Stabilization of a Nonlinear Underactuated Autonomous Airship-A Combined Averaging and Backstepping Approach

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Abstract

A strategy to design a time-varying stabilizing controller of the position and the orientation of an underactuated autonomous airship is proposed. The dynamic modelling of the airship involves six equations with only three inputs. This airship cannot be stabilized to a point using continuous pure-state feedback law. However, the stabilization problem is solved with an explicit homogeneous time-varying control law, based on an averaging approach. We prove that the origin of the system is locally exponentially stable.
Key-words: Autonomous Airships, Homogeneous Time-Varying Stabilization, Averaging Approach.

1 Introduction

The dynamic analysis and control of aerial vehicles is a challenging problem. Their capability is considerable in increasing the manoeuvrability for tasks such as transportation, surveillance and military applications [7] [9]. Airships are member of the family of under-actuated systems because they have fewer inputs than degrees of freedom.
In some studies such as [4] [9], motion is referenced to a system of orthogonal body axes fixed in the airship. The model used was written originally for a buoyant underwater vehicle [4]. It was modified later to take into account the specificity of the airship [9]. In this paper, we propose to control the model given in [8]. This dynamic model has the particularity that the origin of the body fixed frame is the center of gravity, while in the cited works, it is located in the center of volume. An airship is propelled by thrust. In this paper, we consider the stabilizing control problem using only the three available inputs: the main and tail thrusters and the tilt angle of the propellers. The roll is totally unactuated. The same input controls both pitch and surge, while yaw and sway are related. The unactuated dynamics implies constraints on the accelerations. The dynamic positioning control problem consists of finding a feedback control law that asymptotically stabilizes both position and orientation to fixed constant values. Most small time locally controllable systems can be stabilized by continuous time-varying feedback. Homogeneous approximations and high gain feedback control have been applied to systems with drift. These applications can be found for instance in Morin and Samson [3] for the attitude stabilization only, or in Pettersen and Egeland [5] for the stabilization of both position and attitude but with four controls.

Averaging is an important tool used in the analysis of time-varying systems. An auxiliary time-invariant dynamical system: $\dot{x} = f_{av}(x)$ called the average, is used to investigate properties of a time-varying dynamical system $\dot{x} = f(t/\varepsilon, x)$ that depends on a small parameter ε. The search for locally exponentially stabilizing feedback has been reduced to looking for time periodic asymptotically stabilizing homogeneous degree one functions for the approximate system.
In this paper, a periodic time-varying feedback law is developed. The feedback control law is derived using averaging theory and homogeneity properties. It is based on a quaternion representation of the orientation. We proved that it stabilizes asymptotically both the position and orientation of the airship using only the three available control inputs. Moreover, the convergence to the equilibrium point is proved to be exponential.

2 Model of the Airship

The forces and moments are referred to a system of body-fixed axes, centered at the blimp center of gravity. We assume that the earth fixed reference frame is inertial, the gravitational field is constant, the airship is supposed to be a rigid body, meaning that it is well inflated, the aero-elastic effects are ignored, the density of air is supposed to be uniform, and the influence of gust is considered as a continuous disturbance, ignoring its stochastic character.

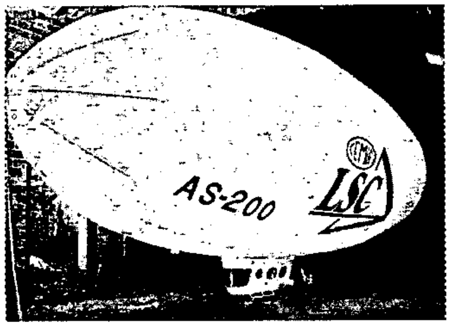

Figure 1: The CEMIF'AS200 technology

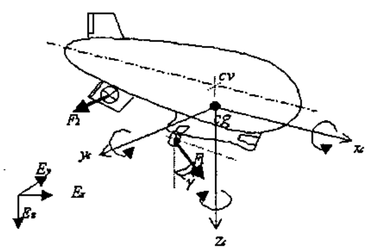

Figure 2: The airship parameterization

2.1 Kinematics

Two reference frames are considered in the derivation of the kinematics and dynamics equations of motion. There are the Earth frame R_f : $\{O - E_x E_y E_z\}$ and the body fixed frame R_m : $\{Cg - x_c y_c z_c\}$. The position and orientation of the vehicle should be described relative to the inertial reference frame while the linear and angular velocities of the vehicle should be expressed in the airship-fixed coordinated system. The origin cg of R_m coincides with the center of gravity of the airship (figure 2). Its axes (x_c, y_c, z_c) are the principal axes of symmetry when available. They must form a right handed orthogonal normed frame. The position of the airship'Cg in R_f can be described by $\eta_1 = (x\ y\ z)^T$. While, the orientation is described by the unit quaternions which are represented by a normalized vector of four real numbers. Let e denote the Euler parameters which are expressed by the rotation axis n and the rotation angle δ about the axis as follows:

$$\eta_2 = \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos(\frac{\delta}{2}) \\ \sin(\frac{\delta}{2}) n_x \\ \sin(\frac{\delta}{2}) n_y \\ \sin(\frac{\delta}{2}) n_z \end{pmatrix}, \quad 0 \leq \delta \leq 2\pi \qquad (1)$$

Let us consider $\eta = (\eta_1, \eta_2)^T$. We introduce $V = (u\ v\ w)^T$, as the linear velocity of the origin and $\Omega = (p\ q\ r)^T$, as the angular velocity which are expressed in the airship-fixed frame. The kinematics of the airship can be expressed by the following matrix form:

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} R(\eta_2) & 0 \\ 0 & J(\eta_2) \end{pmatrix} \nu \qquad (2)$$

where $\nu = (V^T, \Omega^T)^T$.
The orientation matrices R and J are as [5]:

$$R(\eta_2) = \begin{pmatrix} 1-2(e_2^2+e_3^2) & 2(e_1e_2 - e_3e_0) & 2(e_1e_3 + e_2e_0) \\ 2(e_1e_2 + e_3e_0) & 1-2(e_1^2+e_3^2) & 2(e_2e_3 - e_1e_0) \\ 2(e_1e_3 - e_2e_0) & 2(e_2e_3 + e_1e_0) & 1-2(e_1^2+e_2^2) \end{pmatrix} \qquad (3)$$

$$J(\eta_2) = \frac{1}{2} \begin{pmatrix} -e_1 & -e_2 & -e_3 \\ e_0 & -e_3 & e_2 \\ e_3 & e_0 & -e_1 \\ -e_2 & e_1 & e_0 \end{pmatrix} \qquad (4)$$

We assume in the control section that $|\delta| < \pi$, i.e. $e_0 > 0$.

2.2 Dynamics of the airship

First, let us introduce the following notations: B is the magnitude of the buoyancy force [4]. g is the constant gravity acceleration. $\bar{P_i G}$ represents the position of the ith propeller.
Thus in building the non linear six degrees of freedom mathematical model, the additional following assumptions are made: $\bar{P_1 G} = (0\ 0\ P_1^3)^T$ and $\bar{P_2 G} = (-P_2^1\ 0\ 0)^T$. The vector $\bar{BG} = (0\ 0\ z_b)^T$ denotes the position of the center of buoyancy with respect to the airship-fixed frame. m is the mass of the airship, the propellers and the actuators. Propellers are designed to exert thrust to drive the airship forward.
An airship is an under-actuated system with two types of control: forces generated by thrusters and angular inputs controlling the direction of the thrusters (μ is the tilt angle of the main propeller) :

$$F_1 = \begin{pmatrix} T_m \sin\mu \\ 0 \\ T_m \cos\mu \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 \\ T_t \\ 0 \end{pmatrix} \qquad (5)$$

where T_m and T_t represent respectively the main and tail thrusters.
For the following the control parameters are taken as:

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} T_m \sin(\mu) \\ T_t \\ T_m \cos(\mu) \end{pmatrix} \qquad (6)$$

Remark 1. *In the case where we consider $T_m \neq 0$, the transformation $(\tau_1, \tau_2, \tau_3) \mapsto (T_m, \mu, T_t)$ is a diffeomorphism. Then we can maintain (τ_1, τ_2, τ_3) as a control vector for the airship.*

3 Stabilization

With the previous assumptions, the dynamics and kinematics of a small airship can be written in the following compact form [8]:

$$M_\nu \dot{\nu} + C_\nu(\nu)\nu + D_\nu(\nu)\nu + g_\nu(\eta_2) = B_\tau \tau \quad (7)$$

$$\dot{\eta} = \mathrm{J}(\eta_2)\nu \quad (8)$$

where M_ν is the inertia matrix which is block-diagonal and constant matrix (symmetric and definite positive):

$$M_\nu = \begin{pmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{11} & 0 & I_{13} \\ 0 & 0 & 0 & 0 & I_{22} & 0 \\ 0 & 0 & 0 & I_{13} & 0 & I_{33} \end{pmatrix} \quad (9)$$

and the centrifugal and Coriolis matrix:

$$C_\nu(\nu) = \begin{pmatrix} 0 & -m_{22}r & m_{33}q & 0 & 0 & 0 \\ m_{11}r & 0 & -m_{33}p & 0 & 0 & 0 \\ -m_{11}q & m_{22}p & 0 & 0 & 0 & 0 \\ 0 & (Y_v - Z_z)\omega & 0 & I_{13}q & -I_{22}r & I_{33}q \\ 0 & 0 & (Z_z - X_u)u & -I_{13}p - I_{11}r & 0 & I_{33}p + I_{13}r \\ (X_u - Y_v)v & 0 & 0 & -I_{11}q & I_{22}p & -I_{13}q \end{pmatrix} \quad (10)$$

with $m_{11} = m + X_x$, $m_{22} = m + Y_y$, $m_{33} = m + Z_z$, $m_{13} = X_z$ and $m_{31} = Z_x$ with X_x, Y_y and Z_z are the virtual mass terms of X, Y and Z axes respectively. Further, $I_{11} = I_x + K_x$, $I_{22} = I_y + M_y$, $I_{33} = I_z + N_z$, $I_{31} = -I_{xz} + N_x$ and $I_{13} = -I_{xz} + K_z$ where K_x, M_y and N_z are the virtual inertia terms of X, Y, Z about GX, GY and GZ axes respectively.
The constant definite positive damping matrix D_ν takes the following form:

$$D_\nu(\nu) = diag\{-X_u, -Y_v, -Z_w, -L_p, -M_q, -N_r\} \quad (11)$$

The gravitational vector is

$$g_\nu(\eta_2) = \begin{pmatrix} 2(B - mg)(e_1e_3 - e_0e_2) \\ 2(B - mg)(e_2e_3 + e_0e_1) \\ (B - mg)(1 - 2(e_1^2 + e_2^2)) \\ -2Bz_b(e_0e_1 + e_2e_3) \\ 2Bz_b(e_0e_2 - e_1e_3) \\ 0 \end{pmatrix} \quad (12)$$

and the constant matrix B_τ in (7) represents the directions in which the torques are applied :

$$B_\tau = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ P_1^3 & 0 & 0 \\ 0 & -P_2^1 & 0 \end{pmatrix} \quad (13)$$

3.1 A stabilizing feedback law

We will show first, that it is not possible to stabilize the airship, using a feedback law that is a continuous function of the state only. This follows from results by Brockett [6], Coron and Rosier [2], Coron [1] and Morin [3]. The problem is then not solvable using linearization and linear control theory or classical nonlinear control theory like feedback linearization.

Proposition 3.1.1 *The system (7)-(8) cannot be stabilized by a time invariant smooth pure-state feedback law.*

Proof Let us consider $\epsilon = (\epsilon_1, 0)^T$, from equation (8) we will have $\nu = 0$ since $JJ^T = \frac{1}{4}I_{4\times4}$. Therefore, equation (7) leads to:

$$B_\tau\tau - g_\nu(\eta_2) = M_\nu\epsilon_1 \quad (14)$$

Then if we take $\epsilon_1 = (0, \epsilon_0, 0, 0, 0, 0)^T$ with $\epsilon_0 \neq 0$, we will obtain the following system:

$$\begin{aligned} \tau_1 - 2(B - mg)(e_1e_3 - e_0e_2) &= 0 \\ \tau_2 - 2(B - mg)(e_2e_3 + e_0e_1) &= m_{22}\epsilon_0 \\ \tau_3 - (B - mg)(1 - 2(e_1^2 + e_2^2)) &= 0 \\ 2Bz_b(e_0e_1 + e_2e_3) &= 0 \\ P_1^3\tau_1 + 2Bz_b(e_1e_3 - e_0e_2) &= 0 \\ -P_2^1\tau_2 &= 0 \end{aligned}$$

We can deduce from the last equation that $\tau_2 = 0$. Further, the fourth equation implies $e_0e_1 + e_2e_3 = 0$. As a result: $m_{22}\epsilon_0 = 0$ which is impossible since $\epsilon_0 \neq 0$. Therefore, we cannot stabilize the airship by a continuous pure-state feedback (Brokett's necessary condition [6]). However Coron theorem proves that time periodic continuous feedback is sufficient to stabilize the system to a point.
Instead of working with the original inputs, an approximation that makes sense in terms of stabilization about a reference point is desired. The jacobian linearization of this system about any point is not useful in any control theoretic context since the linearized system is not controllable. However, if the Lie algebra of the set of analytic input vector fields has rank n in then there exists homogeneous degree one approximate system. The use of homogeneous feedback is strongly motivated by the existence of a controllable homogeneous approximate system. Homogeneous degree one control functions should be found such that the origin is uniformly asymptotically stable.
We develop in the sequel a continuous time-varying feedback law. The main result is given by the following proposition.

Proposition 3.1.2 *Consider the function*

$$\begin{aligned} p_d &= -k^r r - k^{e_3}e_3 - k^{e_1}e_1 + \frac{k^v v + k^y y}{\sqrt{|v| + |y|}}\sin(\frac{t}{\varepsilon}) \\ w_d &= -k^z z + 2\sqrt{|v| + |y|}\sin(\frac{t}{\varepsilon}) \\ q_d &= -k^{e_2}e_2 - k^x x - k^u u \end{aligned} \quad (15)$$

furthermore, consider the following time-varying feedback law:

$$\tau_1(\nu,\eta,t) = \frac{1}{P_1^3}\left(-(I_{22}k^q + M_q)q + I_{22}k^q q_d + 2Bz_b e_2\right) \quad (16)$$

$$\tau_2(\nu,\eta,t) = \frac{1}{P_2^1 I_{13}}\left((\Delta k^p + L_p I_{33})p - \Delta k^p p_d - 2Bz_b e_1 I_{33}\right) - \frac{N_r}{P_2^1} r \quad (17)$$

$$\tau_3(\nu,\eta,t) = -(m_{33}k^w + Z_w)w + m_{33}k^w w_d + (B - mg) \quad (18)$$

Where $\Delta = I_{13}^2 - I_{11}I_{33}$. Then, for a suitable choice of the positive parameters k^r, k^{e_3}, k^{e_1}, k^z, k^{e_2}, k^x, and k^u there exists ε_0 such that for any $\varepsilon \in (0, \varepsilon_0]$ and large enough k^q, k^p and k^w the feedback (16)-(18) stabilizes locally exponentially the system (7)-(8). ε is a parameter that we need to adjust.

Proof Let consider the following dilation:

$$\chi_\lambda^\alpha(\nu,\eta,t) = (\lambda u, \lambda^2 v, \lambda w, \lambda p, \lambda q, \lambda r, \lambda x, \lambda^2 y, \lambda z, \lambda e_1, \lambda e_2, \lambda e_3, t) \quad (19)$$

The initial system (7)-(8) can be rewritten as

$$\begin{pmatrix} \dot\nu \\ \dot\eta \end{pmatrix} = f(\nu,\eta,t) + g(\nu,\eta,t) \quad (20)$$

with

$$f(\nu,\eta,t) = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2 + \tau_1) \\ \frac{1}{m_{22}}(Y_v v - 2(B-mg)e_1 + m_{33}pw + \tau_2) \\ \frac{1}{m_{33}}(Z_w w - (B-mg) + \tau_3) \\ \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2Bz_b e_1 I_{33} - P_2^1 I_{13}\tau_2) \\ \frac{1}{I_{22}}(M_q q - 2Bz_b e_2 + P_1^3 \tau_1) \\ \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2I_{13}Bz_b e_1 + P_2^1 I_{11}\tau_2) \\ u \\ v \\ w \\ -\frac{1}{2}(e_1 p + e_2 q + e_3 r) \\ \frac{1}{2}p \\ \frac{1}{2}q \\ \frac{1}{2}r \end{pmatrix} \quad (21)$$

and $g(\nu,\eta,t)$ is the remaining terms.
As the functions τ_1, τ_2 and τ_3 are homogeneous of degree one with respect to the dilation and continuous for $(\nu\ ,\ \eta) \neq 0$, they are continuous at zero. Further, one easily verifies that $f(\nu,\eta,t)$ defines a periodic, continuous homogeneous of degree zero with respect to the dilation. Also, the function $g(\nu,\eta,t)$ is continuous and defines a sum of homogeneous vector field of degree strictly positive with respect to the dilation.

To prove the stability it is well known (see [1]) that it is sufficient to show that the origin of the unperturbed system:

$$\begin{pmatrix} \dot\nu \\ \dot\eta \end{pmatrix} = f(\nu,\eta,t) \quad (22)$$

is locally asymptotically stable.
To this purpose, let us consider the following reduced system obtained from (22), by tacking $q = q_d$, $p = p_d$, and $w = w_d$ as new control variables, and where we have removed the equation corresponding to e_0 as it is uniquely defined by e_1, e_2, and e_3 since the Euler parameters satisfy the equation: $e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$ and we have assumed that $e_0 > 0$

We have obtained the following resulting system:

$$\begin{pmatrix} \dot u \\ \dot v \\ \\ \dot r \\ \dot x \\ \dot y \\ \dot z \\ \dot e_1 \\ \dot e_2 \\ \dot e_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2) + \frac{1}{P_1^3 m_{11}}(-M_q q_d + 2Bz_b e_2) \\ \frac{1}{m_{22}}(Y_v v - 2(B-mg)e_1 + m_{33}p_d w_d) \\ -\frac{1}{P_2^1 I_{13} m_{22}}(-L_p I_{33} p_d + N_r I_{13} r - 2Bz_b e_1 I_{33}) \\ -\frac{1}{I_{13}}(L_p p_d + 2Bz_b e_1) \\ u \\ v \\ w_d \\ \frac{1}{2}p_d \\ \frac{1}{2}q_d \\ \frac{1}{2}r \end{pmatrix} \quad (23)$$

The controls q_d, p_d, and w_d are given by (15). One verifies by application of theorem 3.11. [1] that the origin of the closed loop system is asymptotically stable. Indeed, the vector field associated with right-hand side of the closed loop system is continuous periodic and homogeneous of degree zero with respect to the dilation. Due to the periodic time-variant control, the resulting system is a periodic time-varying system, which can be written in the form:

$$\begin{pmatrix} \dot\nu \\ \dot\eta \end{pmatrix} = h(\nu,\eta,t/\varepsilon) \quad (24)$$

We approximate this system by an averaged system which is autonomous. The averaged system is defined as $(\dot\nu\ \dot\eta)^T = h_0(\nu,\eta)$ where $h_0(\nu,\eta) = \frac{1}{T_T}\int_0^{T_T} h(\nu,\eta,t/\varepsilon)dt$ (T_T is the period). Now, the corresponding averaged system is given by:

$$
\begin{aligned}
\dot u &= \tfrac{1}{m_{11}}(X_u u+2(B-mg)e_2)+\tfrac{1}{P_1^3 m_{11}}(-M_q(-k^{e_2}e_2-k^x x-k^u u)\\
&\quad +2Bz_b e_2)\\
\dot v &= \tfrac{1}{m_{22}}(Y_v v-2(B-mg)e_1-m_{33}k^z z(-k^r r-k^{e_3}e_3-k^{e_1}e_1))\\
&\quad -\tfrac{1}{P_2^1 I_{13} m_{22}}(-L_p I_{33}(-k^r r-k^{e_3}e_3-k^{e_1}e_1)+N_r I_{13} r-2Bz_b e_1 I_{33})\\
\dot r &= \tfrac{1}{I_{13}}(L_p(-k^r r-k^{e_3}e_3-k^{e_1}e_1)+2Bz_b e_1)\\
\dot x &= u\\
\dot y &= v\\
\dot z &= -k^z z\\
\dot e_1 &= \tfrac{1}{2}(-k^r r-k^{e_3}e_3-k^{e_1}e_1)\\
\dot e_2 &= \tfrac{1}{2}(-k^{e_2}e_2-k^x x-k^u u)\\
\dot e_3 &= \tfrac{1}{2} r
\end{aligned} \tag{25}
$$

The linearization of the system (25) about the origin is:

$$
\begin{aligned}
\dot u &= \tfrac{1}{m_{11}}(X_u u+2(B-mg)e_2)\\
&\quad +\tfrac{1}{P_1^3 m_{11}}(-M_q(-k^{e_2}e_2-k^x x-k^u u)+2Bz_b e_2)\\
\dot v &= \tfrac{1}{m_{22}}(Y_v v-2(B-mg)e_1)\\
&\quad -\tfrac{1}{P_2^1 I_{13} m_{22}}(-L_p I_{33}(-k^r r-k^{e_3}e_3-k^{e_1}e_1)+N_r I_{13} r-2Bz_b e_1 I_{33})\\
\dot r &= \tfrac{1}{I_{13}}(L_p(-k^r r-k^{e_3}e_3-k^{e_1}e_1)+2Bz_b e_1)\\
\dot x &= u\\
\dot y &= v\\
\dot z &= -k^z z\\
\dot e_1 &= \tfrac{1}{2}(-k^r r-k^{e_3}e_3-k^{e_1}e_1)\\
\dot e_2 &= \tfrac{1}{2}(-k^{e_2}e_2-k^x x-k^u u)\\
\dot e_3 &= \tfrac{1}{2} r
\end{aligned} \tag{26}
$$

We have transformed the stability analysis in the following two subsystems. The first one is:

$$
\begin{aligned}
\dot u &= \tfrac{1}{m_{11}}(X_u u+2(B-mg)e_2)\\
&\quad +\tfrac{1}{P_1^3 m_{11}}(-M_q(-k^{e_2}e_2-k^x x-k^u u)+2Bz_b e_2)\\
\dot e_2 &= \tfrac{1}{2}(-k^{e_2}e_2-k^x x-k^u u)\\
\dot x &= u
\end{aligned} \tag{27}
$$

and the second subsystem is given by:

$$
\begin{aligned}
\dot v &= \tfrac{1}{m_{22}}(Y_v v-2(B-mg)e_1)\\
&\quad -\tfrac{1}{P_2^1 I_{13} m_{22}}(-L_p I_{33}(-k^r r-k^{e_3}e_3-k^{e_1}e_1)+N_r I_{13} r-2Bz_b e_1 I_{33})\\
\dot r &= \tfrac{1}{I_{13}}(L_p(-k^r r-k^{e_3}e_3-k^{e_1}e_1)+2Bz_b e_1)\\
\dot e_1 &= \tfrac{1}{2}(-k^r r-k^{e_3}e_3-k^{e_1}e_1)\\
\dot e_3 &= \tfrac{1}{2} r\\
\dot y &= v
\end{aligned} \tag{28}
$$

Now, it is clear that for a suitable gain parameters, the origin of the subsystems (27), (28) is obviously asymptotically stable. Therefore, the origin of the system (26) is locally asymptotically stable. Consequently, the origin of the system (25) is asymptotically stable. The asymptotic stability of the origin of the system (23) follows by direct application of corollary 1. [3]. After noticing that the functions q_d, p_d,and w_d are homogeneous of degree one with respect to the dilation, and of class C^1 on $\{\Re^6 \times \Re^3 - (0,0)\} \times \Re$, this ends the proof.

3.2 Simulation results and discussions

The lighter than air platform used for simulations is the AS200 by Airspeed Airships. It is a remotely piloted airship designed for remote sensing. It is a non rigid 6m long, 1.4m diameter and 7.6m3 volume airship. In this section, we present some simulation results. Guided by linear control theory applied to the linearization, we have chosen the following control parameters: $k_p = 0.4$, $k_q = 1.6$, $k_w = 1$, $k_v = 0.5$, $k_r = 0.05$, $k_{e_2} = 1.24$, $k_x = 2$, $k_u = 2$, $k_{e_1} = 0.05$, $k_{e_3} = 0.2$, $k_y = 0.5$, $k_z = 0.8$ and $\epsilon = 0.0001$. The initial position and orientation of the airship are taken as: $[x(0), y(0), z(0), e_1(0), e_2(0), e_3(0]^T = [0.5, -0.4, 0.5, 0.1, 0.1, 0.1]^T$. Initially, the airship was at rest.
Figures 3, 4 and 5 show the time evolution of $x(t)$, $y(t)$ and $z(t)$. However the control inputs are given in figures 7, 8 and 9. Thus we show that the airship converges to the origin.

4 Conclusions

Airships offer a control challenge as they have non zero drift. Their linearization at zero velocity is not controllable. We proved that an airship represented by our model is not stabilizable by continuous state feedback. We have discussed the problem of stabilization of an airship and used the fact that the input vector fields are homogeneous of degree one with respect to some dilation. A feedback that is a homogeneous function of degree one makes the closed loop vector field homogeneous of order zero. In this paper, we have derived an explicit smooth time varying continuous feedback by using time-averaging technique. This feedback being uniformly stabilizing in time then each state may be bounded by a decaying exponential envelope. However, proper modelling of the other aerodynamic effects must be adopted. In this paper, we have studied only local properties. In our future work, we will use the fact that asymptotic stability for the averaged system implies semi-global practical asymptotic stability for the actual system.

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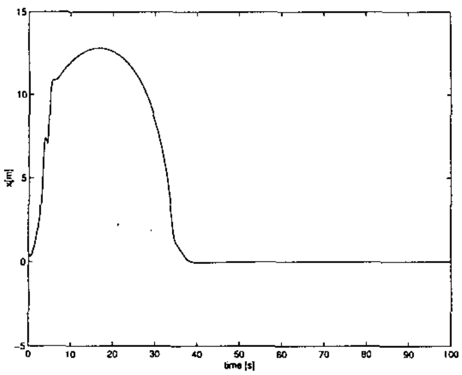

Figure 3: The time evolution of x[m]

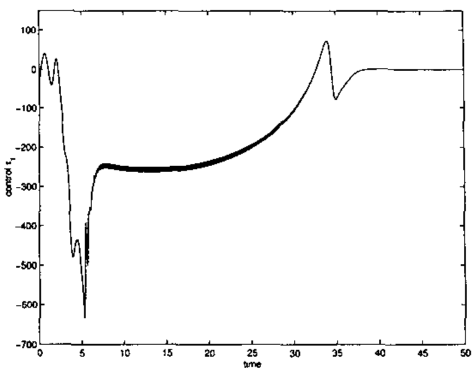

Figure 6: The control input tau1 [N]

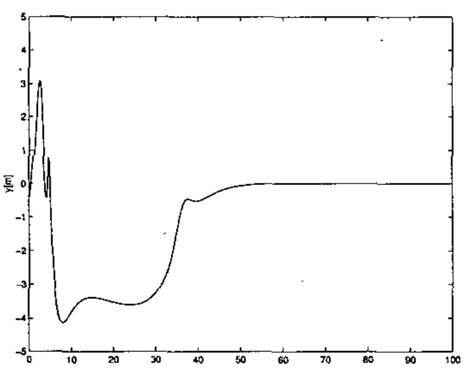

Figure 4: The time evolution of y[m]

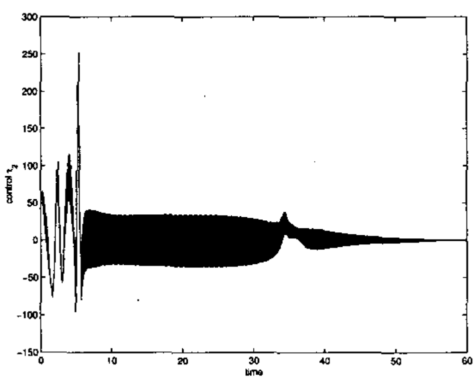

Figure 7: The control input tau2 [N]

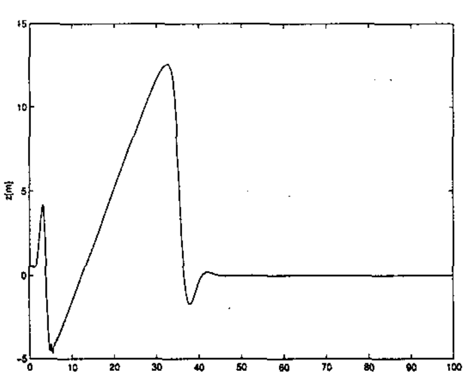

Figure 5: The time evolution of z[m]

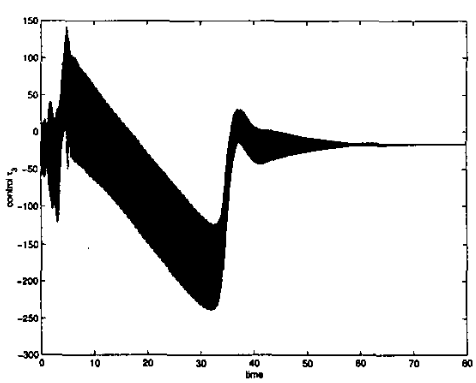

Figure 8: The control input tau3 [N]

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