad (23)$$

The global position of an useful point N_i is then :

$$x^{t}_{N_i} = \tau\left(x^{o}_{N_i}\right) + U_d\left(N^{t}_{i}\right) \qquad (24)$$

$\tau\left(x^{o}_{N_i}\right)$ represent thus the rigid body motion and $U_d\left(N^{t}_{i}\right)$ the deformation at time t. The velocity and the acceleration are computed similarly.

6. Conclusions and future work

Our aim in this paper is to introduce the flexibility of the airship as an extension of the classical structural dynamics taking into account the coupling between the overall rigid body motion and the deformation as well as the interaction fluid-structure. The dynamic analysis of the airships has many specificities compared to traditional theory of underwater vehicles. One main difference is the flexibility of the hull. The study of flexible structures in the space have different performance requirements which may lead to troublesome calculations.
The benefits of our formalism is that the use of an Updated Lagrangian Method applied on "rigid" reference configuration and the uses of an adapted modal synthesis allows to reduce the dynamic equations of a flexible airship to a set of 9 degrees of freedom. Note that we kept constant the mass and stiffness matrices as well as the vibration modes. This is very important in regard to the numerical resolution of the dynamic equation.
We should notice, that in this study, we supposed the overpressure of the boarded helium sufficiently important such that the whole system hull-helium behaves as a flexible structure and the internal pressure is constant.
We will present in future studies variational laws and relationships between the pressure of helium in the airship and the global stiffness and vibration modes of the whole flying system. On the other hand some novel ways will be explored to control such underactuated systems.

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