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ACTUATOR CONSTRAINTS IN POINT TO POINT MOTION PLANNING OF MANIPULATORS

Patrick PLEDEL, Yasmina BESTAOUI
Laboratoire d'Automatique de Nantes, URA CNRS 823
Ecole Centrale de Nantes/Université de Nantes, 1 rue de la Noë, 44072 Nantes Cedex 03, France
email: pledel@lan.ec-nantes.fr bestaoui@lan.ec-nantes.fr

I - INTRODUCTION

Minimum time point to point motion planning has been solved considering kinematical constraints on speed and acceleration [3]. These bounds are approximations and imply the full capability of the robot cannot be utilized. Efficiency can be increased by considering the characteristics of the robot dynamics. [1] has presented a trajectory generation based on optimal control formulation, assuming that joint torques are constrained. [2] have shown that most often the structure of the minimum time control requires that at least one of the actuators is always in saturation whereas the others adjust their torques so that some constraints on motion are not violated while enabling the arm to reach its final desired destination.

Although the obtained results are important, they are not applicable directly to an industrial robot. From an user view point, it would be preferable to have a suboptimal but simpler solution to implement. For this purpose, we have chosen a polynomial trajectory and we find parameters of the trajectory, for a minimal time motion. In this paper, the simple expressions previously obtained [3], are extended to actuator constraints.

II - PROBLEM FORMULATION

2-1 Motion description

For joint variables planning, the time history of all variables are planned to describe the desired motion of robots. The trajectory must be chosen smooth enough not to excite the high frequency unmodelled dynamics. Assume the motion is represented as a 5^{th} degree polynomial interpolation of time between two points :

$$q(t) = q_i + D\,r(t/t_f) \quad \text{with} \quad D = q_f - q_i \tag{1}$$

$$r(t/t_f) = 10(t/t_f)^3 - 15(t/t_f)^4 + 6(t/t_f)^5$$

Using time derivation on equation (1) gives :

$$\dot{q} = \frac{D}{t_f}\frac{dr}{dx}\Big|_{x=t/t_f} \qquad \ddot{q} = \frac{D}{t_f^2}\frac{d^2r}{dx^2}\Big|_{x=t/t_f} \tag{2}$$

2-2 Manipulator model

The manipulator is assumed to be made of rigid links. Its dynamic model can be expressed as :

$$\Gamma = A(q)\ddot{q} + \dot{q}^T B(q)\dot{q} + F(q)\dot{q} + G(q) \tag{3}$$

where q, $\dot{q}$ and $\ddot{q}$ are respectively the joint position, velocity and acceleration, Γ is the joint input torque.

From (2) and (3) we obtain the expression :

$$\Gamma = \frac{1}{t_f^2}\tilde{A}(x) + \frac{1}{t_f}\tilde{B}(x) + G(x) \quad \text{with } x = \frac{t}{t_f} \tag{4}$$

$$\tilde{A}(x) = A(q(x))D\frac{d^2r}{dx^2} + D^T B(q(x))D\left(\frac{dr}{dx}\right)^2 \qquad \tilde{B}(x) = F_v D\frac{dr}{dx}$$

t describes the interval $[0,t_f]$ or equivalently x belongs to [0,1]. Thus, if we use x as a variable, we may write Γ as a separate function of t_f and x.

2-3 Actuator model

In a permanent magnet DC motor, the torque Γ is proportionnal to armature current I. For a non-redundant robot, there are usually as many actuators as the number of d-o-f. Then the actuator dynamics for the whole robot can be characterized in a matrix form as :

$$\Gamma = K\,I \quad \text{and} \quad U = L\frac{dI}{dt} + R\,I + K\dot{q} \tag{5}$$

where L, R and K are diagonal matrices representing inductance, resistance and torque constant of actuators, U being the motor voltage.

The following equations will be used :

$$\frac{d\Gamma}{dt} = \frac{1}{t_f^3}\tilde{C}(x) + \frac{1}{t_f^2}\tilde{E}(x) + \frac{1}{t_f}\tilde{F}(x) \quad \text{with} \tag{6}$$

$$\tilde{C}(x) = D^T B(q(x))D\frac{dr}{dx}\frac{d^2r}{dx^2} + D^T B(q(x))D\frac{dr}{dx}\frac{d^2r}{dx^2} + D^T\frac{dB}{dq}(q(x))D^2\left(\frac{dr}{dx}\right)^3 + A(q(x))D\frac{d^3r}{dx^3} + \frac{dA}{dq}(q(x))D^2\frac{d^2r}{dx^2}$$

$$\tilde{E}(x) = F_v D\frac{d^2r}{dx^2} \qquad \tilde{F}(x) = \frac{dG}{dq}(q(x))D\frac{dr}{dx}$$

2-4 Actuator constraints

Capabilities of DC motors are mainly limited by the heat generation and dissipation characteristics. One actuator constraint consists of the limitation of the motor current to avoid demagnetization. The motor voltage is also constrained to a maximum. Besides, the electric drivers are constrained to a maximum current derivative value. Finally we consider the constraint on the current root mean square value to prevent overheating :

$$|I_j| \le I_{max,j} \qquad |U_j| \le U_{max,j} \tag{7}$$

$$\left|\frac{dI_j}{dt}\right| \le dI_{max,j} \qquad \frac{1}{t_f}\sqrt{\int_0^{t_f} I_j^2(t)\,dt} \le Ieff_{max,j}$$

2-5 Problem formulation

The problem may be formulated as follows :

$$\text{Min}\ \{t_f\} \ \text{subject to} \tag{8}$$

$$t_f \in \left\{ \begin{array}{l} t_f \in \mathbf{R}^+ / |I_j| \le I_{max,j}, |U_j| \le U_{max,j}, \\ \left|\frac{dI_j}{dt}\right| \le dI_{max,j}, \frac{1}{t_f}\sqrt{\int_0^{t_f} I_j^2(t)\,dt} \le Ieff_{max,j} \end{array} \right\}$$

III - RESOLUTION METHOD

The optimization theory gives the solution of (8). It is located in the vertex of the admissible set. Assume the robot moves using the maximum motor capabilities. Current bounds lead to the 2^{nd} degree equation in t_f :

$$\left(\pm K I_{max} - G(x)\right) t_f^2 - \tilde{B}(x)\, t_f - \tilde{A}(x) = 0 \qquad (9)$$

This equation always admits two solutions for every joint. Let $t_{fj/I}(x)$ be the smallest real positive root of (9). Thus the candidate time t_f possible for the motion (1) is given by :

$$t_{f/I} = \underset{1\le j\le n}{\mathrm{Max}}\left(t_{fj/I}(x) / x \in [0,1]\right) \qquad (10)$$

The candidate times t_f for the constraints on current derivatives and voltages are also obtained when one of these values reaches its bounds. We then have :

$$\begin{aligned}
&\pm dI_{max} t_f^3 - K^{-1}\tilde{F}(x) t_f^2 - K^{-1}\tilde{E}(x) t_f - K^{-1}\tilde{C}(x) = 0\\
&\pm U_{max} t_f^3 - \hat{A}(x) t_f^2 - \hat{B}(x) t_f - \hat{C}(x) = 0\\
&\text{with}\quad \hat{A}(x) = KD\frac{dr}{dx} + LK^{-1}\tilde{F}(x) + RK^{-1}\tilde{B}(x)\\
&\hat{B}(x) = LK^{-1}\tilde{E}(x) + RK^{-1}\tilde{A}(x) \qquad \hat{C}(x) = LK^{-1}\tilde{C}(x)
\end{aligned} \qquad (11)$$

These 3^{rd} degree equations can be solved analytically. The respective solutions are called $t_{f/dI}$ and $t_{f/U}$.

Constraint concerning the current root mean square value leads to a 5^{th} degree equation in t_f :

$$\begin{aligned}
&Ieff_{max}^2 t_f^5 - t_f^4\int_0^1 \vec{E}(x)dx - t_f^3\int_0^1 \vec{D}(x)dx -\\
&t_f^2\int_0^1 \vec{C}(x)dx - t_f\int_0^1 \vec{B}(x)dx - \int_0^1 \vec{A}(x)dx = 0
\end{aligned} \qquad (12)$$

$$\vec{A}(x) = \left(K^{-1}\tilde{A}(x)\right)^2 \quad \vec{D}(x) = 2K^{-1}\tilde{B}(x)K^{-1}G(x) \quad \vec{E}(x) = \left(K^{-1}G(x)\right)^2$$

$$\vec{B}(x) = 2K^{-1}\tilde{B}(x)K^{-1}\tilde{A}(x) \quad \vec{C}(x) = \left(\left(K^{-1}\tilde{B}(x)\right)^2 + 2K^{-1}G(x)K^{-1}\tilde{A}(x)\right)$$

Such an equation can be solved numerically, giving all the solutions, the smallest real positive root is $t_{f/Ieff}$.

Numerical implementation consists in choosing start and end points. We then calculate the solutions of (9) and (11) for $x \in [0,1]$ with a sufficient discretization. The minimum time is the following maximum value :

$$t_{f/I} = \underset{1\le j\le n}{\mathrm{Max}}\left(t_{fj/I}(x), t_{fj/dI}(x), t_{fj/U}(x) / x \in [0,1]\right) \qquad (13)$$

Besides, the calculus of the coefficient of (12) gives :

$$t_{f/Ieff} = \mathrm{Max}(\text{real root of } (12)) \qquad (14)$$

The final solution t_f is then the maximum of all previous values.

Note that the differents matrices $\tilde{A}$, ..., $\tilde{F}$, $\vec{A}$, ..., $\vec{E}$ are obtained analytically with a low number of operations.

IV - NUMERICAL EXAMPLES

We performed numerical simulations with a 2-rotational d-o-f robot. In this paper, we only present the case of constraints on current : $I_{max} = [10.0,10.0]$ A, $\dot{q}_{max} = [7,10]$ rad/s and $\ddot{q}_{max} = [3.8,50]$ rad/s^2.

We perform various trajectories whose start point is [0.5,0.3] m, and end point belongs to the plane (X > 0, Y > 0). We plot the ratio between the times t_{fk} and t_{fc} found for kinematical and current constraints (i.e. 100 $(t_{fc} - t_{fk}) / t_{fc}$), as a function of the position of the end point (figure 1).

The minimum and maximum values of the ratio are respectively -25 % and +50 %. Negative values seem to show that kinematical constraints could lead to better results, but in fact the currents exceed their bounds. The motions are physically impossible. Then, kinematical constraints obtained with approximations on (3) and (5), are not acceptable, and do not correspond to the worst admissible cases. Besides, when they are acceptable approximations, there exist trajectories for which the gain is non negligible (up to 50%).

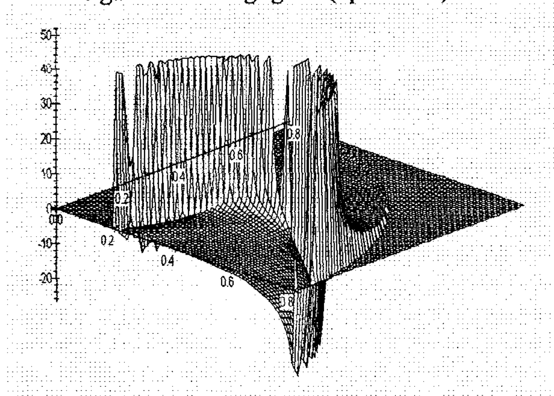

figure 1

Even if the computation time for our formulation is longer and depends on the discretisation adopted, it leads to good results. Using equations (11) and (12) this work will be extended to the more general constraints (7).

V - CONCLUSIONS

In specifying a trajectory, the physical limits of the system must be considered. It is common to model these limits as constant maximum values for acceleration and velocity. The trajectory goes from the initial to the final position with initial and final velocities zero, subject to limits on speed and acceleration. These assumptions are often unrealistic. These considerations mean that even for joint level trajectories, any assumptions about fixed acceleration limits must be based on the worst case. This results in motions that are usually slower than necessary or else the actuators may be unable to follow the requested trajectory. A more realistic assumption means that even for joint level trajectories, any assumption is that the limits on the amount of voltage and current a motor may generate are given limits.

The proposed motion generation algorithm uses the solution of polynomial equations to find the predicted arrival time. This polynomial interpolation with only one parameter (t_f), allows to generate easily the path on line.

VI - REFERENCES

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