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AN OPTIMAL MOTION PLANNING ALGORITHM FOR MANIPULATORS

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Abstract

The optimal motion generation problem is solved subject to actuator constraints while the motion is constrained to an arbitrary path. The considered objective function is a weighted time-energy function. We present some results using a mathematical programming technique.

1. Introduction

Motion along a predefined path is common in robotics. The path is given from the application and a first step is to obtain a nominal motion specification. In this paper, the emphasis is put on the optimal planning of manipulators trajectories, in joint space. For rigid robots, the minimal time optimization along a predefined path can be solved using phase-plane techniques [6]. This method, however cannot be extended to the case under interest with a time-electric energy performance index. In this paper, the determination of the desired robot motion as a function of time involves a nonlinear optimal problem. We will focus on manipulators actuated by DC motors which operate over a wide speed range and have excellent control characteristics.

2. Modelling

2.1 Manipulator Model

For a manipulator with n joints, the dynamic model can be expressed using the Lagrangian equation as:

$$\Gamma = A(q)\ddot{q} + \dot{q}^T B(q)\dot{q} + G(q) + F(q)\dot{q} \tag{1}$$

where the (n,1) vectors q, $\dot{q}$ and $\ddot{q}$ are the joint position, velocity and acceleration, the (n,1) vector Γ is the joint input torque, G is the (n,1) gravitational force vector, B is the (n,n,n) Coriolis and Centrifugal force matrix, F is the viscous friction and A is the (n,n) inertial matrix.

2.2 Actuator Model

For a non-redundant multi-degrees-of-freedom robot, there are usually as many actuators as the number of d-o-f. In a permanent magnet DC motor, the actuator dynamics, giving the voltage U as a function of the current I are:

$$I = K^{-1}\Gamma \qquad and \qquad U = L\frac{dI}{dt} + RI + K\dot{q} \tag{2}$$

where L, R and K are square regular diagonal matrices representing the inductance, resistance and torque constant.

2.3 Path Description

The path describes the robot motion in space and is represented as a parameterized curve, q = q(s), where s is a scalar path parameter. The optimal path planning problem which consists in finding q(t) is then replaced by the optimal search of s(t) on the interval [0,T].

3. Motion Generation Problem

3.1 Introduction

Most motion generation laws are developed based on kinematical constraints, obtained for the most unfavorable configurations. Thus, to define maximal torques, accelerations and velocities, we have to neglect some terms of the dynamic models (1) and (2). However, the determination of such values allows to propose simple motion generation laws [4].

3.2 Optimal Problem Formulation

Equation (1) can be written as:

$$\Gamma = A_1(s)\ddot{s} + B_1(s)\dot{s}^2 + F_1(s)\dot{s} + G_1(s) \tag{3}$$

$$A_1(s) = A(q(s))\, q_s \qquad F_1(s) = F_v(q(s))\, q_s$$

$$B_1(s) = q_s^T B(q(s))\, q_s + A(q(s))\, q_{ss} \qquad G_1(s) = G(q(s))$$

where q_s and q_{ss} are the derivatives of q with respect to s.

The capabilities of a DC motor are mainly limited by the heat generation and dissipation characteristics. Then the actuator constraint limits the torque (or force), joint speeds and accelerations, applied to each link. The state and control variables are relevent to the path parameter, and chosen to be $x = [s, \dot{s}, \ddot{s}]^T$ and $u = \dddot{s}$, so the differential equation describing the system is given by:

$$\dot{x} = Ax + Bu \text{ where } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

The optimization of the motion along a specified path can be stated as the following problem:

$$Min \quad J = \int_0^T [(1-\alpha) + \alpha U(t)I(t)]dt \qquad Subject\ to$$

$$\dot{x} = Ax + Bu \quad \text{with the actuators limitations:}$$

$$-I_{max} \le I \le I_{max} \qquad -U_{max} \le U \le U_{max} \tag{5}$$

$$-Dimax \le \frac{dI}{dt} \le Dimax \qquad \frac{1}{T}\sqrt{\int_0^T I^2(t)dt} \le Ieffmax$$

where Imax, Dimax and Ieffmax are respectively the maximum absolute values of the motor current instantaneous value, slew rate and square mean value and Umax is the maximum absolute value of the voltage supply. The parameter α is chosen by the user to give more or less weight to time or energy.

4. Resolution Method

4.1 Introduction

In theory, any optimal control problem can be solved analytically by employing Pontryagin's minimum principle. However, it is impracticable to do so if the state vector dimension is higher than two. Then optimal control problems should be solved numerically [1], [2].

All of the available numerical methods first discretize the given continuous system with a fixed sampling period. The period should be small enough so that no significant discretization error is introduced. Since in the discrete domain, the number of variables is itself a variable, this problem can only be considered by solving exhaustively a sequence of fixed time problems [3]. This makes the search iteration count long. This paper fixes the count of steps N and treats the sampling period τ as an optimization variable.

4.2 Discrete Counterpart

With a period τ being a small constant, Euler's first order approximation of the time derivative of the state vector gives a discrete state equation $x_{i+1} = (I+\tau A)x_i + \tau B u_i$. Solving it, gives the state expression at general time k in terms of the initial state and the intermediate inputs:

$$x_k = (I + \tau A)^k x_0 + \tau \sum_{i=0}^{k-1} (I + \tau A)^{k-1-i} B u_i \tag{6}$$

Considering the sampling period as a variable, this problem is transformed into a discrete form as follows:

$$X = [u_0, u_1, ..., u_{N-1}, \tau]^T \quad with \quad dim(X)= N+1 \tag{7}$$

The performance index: $(1-\alpha)\,(N-2)\tau + \alpha\,\tau \sum_{i=0}^{N-2} U_i I_i$

The equality constraints:

$$x_f - (I + \tau A)^{N-2} x_0 - \tau \sum_{i=0}^{N-2} (I + \tau A)^{N-2-i} B u_i = 0$$

Numerous simulations were performed using the NPSOL software, using a Sequential Quadratic Programming technique [5]. The choice of solution method will depend on the information available about the problem functions. For our concrete problem, we have given the gradients of the functions computed with the MAPLE software.

5. Simulation Results

5.1 Simulations

The algorithm was applied to a two d-o-f robot. The proposed path, defined by four crossing points, [0.5,0.3]m, [0.2,0.5]m, [0.0,0.5]m and [0.0,0.4]m, is represented by parts of fifth degree s-polynomials. The dimension of vector X is chosen to be 201, There exists then 1205 constraints (i.e. 3nN+3+n). The bounds of the voltage and current are the different values of the robot of our laboratory : Umax=(30,20)V, Imax=(10,10)A, Ieffmax=(12,10)A, Dimax=(10^4,10^4)A/s.
Most of the nonlinear programming algorithms require a feasible solution set of the optimization variables to start the optimization. We describe a way to deal with this problem. As the different solutions of general minimum-time problems are usually made of phases with positive and negative accelerations, we choose an initial solution of this shape for $[\ddot{s}_k]_{k\leq N-1}$. Moreover, experiences show that convergence was easier if the initial solution is continuous and satisfy $u_0=u_f=0$. So, we finally choose an initial solution X0 varying from umax to -umax. The number of discretization points and umax are user dependable parameters. τ is obtained such as $s(T)=s_f$.
The optimal times obtained respectively for the maximal velocity limit curve method [6] and the one we propose, are T=1.2s and T=1.22s. When the objective function is a weighted time energy function, the maximal velocity limit curve method cannot be used. Using our method, the final time for α=0.1 is T=3.1s and for α=0.5 is T=5.9s.
For α=0, the solution is bang-bang i.e. one of the current (torque) is always saturated, but the voltage sometimes also saturate (figure1). This ensures the limits on joint speeds and accelerations.

5.2 Discussions

The maximal velocity limit curve method [6] gives good results for simple cases. The forward and backward integration must be done with a very low period. Moreover, on a singular arc, the search must be very precise. The nature of possible switching points being very different (tangency, discontinuity and critical), the behaviour of the robot in these points is also very different. The implementation of the algorithm involves the determination of many precision parameters, depending on each followed path. This takes a certain amount of time and needs some numerical experience. Velocity and torques constraints are not sufficient to ensure a safe behavior of the robot, in some cases, this algorithm gives an infinite curvilinear acceleration and thus the real actuators constraints are not fulfilled.
In the authors knowledge, weighted time-electric energy problem is solved for the first time and the obtained results are original. Solutions are smoother than for minimal time approach (figure 2). As α grows from 0 to 1, the motion is slower. The minimal energy approach may be very interesting in some cases.

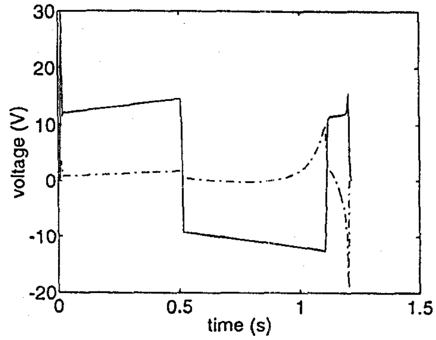

figure 1

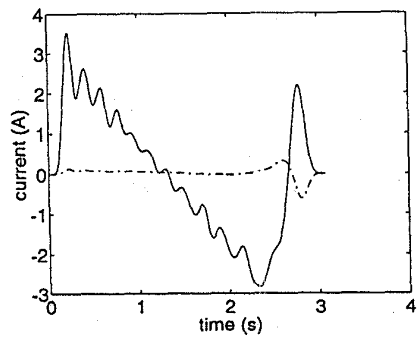

figure 2

6. Conclusions

This paper considers a solution to the problem of moving a manipulator, with weighted time-energy performance index along a specified geometric path subject to voltage and current constraints, taking into account the viscous friction. The motion generation algorithm uses the solution of an optimal problem to find the predicted arrival time as well as the joint acceleration, velocity and position versus time. In contrast to traditional methods in which the count of the control steps is chosen as the variable and an exhaustive sequential search is used to find the minimum time, the proposed approach considers the sampling period as a variable. The optimization problem is solved using the NPSOL software. A weighted time-energy performance index is of great interest since it allows the use of smooth controls while existing methods are only time-optimal ones. We have proposed some comparison remarks.

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