AN ADAPTIVE CONTROL METHOD OF AUTOMATED VEHICLES WITH INTEGRATED LONGITUDINAL AND LATERAL DYNAMICS IN ROAD FOLLOWING

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Abstract: This paper proposes adaptive control using integrated both longitudinal and lateral dynamics models of automated vehicles. Dynamics are described by a nonlinear non-holonomic model. We show that the dynamics of the dc-actuated vehicle is asymptotically stable, with well updated dynamic parameters.

I. INTRODUCTION

This paper addresses the problem of road following : automatic movement along a predefined path. While an automated vehicle travels at a relatively low speed, controlling it with only a kinematics model may work. However, as automated vehicles are designed to travel at higher speeds, dynamics modeling becomes important. An important characteristic of many of the studies concerning the automated vehicle modeling and control [Cha95, Cho98, Fre97] is that they deal only with some simplified low order linear models. These models are too simple for studying the integrated longitudinal and lateral dynamics. Traditionally, the nonlinear model is considered useful in a simulation environment while the linear model is used for control design.

We are interested in control design for an automated vehicle represented by a full non linear model. [Fre97] have used pole placement technique. However, this technique is known to be sensitive to uncertainties. To take into account uncertainties on aerodynamic parameters and tire – road contact parameters we have chosen an adaptive control method.

II. MODELLING.

A single track model that includes the transverse and longitudinal dynamics, neglects roll and pitch angles and groups the front and rear wheels as a single wheel [Fre97], is considered in this paper. The guidance system operates the steering wheel causing some wheels to work with a sideslip and to generate lateral forces. These forces cause a change of attitude of the vehicle and then a sideslip of all wheels. The resulting forces bend the trajectory. The important dynamical variables are (figure 1): the vehicle orientation ψ, the longitudinal velocity v and the sideslip angle β. In normal road conditions, particularly if radial tires are used, the sideslip angles become large only when approaching the limit lateral forces [Sha 2000].

The actual position of the center of gravity is determined by the Cartesian coordinates x and y in the absolute position. The quantities S_v and S_h represent respectively the front and rear side forces. The rear and front longitudinal forces H and V respectively are resulting from the power train and brakes. The air resistance is represented by T and the steering angle is represented by δ. The following constants are also used : vehicle mass m, moment of inertia I and the distance l_v (l_h) between the front (rear) wheels and the center of gravity.

The dynamic equations are given as [Fre97]:

$$\begin{aligned}
\dot{x} &= v.\cos(\psi-\beta) \\
\dot{y} &= v.\sin(\psi-\beta) \\
\dot{\beta} &= \dot{\psi} - \frac{1}{mv}\begin{Bmatrix}(H-T)\sin\beta + V\sin(\delta+\beta) + S_v\cos(\delta+\beta) \\ + S_h\cos\beta\end{Bmatrix} \\
\ddot{\psi} &= \frac{1}{I}\{S_v l_v\cos\delta - S_h l_h\} \\
\dot{v} &= \frac{1}{m}\{(H-T)\cos\beta + V\cos(\delta+\beta) - S_v\sin(\delta+\beta) - S_h\sin\beta\}
\end{aligned} \tag{1}$$

with the following relations

$$\begin{aligned}
S_v &= \frac{\Gamma_{vm}}{\alpha_{vm}}\left(\beta - l_v\frac{\dot{\psi}}{v} + \delta\right) \\
S_h &= \frac{\Gamma_{hm}}{\alpha_{hm}}\left(\beta + l_h\frac{\dot{\psi}}{v}\right) \\
T &= \frac{1}{2}CA\rho v^2
\end{aligned} \tag{2}$$

where $v = ds/dt$ and $\omega = d\delta/dt$. The aerodynamic resistance coefficient C, atmospheric density ρ and the vehicle cross sectional surface A have been included. Here, $\Gamma_{h,v}/\alpha_{h,v}$ represent the characteristic curves of the tires, with $\Gamma_{vm} = \max(\Gamma_v)$ and $\alpha_{vm} = \max(\alpha_v)$, etc. The characteristic lines include the limitations and the descending behavior for high values in the argument [Fre97].

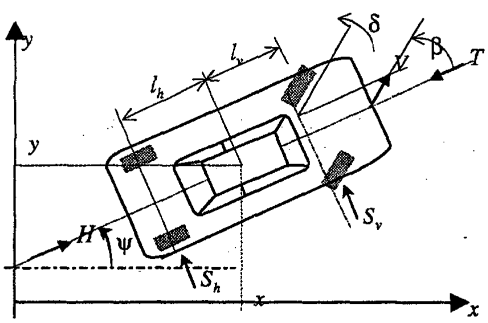

Figure 1. The vehicle parameterization

The steering angle δ of the front wheel is the input for the vehicle in [Fre97]. However, the real input of the guidance system is the torque applied to the steering wheel, thus giving a steering wheel angle resulting in the front wheel orientation.

The sources of non linearities are mainly 3 : the presence of products of the variables of motion in the equation, the presence of trigonometrical functions and the nonlinear nature of the forces due to the tires. All these non linearities are often neglected, such as in [Fre97] [Gen97]. There, the steering angle and the sideslip angles of the wheels and of the vehicle are supposed very small.

In our model, the interaction between longitudinal and lateral forces due to the tires is not neglected.

Let τ_r denotes the rear axle torque and τ_b the steering torque. Then we can write using (1-2)

$$\left.\begin{aligned} \tau_r &= r\left(\begin{aligned} & m\dot{v} + T + mv(\dot{\beta} - \dot{\psi})\cot g(\delta) \\ & \quad + S_v \cot g(\delta)\cos(\delta) + S_h \cot g(\delta) \end{aligned}\right) + \frac{1}{r}(I_r + I_f)\dot{v} \\ \tau_b &= \frac{1}{n} I_{dw}\ddot{\delta} + \frac{1}{n} f_\omega \dot{\delta} \end{aligned}\right\} \quad (3)$$

where $\cot g(\delta) = 1/\tan(\delta)$, r is the wheels radii. I_r regroups the rear axle polar moment of inertia and the motor moment of inertia. I_f is the front axle polar moment of inertia. τ_b is the torque for steering intervention. We exclude here uses of differential brake, and the two wheels' δ is assumed to be identical. For vehicle steering intervention through differential braking we can refer to Pilutti [Pil98].

The handling of the vehicle can be studied using these two equations. Aerodynamics forces being considered in this study, they introduce a strong dependence on v^2. A similar effect but far less important is due to rolling resistance.

The actuator of the steering column is an electric motor with dc-current available on certain standard vehicles. Figure 2 shows the site of the engine on the steering column as well as the sensor of couple to measure the effort exerted on the wheel.

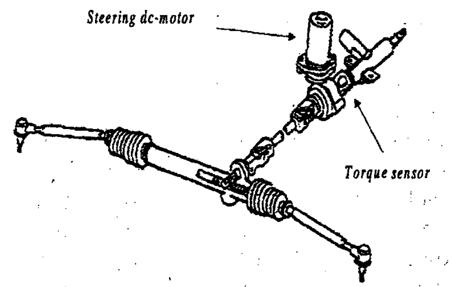

Figure 2. Steering angle control

For a permanent magnet dc-motor, the torque τ is proportional to the armature current J.

Thus actuator dynamics can be characterized in a matrix form as [Bes99] [Bej97]:

$$\left.\begin{aligned} u &= L\dot{J} + R.J + K(v \quad \omega)^T \\ \tau &= (\tau_r \quad \tau_b)^T = K.J \end{aligned}\right\} \quad (4)$$

where L, R and K are 2x2 regular diagonal matrices representing respectively the inductance, resistance and torque constants of the actuators. u is the motor voltage vector. We assume that the transmission from the motors to the mechanism to be perfectly rigid, *i.e* the transmission does not suffer from backlash or flexibility.

III. ADAPTIVE CONTROL.

Interest in adaptive control of nonlinear systems was stimulated by major advances in the differential geometric theory of nonlinear feedback control. A through treatment of this theory was given by Kristic [Kri95]. The control problem formulated here, consists of finding a control law to achieve tracking of a reference trajectory in task space with constant parametric uncertainties of the vehicle.

In fact, it is not easy to measure some physical parameters such that aerodynamics parameters, parameters of the characteristic line describing the side force values in the longitudinal and lateral directions, viscous friction and moment of inertia of the steering wheel around the center of gravity. We rewrite the vehicle kinodynamics model combined with actuators dynamics such that the model is linear in the updated parameters

We start with the following system (5) which is obtained from (3)

$$\left.\begin{aligned} N(y)\ddot{y}+Z(y,\dot{y}) &= \tau \\ \dot{\tau} &= L_{dc}u+R_{dc}.\tau+K_{dc}\dot{y} \end{aligned}\right\} \quad (5)$$

where $y=(s \quad \delta)^T$, $\dot{y}=(v \quad \omega)^T$ and $\ddot{y}=(\dot{v} \quad \dot{\omega})^T$. L_{dc}, R_{dc} and K_{dc} are function of the dc-motor parameters (see appendix). The expressions of $N(.)$ and $Z(.)$ are also given in the appendix.

The uncertainty parameters are regrouped in the following vector :

$$\theta=\begin{pmatrix} rm+\frac{1}{r}(I_r+I_f) \\ r\frac{1}{2}CA\rho \\ rm \\ r\frac{\Gamma_{vm}}{\alpha_{vm}} \\ r\frac{\Gamma_{hm}}{\alpha_{hm}} \\ \frac{1}{n}I_{d\omega} \\ \frac{1}{n}f_{\omega} \end{pmatrix} \quad (7)$$

We are interested in the unknown vehicle parameters (aerodynamic parameters and tire – road contact parameters). Parameters related to the dc-motors can be estimated separately. These parameters should be written linearly in the model in order to use the adaptive control procedure. To confirm the dependence in the constant parameters, we write the dynamics of the vehicle as

$$N(y,\theta)\ddot{y}+Z(y,\dot{y},\theta)=\tau \quad (8)$$

which takes the following compact form

$$\Phi(y,\dot{y},\ddot{y})\theta=\tau_d+e_\tau \quad (9)$$

The form of $\Phi(.)$ is detailed in appendix. $e_\tau=\tau-\tau_d$ denotes the error in torque which can be viewed as a perturbation to the vehicle' dynamics. τ_d is a suitable reference torque which will be specified later. In fact, the dynamic of e_τ is generated by the actuator' dynamics

$$\begin{aligned} \dot{e}_\tau &= \dot{\tau}-\dot{\tau}_d \\ &= -\dot{\tau}_d+K_{dc}\dot{y}+R_{dc}.\Phi(y,\dot{y},\ddot{y})\theta+L_{dc}u \end{aligned} \quad (10)$$

where u is the new input in armature' voltages. The aim of the control is to guarantee $e_\tau \to \infty$ as $t \to \infty$ (time), and to choose τ_d.

Let us introduce the following notations : $\tilde{y}=y_d-y$, where $y_d=(s_d \quad \delta_d)^T$, s_d is an curvilinear abscissa and δ_d is the desired steering angle. $\tilde{\theta}=\hat{\theta}-\theta$, $\hat{\theta}$ is the estimation of θ, $s_y=\dot{\tilde{y}}+\Lambda\tilde{y}$ which is a filter to $\tilde{y}$ with $\Lambda=\Lambda^T>0$.

Lemma 1

The open-loop system dynamics represented by the state vector $x=(\tilde{y} \quad s_y \quad \tilde{\theta} \quad e_\tau)^T$ is as

$$\dot{x}=\begin{pmatrix} \dot{\tilde{y}} \\ \dot{s}_y \\ \dot{\tilde{\theta}} \\ \dot{e}_\tau \end{pmatrix}=\begin{pmatrix} \dot{y}_d-\dot{y} \\ \ddot{y}_r+N(y,\theta)^{-1}[-\tau_d+Z(y,\dot{y},\theta)] \\ \dot{\hat{\theta}} \\ f(e_\tau)+F(e_\tau)\theta+L_{dc}u \end{pmatrix} \quad (11)$$

Proof

The proof is immediate if we consider that $\dot{\theta}=0$ (constant parameters). Further $f(e_\tau)=-\dot{\tau}_d+K_{dc}\dot{y}$ and $F(e_\tau)=R_{dc}\Phi(y,\dot{y},\ddot{y})$.

$\dot{\hat{\theta}}$ is an appropriate adaptive law should guarantee the vehicle dynamic stability.

Lemma 2

First, we consider the following notations

$\Phi \equiv \Phi(y,\dot{y},\ddot{y})$ and $\Phi_r \equiv \Phi(y,\dot{y},\ddot{y}_r)$

Under the following control laws in

$$\tau_d=\Phi_r\hat{\theta}+K_p\tilde{y}+K_v\dot{\tilde{y}} \quad (12)$$

and

$$u=-L_{dc}^{-1}\begin{pmatrix} \Lambda_\tau e_\tau+f(e_\tau)+R_{dc}\Phi_r\hat{\theta} \\ +R_{dc}\left(K_p\tilde{y}+K_v\dot{\tilde{y}}\right) \end{pmatrix} \quad (13)$$

with $K_p=K_p^T>0$, $K_v=K_v^T>0$, the dynamics of the closed loop system is

$$\dot{x}=\begin{pmatrix} \dot{\tilde{y}} \\ \dot{s}_y \\ \dot{\tilde{\theta}} \\ \dot{e}_\tau \end{pmatrix}=\begin{pmatrix} \dot{y}_d-\dot{y} \\ -N(y,\theta)^{-1}\left(\Phi_r\tilde{\theta}+K_p\tilde{y}+K_v\dot{\tilde{y}}\right) \\ \dot{\hat{\theta}} \\ -\Lambda_\tau e_\tau \end{pmatrix} \quad (14)$$

Note that the origin $(\tilde{y}=0, s_y=0, \tilde{\theta}=0, e_\tau=0)$ is an equilibrium point of (14).

Proof

First, we note that

$$\Phi(y,\dot{y},\ddot{y}_r)\hat{\theta} = N(y,\hat{\theta})\ddot{y}_r + Z(y,\dot{y},\hat{\theta})$$

Then, if we consider the form of $\dot{s}$, we get

$$\begin{aligned} N(y,\theta)\dot{s}_y &= N(y,\theta)\ddot{y}_r - N(y,\hat{\theta})\ddot{y}_r \\ &\quad + Z(y,\dot{y},\theta) - Z(y,\dot{y},\hat{\theta}) - K_p\tilde{y} - K_v\dot{\tilde{y}} \\ &= -\Phi_r\tilde{\theta} - K_p\tilde{y} - K_v\dot{\tilde{y}} \end{aligned} \quad (15)$$

Now, the input in voltages can be substituted to obtain the dynamic of $\dot{e}_\tau$. Thus,

$$\dot{e}_\tau = R_{dc}\left(\Phi\theta - \Phi_r\hat{\theta}\right) - R_{dc}\left(K_p\tilde{y} + K_v\dot{\tilde{y}}\right) - \Lambda_\tau e_\tau \quad (16)$$

We compute

$$\begin{aligned} \Phi\theta - \Phi_r\hat{\theta} &= N(y,\theta)(\ddot{y}_r - \dot{s}_y) + Z(y,\dot{y},\theta) \\ &\quad - N(y,\hat{\theta})\ddot{y}_r - Z(y,\dot{y},\hat{\theta}) \\ &= -\left(N(y,\hat{\theta}) - N(y,\theta)\right)\ddot{y}_r \\ &\quad - \left(Z(y,\dot{y},\hat{\theta}) - Z(y,\dot{y},\theta)\right) - N(y,\theta)\dot{s}_y \\ &= -\Phi_r\tilde{\theta} - N(y,\theta)\dot{s}_y \end{aligned} \quad (17)$$

Substituting the expression of $N(y,\theta)\dot{s}_y$ which is given by (15), in (17), leads to

$$\Phi\theta - \Phi_r\hat{\theta} = K_p\tilde{y} + K_v\dot{\tilde{y}} \quad (18)$$

So it is straightforward to verify that $\dot{e}_\tau = -\Lambda_\tau e_\tau$.

Our stability results are formulated in the following Theorem.

Theorem

The parametric model of the dc-actuated longitudinal-lateral vehicle given by (9) and (10)

$$\left.\begin{aligned} &\Phi(y,\dot{y},\ddot{y})\theta = \tau_d + e_\tau \\ &\dot{e}_\tau = -\dot{\tau}_d + K_{dc}\dot{y} + R_{dc}.\Phi(y,\dot{y},\ddot{y})\theta + L_{dc}u \end{aligned}\right\} \quad (19)$$

having as inputs τ_d and u given by (12) and (13), respectively, and the update law

$$\dot{\hat{\theta}} = \Gamma\phi_r^T(\dot{\tilde{y}} + \Lambda\tilde{y}) \quad (20)$$

is asymptotically stable. Then $\tilde{y} \to 0$ as $t \to \infty$. $\Gamma = \Gamma^T > 0$.

Proof

The Lyapunov function is chosen as

$$\begin{aligned} V(x) &= \tfrac{1}{2}s_y^T N(y,\theta)s_y + \tfrac{1}{2}\tilde{y}^T\left(K_p + \Lambda K_v\right)\tilde{y} \\ &\quad + \tfrac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \tfrac{1}{2}e_\tau^T e_\tau \end{aligned} \quad (21)$$

Note that $N(y,\theta)$ depends only in θ, so it is time derivative is equal to zero. The skew symmetry property which is not available here can be ignored. Therefore, the time derivative of V is given by :

$$\begin{aligned} \dot{V}(x) &= s_y^T N(\theta)\dot{s}_y + \tilde{y}^T\left(K_p + \Lambda K_v\right)\dot{\tilde{y}} \\ &\quad + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + e_\tau^T\dot{e}_\tau \end{aligned} \quad (22)$$

which takes this form

$$\begin{aligned} \dot{V}(x) &= (\dot{\tilde{y}} + \Lambda\tilde{y})^T\left[-\Phi_r\tilde{\theta} - K_p\tilde{y} - K_v\dot{\tilde{y}}\right] + \tilde{y}^T K_p\dot{\tilde{y}} \\ &\quad + \tilde{y}^T\Lambda K_v\dot{\tilde{y}} + \tilde{\theta}^T\Phi_r^T\left(\dot{\tilde{y}} + \Lambda\tilde{y}\right) - e_\tau^T\Lambda_\tau e_\tau \end{aligned} \quad (23)$$

some simplifications permits to write

$$\dot{V}(x) = -\tilde{y}^T\Lambda K_p\tilde{y} - \dot{\tilde{y}}^T K_v\dot{\tilde{y}} - e_\tau^T\Lambda_\tau e_\tau \quad (24)$$

which is negative semi-definite (globally) meaning that the origin $(\tilde{y}=0, s_y=0, \tilde{\theta}=0, e_\tau=0)$ is stable but not asymptotically. The LaSalle-Yoshizawa theorem permits to conclude the solution of $\dot{V}(x)=0$ contains $x=0$. To do this, from $\dot{s}$ for $\tilde{y}=\dot{\tilde{y}}=e_\tau=0$, we get $\Phi(y_d,\dot{y}_d,\ddot{y}_d)\tilde{\theta}=0$ which means that $\tilde{\theta}=0$. Therefore the origin is asymptotic stable.

Now to determine a bound of $\tilde{y}$ it is sufficient to prove, referring to the Barbalat' lemma, that

$$\int_0^\infty \|\tilde{y}\|^2 dt \le \frac{V\left(\tilde{y}(0), s_y(0), \tilde{\theta}(0), e_\tau(0)\right)}{\lambda_m(Q)}$$

where $Q = diag\left(\Lambda K_p, K_v, \Lambda_\tau\right)$ and λ_m is the Q' minimum eigenvalue.

As a result $\tilde{y} \to 0$ as $t \to \infty$

IV. NUMERICAL EXAMPLES.

4.1. vehicle characteristics.

Many simulations were performed with a vehicle that characteristics are [Gen97]: $m = 1000Kg$, $I = 1210Kgm^2$, $l_v = 0.87m$ $l_h = 1.29m$

Both motors are identical: $K_{dc} = 1.Nm/A$; $n=1$; $I_{max} = 10A$; $dI_{max} = 10000A$; $V_{max} = 25ms^{-1} = 90Kmh^{-1}$.

4.2. Simulation results.

The simulations are performed using MATLAB software. The parameters of the regulator are chosen as :

K_p = 10I; K_v = I; Γ = I; Λ=I; I is the identity matrix.

The initial conditions are $e_\tau(0) = s_y(0) = \tilde{y}(0) = \tilde{\theta}(0) = 0$. The reference trajectory is calculated using the approach presented in [Bes2000]. We suppose that we have no knowledge about the aerodynamic parameters and the front and rear road-tire contact parameters.

$\hat{\theta}_{2^\wedge init} = 0 \quad \hat{\theta}_{4^\wedge init} = 0 \quad \hat{\theta}_{5^\wedge init} = 0$

The length of the path is 10m. Three cases are tested :

- A straight line (the curvature K(s)=0).
- An arc of circle (the curvature K(s)=0.5).
- A clothoid (the curvature K(s)=0.5*s) : Figure 3 (due to space limit only this figure is presented).

In each figure, appear 12 schemes :

a - the path (y versus x),

the eleven left are all versus the time,

b - the position errors ($x-x_d$; $y-y_d$);

c – the desired and real velocity on the path;

d – the desired and real acceleration;

e – the desired and real jerk;

f – the voltage of the motors;

g – the angle δ ;

h – the angular velocity ω;

i – the desired and real longitudinal torques;

j - the desired and real lateral torques;

k – the variation of the three estimated parameters $\hat{\theta}_2$; $\hat{\theta}_4$ $\hat{\theta}_5$;

l – the derivative of these three estimated parameters.

For the three cases, we obtain the following estimation of these three parameters :

Straight line

$\hat{\theta}_2 = 2.0018 \quad \hat{\theta}_4 = 0 \quad \hat{\theta}_5 = -57.0281$

Arc

$\hat{\theta}_2 = 2.0038 \quad \hat{\theta}_4 = 2.1751 \quad \hat{\theta}_5 = -57.1132$

Clothoid

$\hat{\theta}_2 = 2.001 \quad \hat{\theta}_4 = 7.3899 \quad \hat{\theta}_5 = -57.1991$

V. CONCLUSIONS AND FUTURE WORK.

This paper has presented a vehicle dynamic models suitable for path planning and control studies. We have used an adaptive Lyapunov approach to propose a controller. The aerodynamic coefficients and the rear road-tire contact parameters seem to be constant versus the curvature of the road, while the front road-tire contact parameter is greatly dependent on the curvature of the road.

Although dc-motors have been considered, other actuators such as ac-machines present the same kind of constraints on both the current and voltage.

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Appendix

The dc-motor parameters are the following $R_{dc} = R(nK_t)^{-1}$; $K_{dc} = K_b n$; $L_{dc} = L(nK_t)^{-1}$. R and L are diagonal positive matrices of the armature resistance and inductance respectively. K_b is a diagonal matrix of the back *e.m.f* constant of the motors. u is the armature' input voltages. $n \in R^{2x2}$ is a diagonal matrix of the gear ratios ($n > 0$). $K_t \in R^{2x2}$ ($K_t > 0$) is a diagonal matrix of the motor torque constants.

The expressions of coefficients given in (6)

$$N(y) = \begin{pmatrix} rm + \frac{1}{r}(I_r + I_f) & 0 \\ 0 & I_{d\omega} \end{pmatrix}$$

$$Z(y, \dot{y}) = \begin{pmatrix} r\begin{pmatrix} \frac{1}{2} CA\rho v^2 + mv(\dot{\beta} - \dot{\psi}) \cot g(\delta) \\ + S_v \cot g(\delta) \cos(\delta) + S_h \cot g(\delta) \end{pmatrix} \\ f_\omega \omega \end{pmatrix}$$

The form of $\Phi(.)$ is as follows :

$$\Phi^t(y, \dot{y}, \ddot{y}) = \begin{pmatrix} 0 & \dot{v} \\ 0 & \frac{1}{2} v^2 \\ 0 & v(\dot{\beta} - \dot{\psi}) \cot g(\delta) \\ 0 & (\beta - l_v \frac{\dot{\Psi}}{v} + \delta) \cot g(\delta) \cos(\delta) \\ 0 & (\beta + l_h \frac{\dot{\Psi}}{v}) \cot g(\delta) \\ \ddot{\delta} & 0 \\ \dot{\delta} & 0 \end{pmatrix}$$

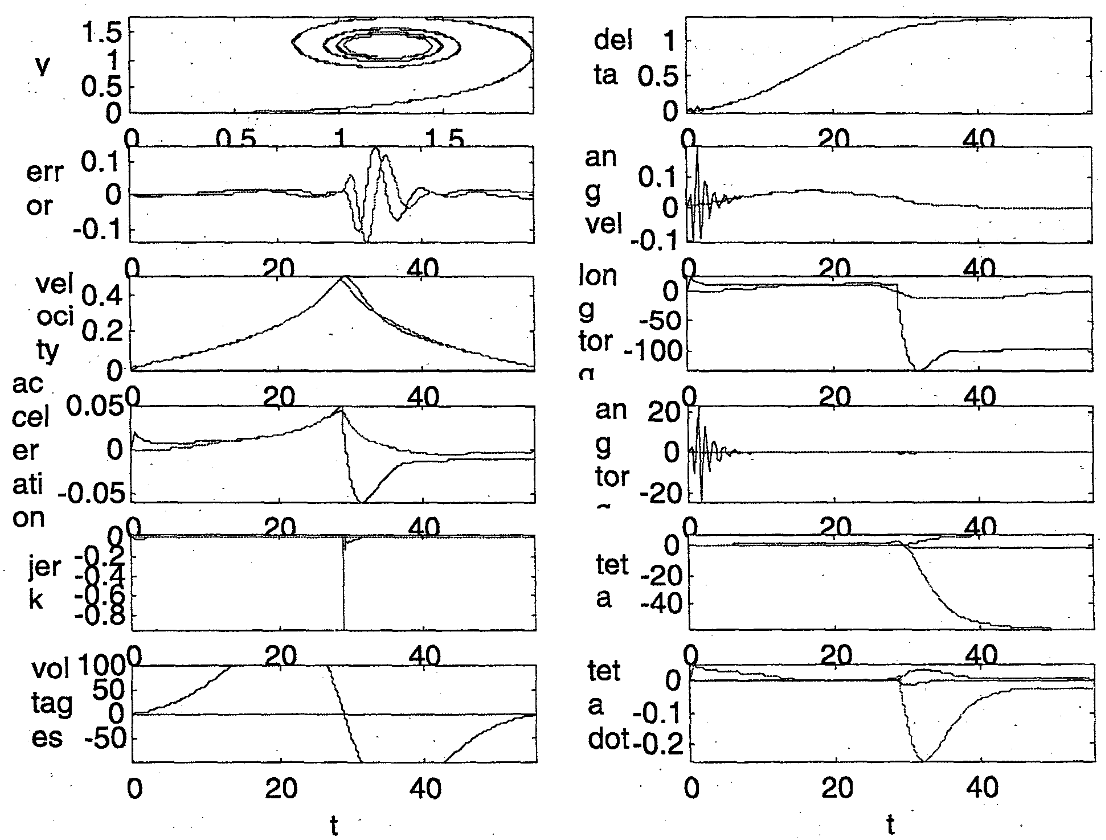

Figure 3. A clothoide (the curvature K(s)=0.5s)