Actuator Constraints in Optimal Motion Planning of Manipulators

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Abstract

The optimal motion generation problem is solved subject to various actuator constraints while the motion is constrained to an arbitrary path. The considered objective function is a weighted time energy function while most of existing methods consider only the time-optimal problem. We present some simulation results using a mathematical programming technique (Sequential Quadratic Programming) existing in the NPSOL software.

1 Introduction

Motion along a predefined path is common in robotics, for instance in the case of spraying, cutting or welding. It is natural to look for an optimal solution along the path [9]. The path is given from the application and a first step is to obtain a nominal motion specification. A required path is normally expressed analytically in cartesian space. The resulting trajectory can be expressed either in cartesian space or joint space and the advantages and disadvantages of either representation are consistent with the interpolation method [5]. The geometric path does not contain any timing information but includes only spatial positions and orientations. When a continuous correspondence is made between a trajectory described in cartesian space and joint space, problems may appear related to workspace and singularities. The trajectory must be planned so as to remain in the manipulator workspace. In this case, path generation in joint space could be easily executed but a cartesian trajectory would fail. In this paper, the emphasis is put on the optimal planning of manipulators trajectories, in joint space. The trajectory must be optimal with respect to a specified performance index.

For rigid robots, the minimal time optimization along a predefined path can be solved using phase-plane techniques. This algorithm can however not be extended to the case under interest, the objective function being a weighted time electric energy function. The determination of the desired robot motion as a function of time involves a nonlinear optimal control problem.

We will focus on manipulators actuated by DC motors which operate over a wide speed range and have excellent control characteristics. The motor current of a DC motor is proportional to the torque it generates.

The remainder of this paper is divided into four sections. The problem is formulated in the following paragraph. Then numerical resolution is introduced in the third paragraph while, some simulation results are presented in the fourth section. Finally, general conclusions are given in the last paragraph.

2 Problem formulation

2.1 Manipulator model

The manipulator is assumed to be made of rigid links. For a manipulator with n joints, the dynamic model can be expressed using the Lagrangian equation as:

$$\Gamma = A(q)\ddot{q} + \dot{q}^T B(q)\dot{q} + G(q) + F(q)\dot{q} \qquad (1)$$

where the n x 1 vectors q, $\dot{q}$ and $\ddot{q}$ are respectively the joint position, velocity and acceleration, the n x 1 vector Γ is the joint input torque, G is the n x 1 gravitational force vector, B is the n x n x n Coriolis and Centrifugal force matrix, F is the viscous friction and A is the n x n inertial matrix.

2.2 Actuator model

In a permanent magnet DC motor [8], the magnetic field is developed by permanent magnets. For such a motor, the torque Γ is proportionnal to armature current I. For a non-redundant multi-degrees-of-freedom robot, there are usually as many actuators as the number of degrees-of-freedom. Then we consider the actuator dynamics for the whole robot being characterized in a matrix form as :

$$\Gamma = K I \quad \text{and} \quad U = L\frac{dI}{dt} + R I + K \dot{q} \qquad (2)$$

where L, R and K are square regular diagonal matrices representing the inductance, resistance and torque constant of the robot. U is the motor voltage .

Some additional assumptions are made :

- the kinetic energy of the motor is due mainly to its own rotation. Equivalently, the motion of the rotor is a pure rotation with respect to an inertial frame.

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- the motor/transmission inertia is symmetric about the motor shaft axis of rotation, so that the gravitational potential of the system and also the velocity of the motor center of mass are both independent of motor position.

2.3 Path description

The path describes the robot motion in space. In practice, analytical function are seldom directly available. Usually, the path is specified as a finite number of points which have to be passed in a given order [5]. Assume the path is represented as a parameterized curve q=q(s), where s is a scalar path parameter. A trajectory is obtained from the path by specifying the path parameter as a function of time. The function s(t) is defined on the interval [0,T] where $s(0)=s_0$ and $s(T)=s_f$. Since we assume that the path is fixed, i.e. the function q(s) is given, the trajectory q(s(t)) can be represented by the path parameter s(t). We assume that the path parameter s(t) is piecewise twice differentiable with respect to t. Using the chain rule for differentiation gives :

$$\dot{q} = q_s \dot{s} \quad \text{and} \quad \ddot{q} = q_{ss} \dot{s}^2 + q_s \ddot{s} \qquad (3)$$

where q_s is the vector tangent to the path and q_{ss} is the curvature vector obtained by differentiating q_s with respect to s.

2.4 Rewriting the robot dynamics

Equation (1) describing the robot dynamics, can be substituted, using equations (3) to get :

$$\Gamma=A_1(s)\ddot{s}+B_1(s)\dot{s}^2+F_1(s)\dot{s}+G_1(s) =A_1(s)\ddot{s}+A_2(s,\dot{s}) \qquad (4)$$

where :

$$A_1(s) = A(q(s))\, q_s \quad B_1(s) = q_s^T B(q(s))\, q_s + A(q(s))\, q_{ss}$$
$$F_1(s) = F_v(q(s))\, q_s \quad G_1(s) = G(q(s)) \qquad (5)$$

The parameters A_1, B_1, G_1 and F_1 are path specific, representing the inertia and centrifugal-Coriolis, gravity and friction forces reflected at the joints for a given point along the path.

We write the following equation which will be used in the sequel having time derivative of Γ :

$$\frac{d\Gamma}{dt} = A_1(s)\dddot{s} +[A_{1s}(s)+2B_1(s)]\dot{s}\ddot{s}+B_{1s}(s)\dot{s}^3 +F_{1s}(s)\dot{s}^2+G_{1s}(s)\dot{s}+F_1(s)\ddot{s} \qquad (6)$$

2.5 Optimal problem formulation

The optimization algorithm ([7], [6]) is a tractable way to obtain the minimum-time solution where the constraints are obtained by combining the kinematical and torque constraints, the robot dynamics and the path constraint. In fact the capabilities of a DC motor are mainly limited by the heat generation and dissipation characteristics. One actuator constraint consists of the limitation of the absolute value of the motor current I_{max} to prevent overheating :

$$\|I\| \le I_{max} \qquad (7)$$

The voltage supplied to the motor is also constrained to a maximum U_{max} determined by the value of the motor supply. So another actuator constraint is given by :

$$\|U\| \le U_{max} \qquad (8)$$

Then, these actuator constraints limit the torque (or force) applied to each link. This limitation will also result in limitations of the link speeds and accelerations which maximum values are usually obtained making approximations. In order to avoid those approximations, in this paper we consider constraints (7) and (8).

The state and control variables are respectively chosen to be $x=[s;\dot{s};\ddot{s}]^T$ and $u=\dddot{s}$ so, the differential equation describing the system are given by :

$$\dot{x}= \mathbb{A}\, x + \mathbb{B}\, u \text{ with } \mathbb{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \mathbb{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (9)$$

this system is a linear one. However, the other functions involved are highly nonlinear ones.

The optimization of the motion along a specified path can be stated as the following problem :

$$\text{Min}_{/T} \{J = \int_0^T [(1-\alpha)+\alpha U(t)I(t)]\, dt\} \qquad (10)$$

Subject to

- dynamical model: $\dot{x}= \mathbb{A}\, x + \mathbb{B}\, u$ (11)
- state (position) constraint: $s(0)=0;s(T)=s_f$ (12)
- actuators limitations: $-I_{max} \le I \le I_{max}$ (13)

$-U_{max} \le U \le U_{max}$ (14)

The parameter α is chosen by the user to give more or less weight to time or electric energy. The actuators constraints (eq13-14), using equations (2) and (6), may also be written as:

$$\|A_1(s)\, \ddot{s} + A_2(s, \dot{s})\| \le K\, I_{max} \qquad (15)$$

$$\|LK^{-1}\{A_1(s)\dddot{s} +[A_{1s}(s)+2B_1(s)]\dot{s}\ddot{s}+B_{1s}(s)\dot{s}^3+ F_{1s}(s)\dot{s}^2+G_{1s}(s)\dot{s}+F_1(s)\ddot{s}\}+ RK^{-1}(A_1(s)\ddot{s}+A_2(s, \dot{s}))+K\dot{q}(s)\| \le U_{max} \qquad (16)$$

3 Resolution method

3.1 Introduction

In theory, any optimal control problem can be solved analytically by employing Pontryagin's minimum principle. However, it is impracticable to do so if the dimension of the state vector is higher than two. In optimal control, there are direct and indirect solution methods. Direct means to affect a control history directly by varying a finite set of defining parameters. Indirect means to solve the two point boundary values problems constituted by the necessary conditions of optimality. Such methods are for example : Differential Dynamic Programming technique

[1], or multiple shooting technique [2]. However, they require an a priori identification of singular arcs.

Therefore as practical alternatives, many numerical methods have been developed. All of the available numerical methods first discretize the given continuous system with a fixed sampling period. The period should be small enough so that no significant discretization error is introduced. Then the minimum count of steps are searched for, such that the system reaches the desired final state. Since in the discrete domain, the number of variables is itself a variable, this problem can only be considered by solving exhaustively a sequence of fixed time problems. The difficulties of this method lie in the mechanism of sequencing the fixed time problems and in knowing when to stop [3].

Another difficulty of the discretize-then-search method is the conflicting effect of the sampling period on the discretization error and on the complexity of the optimization problem. Since the final time is fixed for the original continuous system, although it is unknown yet, it should be equal to the product of the sampling period used in the discretization and the count of steps found in the optimization. Thus, if the sampling period is reduced for better accuracy, then the minimum count of steps increases inversely. This makes the search iteration count long, and the problem size becomes larger near the end of the search process. In order to avoid the exhaustive iteration and to overcome the conflicting effect of the sampling period for the discretization error and on the computational complexity, this paper fixes the count of steps and treats the sampling period for the discretization error as an optimization variable. The optimization process minimizes the sampling period and determines the corresponding fixed number of steps to achieve a desired state transition under constraints. Any change of the sampling period during the optimization changes the dynamics of the discrete system which is a counterpart of the original continuous system. Since the usual optimization process is performed on a system whose dynamics are fixed, the proposed approach seems unreasonable. However, the goal of optimization is not only to reduce the sampling period itself, thus reducing the final time (the product of the count of steps and the minimum sampling period obtained in the optimization), but for the discrete counterpart to best approximate the original continuous system.

3.2 Discrete counterpart

With a period τ being a small constant, Euler's first order approximation of the time derivative of the state vector (eq 9) is given as $\dot{x}(t)= [x(t+\tau)-x(t)]/\tau$. This gives a discrete state equation :

$x_{i+1} = (I+\tau\mathbb{A})x_i+\tau\mathbb{B}u_i$ with $x_i=x(i\tau)$ and $u_i=u(i\tau)$ (17)

Solving (17) gives the state expression at general discretized point k in terms of the initial state and the intermediate inputs as :

$$x_k = (I + \tau\mathbb{A})^k x_0 + \tau \sum_{i=0}^{k-1} (I + \tau\mathbb{A})^{k-1-i} \mathbb{B} u_i \quad (18)$$

where $(I+\tau\mathbb{A})^k = \begin{bmatrix} 1 & k\tau & k(k-1)\tau^2/2 \\ 0 & 1 & k\tau \\ 0 & 0 & 1 \end{bmatrix}$ and

$$(I+\tau\mathbb{A})^{k-1-i}\mathbb{B} = \begin{bmatrix} (k-i-1)(k-i-2)\tau^3/2 \\ (k-i-1)\tau^2 \\ \tau \end{bmatrix} \quad (19)$$

With this discretization, the original problem is restated in a new manner. If the original continuous system has a (theoretical) optimal time T, the minimum count of the discrete steps will be $\mathbb{N} = \mathrm{Int}(T/\tau)$.

Considering the sampling period as a variable, this problem is transformed into a discrete form as follows:

The variable $X = [u_0,u_1,...,u_{\mathbb{N}-1},\tau]^T$ (20)

The performance index: $(1-\alpha)(\mathbb{N}-2)\tau+\alpha\,\tau \sum_{i=0}^{\mathbb{N}-2} U_iI_i$ (21)

The equality constraints:

$$x_f - (I+\tau\mathbb{A})^{\mathbb{N}-2} x_0 - \tau \sum_{i=0}^{\mathbb{N}-2} (I+\tau\mathbb{A})^{\mathbb{N}-2-i}\mathbb{B}u_i = 0 \quad (22)$$

The inequality constraints

$-I_{max} \le I_k(X) \le I_{max}$ $\quad$ $-U_{max} \le U_k(X) \le U_{max}$

$-\dot{s}_k \le 0$ $\quad$ for all $0 \le k \le \mathbb{N}-1$ (23)

Then there are $(2n+1)\mathbb{N}+3$ constraints.

4 Simulation results

Numerous simulations were performed using the NPSOL software [4]. NPSOL uses the Sequential Quadratic Programming (SQP) technique which belongs to the class of Projected Lagrangian methods [4]. This class includes algorithms that contain a sequence of linearly constrained subproblems based on the Lagrangian method. The idea of linearizing nonlinear constraints occurs in many algorithms for non linearly constrained optimization including the reduced gradient type methods. The subproblems involves the minimization of a general non linear function subject to linear equality constraints and can be solved using an appropriate technique [4]. The choice of solution method will depend on the information available about the problem functions (i.e. the level and cost of derivatives information) and on the problem size. For our concrete problem, we have given the gradients of the functions (eq 21-23) computed with the MAPLE software, to the NPSOL program.

The algorithm was applied to a two-degree-of-freedom robot (with two rotational joints). The proposed path, defined by four crossing points, is represented by parts of fifth degree s-polynomials. Table 1 gives the crossing points coordinates:

Points	1	2	3	4
Axis X (m)	0.5	0.2	0.0	0.0
Axis Y (m)	0.3	0.5	0.5	0.4

table 1

The dimension of vector X is chosen to be 201. There exists then 1003 constraints. The bounds of the voltage and current are the different values of the SCARA robot of our laboratory : $U_{max}=(30,20)^T$V and $I_{max}=(10,10)^T$A.

Most of the nonlinear programming algorithms require a feasible solution set of the optimization variables to start the optimization. However, in many cases, it is difficult to find a feasible solution. It is known from SQP that it converges quadratically if the initial estimate is sufficiently close to the solution and the hessian of the Lagrangian is positive semi-definite. When the restrictive conditions mentioned above, are not verified, SQP algorithm will in general fail to converge. One reason is that the correct active constraints set must somehow be determined. Another is that the subproblems may be defective because of incompatible constraints. SQP can be viewed as a Newton-method and it is known that an unsafeguarded Newton method is not a robust algorithm. SQP is good in terms of its local convergence properties but not in terms of guaranteed convergence.

We describe a way to deal with this problem. Classical approaches ([6]) show that most solutions in minimum time are bang-bang. Then, for a motion between two points, the motion is made of phases with respectively, positive and negative accelerations. If we approximate this motion with a continuous curve we may obtain the following curves for $\ddot{s}$ and $\dddot{s}$:

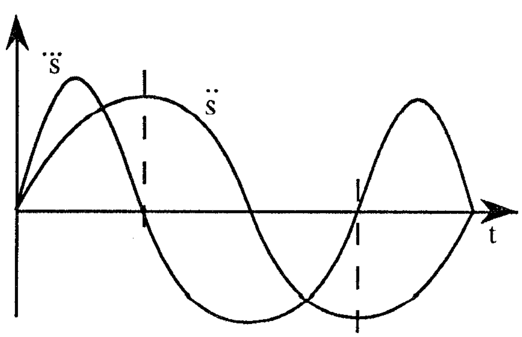

figure 1

In order to generalize this, in the case of several crossing points, we use a polynomial interpolation curve, where the user chooses the number of variations and the maximum value of $\dddot{s}$ = u :

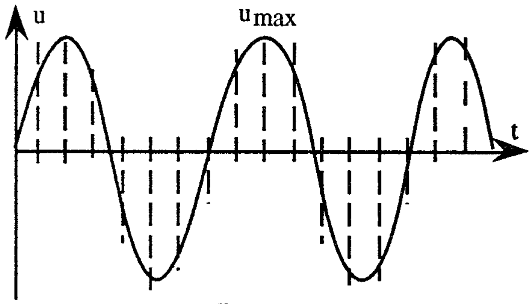

figure 2

The $\mathbb{N}$ first estimated elements of X (see eq. (23)) are then obtained discretizing the previous curve, and τ is solution of $s(T)=s_f$, which gives :

$$\tau = \sqrt[3]{\sum_{i=0}^{\mathbb{N}-2} \frac{2\, s_f}{(\mathbb{N}-i-2)(\mathbb{N}-i-3)\, u_i}} \qquad (24)$$

We performed some simulations with different values of L and R. Table 2 presents the number of iterations and the value of the performance index for different cases of R and L, when $\alpha=0$. The identified values of our robot are L0=[0.006, 0.002]H and R0=[1.2, 2.0]Ω. Those examples were performed with curvilinear accelerations and velocities equal to zero at the extremities:

Example	R (Ω)	L (H)	Nb It	T (sec)
1	R0	L0	181	1.21
2	2.5 R0	2 L0	370	1.27
3	3 R0	2 L0	227	1.34

table 2

We give for every example the shape of the current and the voltage. The first axis is represented with a continuous line, while the second one is plotted with a dotted line. All these figures show the importance of the DC actuators constraints (13-14), particularly for the primary degrees of freedom, for which the motors are more powerful, with non negligeable resistances and inductances. Sometimes, currents are saturated, sometimes the voltages and sometimes both of them. Example 1 shows that the current (torque) of the first axis is always saturated, we then have a bang-bang solution (figure 3). But in transition phases, the voltage is also saturated (axis 1 at the begining, and axis 2 at the end of the motion: figure 4). Thus equation (8) cannot be simplified as $\dot{q}_{max}=K^{-1}(Umax-RImax)$ [9]. Example 2 shows that both constraints on the current and the voltage can saturate (figures 5, 6). And in example 3 only the voltage saturates (figure 8). This actually shows that usual constraints on torques are not sufficient although.

In addition, although there are a lot of constraints and the path is non trivial, the convergence seems to be correct, with a good choice of the initial estimate. In fact, the choice of the initial estimate remains the main problem. Those simulations were performed with an initial curve for u with 11 variations and u_{max}=300. Then an initial value for τ is 14 ms. The choice of s_f=15 was made to contract or expand the curve, so as not to have numerical problems in the calculus of the gradient constraints. Although such a choice has been made, some problems remain (figures 5-8): the final solution can not be improved by NPSOL; it is the reason why some irregularities appear in the last part of the previous figures. After a manual search of all those parameters, the time execution was about two hours on a P.C. with a Pentium 90 processor with 24 Mo RAM.

In the case of a mixed objective function, time execution is the same but the initial estimate involves 27 variations with u_{max}=150. s_f is chosen to be equal to 40. The final shape of the curve obtained when α=0.9, L=L0 and R= R0, in figure 9, shows the importance of α: the curve is smoother but the final time greater (T=3.1 s).

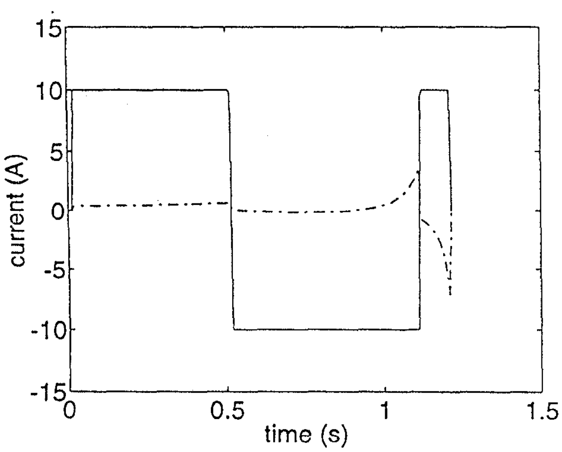

figure 3

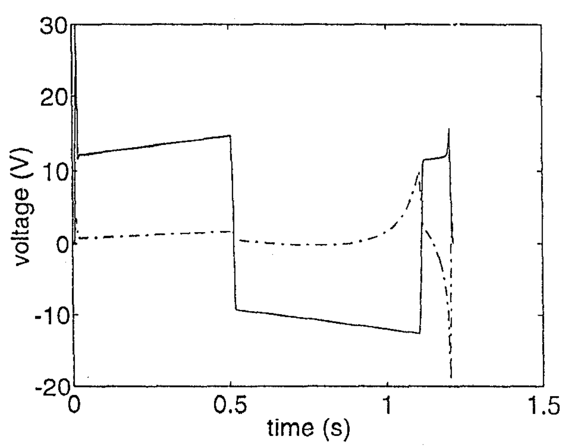

figure 4

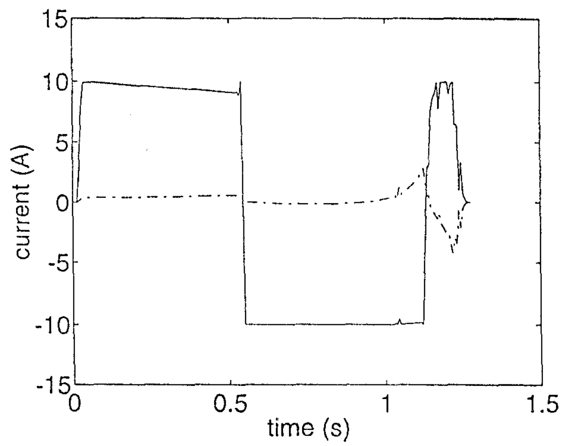

figure 5

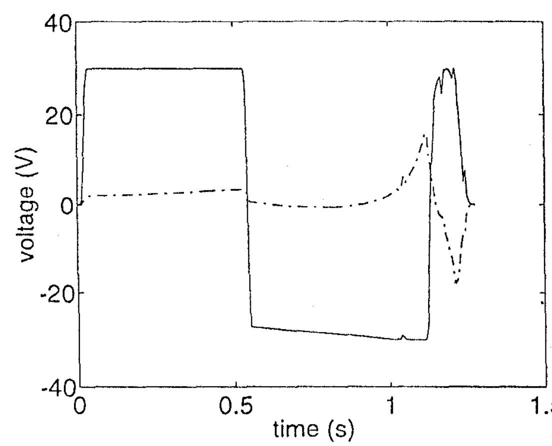

figure 6

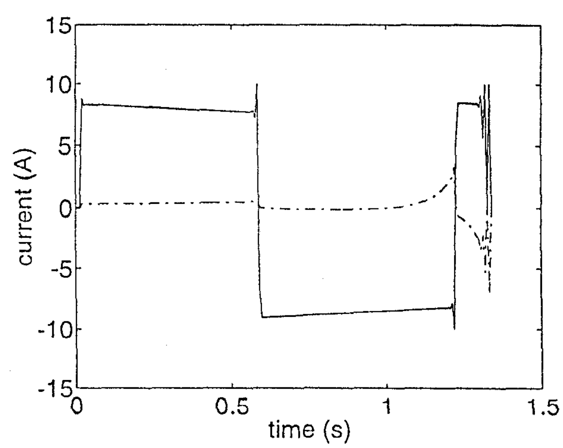

figure 7

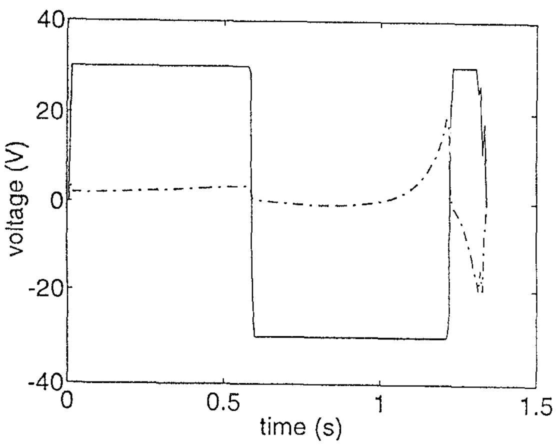

figure 8

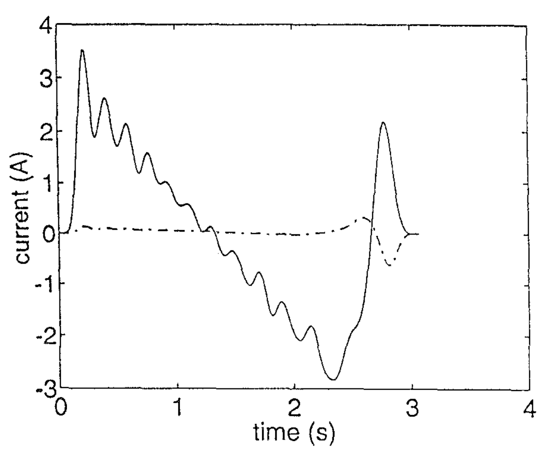

figure 9

5 Conclusions

This paper considers a solution to the problem of moving a manipulator, with weighted time-energy performance index through some given points, subject to voltage and current constraints, taking into account the viscous friction. The motion generation algorithm uses the solution of an optimal problem to find the predicted arrival time as well as the acceleration, velocity and position versus time. The obtained trajectory is continuous on position, velocity and acceleration. A weighted time-energy performance index is of great interest since it allows the use of smooth controls. The current work shows the importance of the actuators constraints on the current and voltage, which should be prefered to the usual constraints on the joint torques, accelerations and speeds. Their maximum values are usually constant and then, do not take into account the nature of the considered path.

In contrast to traditional methods in which the count of the control steps is chosen as the variable and an exhaustive sequential search is used to find the minimum time, the proposed approach considers the sampling period as a variable. The optimization problem is solved using the NPSOL software.

In the future, we will include new constraints on other characteristical elements of the actuators such as the thermic limit on a period given by the current mean square. Besides, drivers do not support infinite slew rate of the actuators. We will also try to orientate our work toward suboptimal solutions in order to have less parameters to adjust. Although we considered DC motors, other actuators generally present the same constraints since usually both the current and voltage of the actuators are bounded .

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