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A DETERMINATION OF ROBOTIC REGULATOR GAINS

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Abstract:
This work presents a stability analysis of the computed torque technique, using the perturbation theory.
Using this analysis a simple nonlinear gain function is proposed for the PID regulators.

I- INTRODUCTION

In this paper, we study the stabilizability problem for robot manipulator control. First, in [4] constant gains were used to stabilize this system . [2] proposed a stability analysis using continuous gains varying linearly and [5] studied constant and variable PD non-linear regulator gains. We finally mention Lie algebra and Lyapunov methods.
Our primary objective in the work described here is to propose a controller that has a simple structure, is easily implemented and is capable of path following for manipulators.

Notations: $\|.\|$ represents the vectorial euclidean norm, $\|.\|_0$ is the associated matricial norm, (.,.) is the scalar product in $\mathbb{R}^n$ and I is the identity matrix.

II- PROBLEM FORMULATION

The dynamics of a multilink articulated robot manipulator can be characterised by a set of nonlinear and coupled second-order differential equation : $\Gamma = D(q)\, q'' + h(q,q')$ (1)

where Γ are n x 1 external applied torques for joint actuators, q, q' and q" are respectively n x 1 joint positions, velocities and accelerations. h(q,q') is the gravitational, Coriolis and Centrifugal force vector and D(q) is the positive definite n x n inertia matrix.
To control the manipulator, the following control law is proposed :

$$\Gamma = \Gamma^d + \Delta\Gamma \quad (2)$$

where $\Gamma^d = \mathcal{D}(q^d)\, q''^d + \mathcal{h}(q^d, q'^d)$ (3)

and

$$\Delta\Gamma = \mathcal{D}(q^d)\,[K_v(t)\,(q'^d - q') + K_p(t)\,(q^d - q) + K_I(t)\int_0^t (q^d(\tau) - q(\tau))\, d\tau] \quad (4)$$

where $\mathcal{D}$ and $\mathcal{h}$ are estimates of D and h respectively, K_v, K_p and K_I are n x n diagonal gain matrices with K_{vj} , K_{pj} and K_{Ij} on the diagonals. Since one does not have access to the exact inverse dynamics, the linearization and the decoupling will not be exact.
The i^{th} joint has as closed loop dynamics:

$$x'''(q,t) + K_v(t)\, x''(q,t) + K_p(t)\, x'(q,t) + K_I(t)\, x(q,t) = J(t,q,q',q'') \qquad x(0) = x'(0) = 0 \quad (5)$$

with $x(q,t) = \int_0^t (q^d(\tau) - q(\tau))\, d\tau$ and

$$J(t,q,q',q'') = \mathcal{D}^{-1}(q^d)\,[\,(\mathcal{D}(q^d) - D(q))\, q'' + \mathcal{h}(q^d,q'^d) - h(q,q')](6)$$

As $\mathcal{D}(q^d)$ is positive definite, its inverse matrix exists.
The functions $\mathcal{D}$, D, $\mathcal{h}$ and h are assumed to be C^1 so they are uniformly bounded when q,q' and q" are bounded, J is also a C^1 function uniformly bounded with respect to its arguments; t>0

III - EXISTENCE AND STABILITY ANALYSIS

The aim of this paragraph is to present a stability analysis of the computed torque technique, giving conditions on the gain parameters.

Theorem : If the gain matrices fulfill the following conditions:

H1) $K_p(t)$, $K_v(t)$ and $K_I(t)$ locally integrable

*) Each coefficient of $K_p(t)$, $K_v(t)$ and $K_I(t)$ is Borel measurable on $C_{r,T}$

**)if $\int_0^t \mathrm{Sup}\,(\,\|K_p(t)\|_0, \|K_v(t)\|_0, \|K_I(t)\|_0\,)\, dt < \infty$

$I_\tau = [0,\tau] \qquad t \in [0,T]$ (7)

H2) $K_p(t)$ is positive definite, uniformly bounded, and its coefficients are derivable,

$\mathrm{Sup}_{t\in I_\tau}(\,\|K_p(t)\|_0\,) < \infty$ and $\mathrm{sup}_{t\in I_\tau}(\,\|K_v(t)\|_0\,) < \infty$ (8)

H3) There exists a constant γ such that:

$K_{vj}\, K_{pj} - K_{Ij} + K'_{pj} = \gamma \qquad j=1,\ldots,n$ (9)

H4) $\|x''(0)\| + \dfrac{\sup(\|J(t,q,q',q'')\|)}{\sup(\,\|K_v(t)\|_0)} \leq \min\, 1, \inf(\,\|K_p(t)\|_0\,)).\ r$ (10)

We have then the following results:

i) Existence and unicity of the solution: The solution x(t) exists and is unique on $C_{r,T}$. a compact subset on $\mathbb{R}^n x[0,T]$

ii) Stability of the solution in the neighbourhood of 0 :

$\forall\, t \in I_\tau,\ \|x(t)\| < r$ (11)

iii) Asymptotic behaviour with respect to K_p.

The solution x(t) of problem (5) is exponentially stable in the neighbourhood of 0.

Proof : The proof of this theorem may be found in [1].

This theorem allows us to analyze the effects of every component appearing in the differential equation on the stability and the asymptotic behaviour of the error e(t). In fact, it shows that if the initial error $\|e(0)\|$ is small enough, then there exists a minimal nonlinear gain such that if $\|K_p(t)\|$ is greater than this gain, then $\|e(t)\|$ stays inside the sphere B(0,r). The reason is that a constant gain may be turned with respect to the most unfavourable case, while a non linear gain can be designed so as to vary with the robot configuration, and assume large values only when it is really needed. Assumptions H1 and H2 of Theorem have more qualitative value than quantitative one. For this reason, we try to determine nonlinear gains functions a priori, starting from practical data and then to verify that they are coherent with the results of the analysis of Theorem. When Coriolis and centrifugal torques are not exactly modelled in the compensating term, their influence increases with the robot velocity. Increasing the $K_{vj}(t)$ gain simultaneously with $q'^d_j(t)$ tends to reduce the related disturbances. We can propose this first law :

$$K_{vj}(t) = |q'^{d}_{j}(t)|,\ K_{pj}(t) = |\alpha_j\ q^{d}_{j}(t)|, \qquad (12)$$

where $\alpha_j > \alpha 0_j$ such that

$$\alpha 0_j = |x''_j(0)| + \frac{\sup(|J(t,q,q',q'')|)}{\sup(|q'^{d}_{j}(t)|)}\ \frac{1}{\inf(|q^{d}_{j}(t)|)}$$

and $\quad K_{Ij}(t) = K_{vj}(t)\, K_{pj}(t) + K'_{pj}(t) + \gamma \qquad (13)$

or this second law :

$$K_{vj}(t) = |e'_j(t)|,\ K_{pj}(t) = |\alpha_j\ e_j(t)| \qquad (14)$$

and K_{Ij} is also given by eqn (13).

V - SIMULATION RESULTS

Many numerical simulations to the first three rotational joints of the robotic manipulator MA23 are illustrated in this section to test the efficiency of the proposed control scheme. The initial and final positions are:

$q_0 = (0,0,0)$rd ; $q_{goal} = (0.5,0.5,0.5)$rd

The velocity and acceleration constraints are:

$q'_{max} = (1,1,1)$ rd/s $\qquad q''_{max} = (3,3,3)$ rd/s^2

The nominal mass of the third axis and the load (m_3=4Kg) is overestimated by 10% with respect to the assigned mass (m_3=4.4Kg). The nominal values were used in the computation of the nonlinear desired feedback. The assigned values were used in the computational implementation of the robot arm.

The Quintic polynomial was chosen as a desired trajectory :

$$q^{d}_{j}(t) = a_{0j} + a_{1j}\ t + a_{2j}\ t^2 + a_{3j}\ t^3 + a_{4j}\ t^4 + a_{5j}\ t^5$$

with null initial and final velocities and accelerations.

The following laws were chosen:

a - TEST 1 : gains are given by eqns (12)-(13) , $[\alpha = 1\ \gamma = 1]$

b - TEST 2 : gains are given by eqns (14) $[\alpha = 10\ \gamma = 1]$

Figures 1 and 2 show the evolution of the first joint PID gains respectively for Tests 1 to 2.

The first and most important simulation result is that the nonlinear gains although very small in regard to the constant gains are sufficient to stabilize the system. This shows the effectiveness of the small gain theorem. Furthermore, the nonlinear control laws perform better when the desired trajectory is very regular. The quintic polynomial gives better results than the third order one. Concerning the robustness, several other simulations were performed that show that the parameter α (see equations 12 and 14) has to be increased to counteract the effect of parameter uncertainties. We also remark that the first law (eqn 12) gives better results than the second law (eqn 14).

VI - CONCLUSIONS

Manipulator's control system based on computer torque technique incorporates a model of the manipulator dynamics. The nominal torque, computed using this mathematical model, does not reflect the effects of unknown loadings and uncertainty in modelling the parameters. An approach is presented in this paper which takes care of this problem. We propose a stability analysis of the computed torque technique using a PID regulator. Then, using the proposed theorem, we employ PID regulators such that the system is asymptotically stable. A method for designing robust controllers that provide guaranteed stability and performance to system with model uncertainties has been presented. As shown by the application of the method to several examples, it seems that the design does not require an important control effort and improves closed-loop system performance.

Simulation results showed that the use of nonlinear gains is quite effective. Only two laws were presented. However, other choices can be made.

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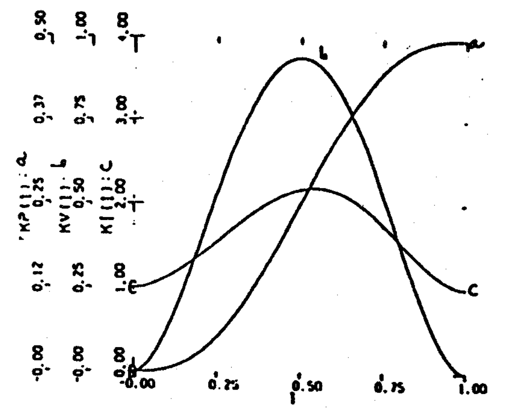

figure 1

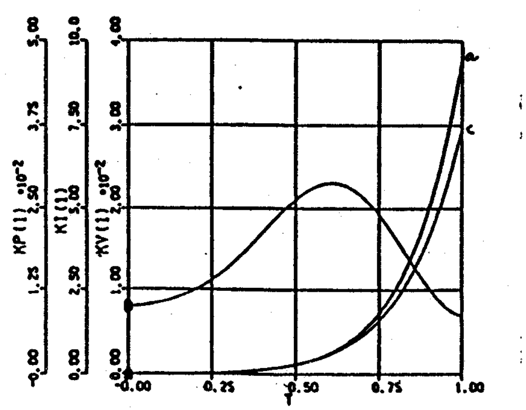

Figure 2