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Polynomial Motion Generation of Manipulators with Technological Constraints

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Abstract

Motion generation gives the joint positions, speeds and accelerations of manipulators, at every moment. We assume that the motion are polynomial trajectories, that ensure joint acceleration continuity. Usual kinematical constraints, obtained with approximations, are not always sufficient, then we will focus on actuators constraints on voltages and currents.

1 Introduction

The minimum time motion generation has been solved in a number of ways, following the usual approach, i.e. taking as the feasible limits purely kinematic constraints on velocity and acceleration [2], [5], [6]. Conventional motion generation in joint space uses a constant bound on the acceleration. This bound must represent the global least upper bound of all operating accelerations so as to enable the manipulator to move under any operating conditions. It implies that the full capabilities of the manipulator cannot be utilized if the conventional approach is taken. The efficiency of the robotic system can be increased by considering the characteristics of the robot dynamics at the motion generation stage. [6] had applied the classical approach of point to point minimum time control to robot arms, where only a linear approximate model was used. [1] has presented a trajectory generation based on optimal control formulation. Assuming that joint torques are constrained and using the Hamiltonian formulation of the dynamic model, a minimum time cost criterion was considered. [3] have shown that most often the structure of the minimum time control requires that at least one of the actuators is always in saturation whereas the others adjust their torques so that some constraints on motion are not violated while enabling the arm to reach its final desired destination.

Several other methods were presented for the resolution of the via points motion problem. [7] used the fact that velocity and acceleration should be as close as possible to their bounds to achieve a time optimal motion. Constant velocity intervals are connected with constant acceleration ones (quadratic arcs). This results in a trajectory of all joints that move close to the given points with hopefully sufficient accuracy. [10] suggested an improvement of the algorithm with respect to the minimum task execution time and accuracy. [8] considered the optimal motion generation problem subject to various actuator constraints while the motion is constrained to an arbitrary path.

Although the obtained results are very important theoretically, practically they are not applicable directly to an industrial robot. From an user view point, it would be preferable to have a somewhat suboptimal but simpler solution to implement. For this purpose, we have chosen, a priori, a polynomial trajectory and we find parameters of the trajectory, for a C^2 minimal time motion. In this paper, using the formal calculus software MAPLE, we will show that the simple expressions previously obtained [5], [6], can be numerically extended when we include different actuator constraints.

The remainder of this paper is divided into six sections. While the models and the proposed problem are formulated in the second and third section, the resolution method is stated in the fourth paragraph. Some simulation results are given in the fifth paragraph and some conclusions added in the last section.

2 Models

2.1 Manipulator Model

The manipulator is assumed to be made of rigid links. For an n joints manipulator, the dynamic model can be expressed as :

$$\Gamma = A(q)\ddot{q} + \dot{q}^T B(q)\dot{q} + F(q)\dot{q} + G(q) \qquad (1)$$

where the vectors q, $\dot{q}$ and $\ddot{q}$ are respectively the joint position, velocity and acceleration, the vector Γ is the joint input torque, G is the gravitational force vector, B is the n x n x n Coriolis and Centrifugal force matrix, F is the viscous friction and A is the n x n inertial matrix. Coulomb frictions are neglected in (1).

2.2 Actuator Model

In a permanent magnet DC motor, the magnetic field is developed by permanent magnets. For such a motor, the torque Γ is proportionnal to armature current I. For a non-redundant multi-degrees-of-freedom robot, there are usually as many actuators as the number of degrees-of-freedom. Then the actuator dynamics for the whole robot can be characterized in a matrix form as :

$$\Gamma = K\,I \quad \text{and} \quad U = L\frac{dI}{dt} + R\,I + K\dot{q} \qquad (2)$$

where L, R and K are square regular diagonal matrices representing the inductance, resistance and torque constant of the robot actuators. U is the motor voltage.

2.3 Actuator Constraints

The capabilities of a DC motor are mainly limited by the heat generation and dissipation characteristics. One actuator constraint consists of the limitation of the absolute value I_{max} of the motor current in order to avoid demagnetization. The motor voltage is also constrained to a maximum U_{max}. Besides, the electric drivers are constrained to a maximum derivative of the current dI_{max}. We consider the constraint on the current mean square value $Ieff_{max}$ to prevent overheating. Finally we also have to fulfill a limitation on joint speeds because of mechanical considerations. Then, for every joint $1 \leq j \leq n$:

$$|I_j| \leq I_{max,j} \qquad (3)$$

$$|U_j| \leq U_{max,j} \qquad (4)$$

$$\left|\frac{dI_j}{dt}\right| \leq dI_{max,j} \qquad (5)$$

$$\sqrt{\frac{1}{t_f}\int_0^{t_f} I_j^2(t)\,dt} \leq Ieff_{max,j} \qquad (6)$$

$$|\dot{q}_j| \leq qp_{max,j} \qquad (7)$$

Most of the time, people use more restricting relations than (3)-(7). They prefer to define maximal accelerations and velocities, making approximations in (1)-(7), in order to find rapidly an upper value for t_f :

$$K_{a,j} = K_j I_{max,j} / \|A(q)\|_{max,j}$$
$$K_{v,j} = K_j^{-1}\left(U_{max,j} - R_j I_{max,j}\right) \qquad (8)$$

3 Problem Formulation

For joint variables generation, the time history of all joint variables and their derivatives are planned to describe the desired motion of manipulators. This has the advantage to give directly the reference trajectories, and not to have to deal with the inverse kinematic models.

The desired trajectory must be chosen smooth enough not to excite the high frequency unmodelled dynamics. It is the reason why we will choose polynomials allowing zero speed and acceleration motion at start and end points.

If we attempt to pass the manipulator through intermediate points with non zero velocity then at all trajectory extremum points, the manipulator could overshoot the trajectory point, as the velocity must change sign either before or after the point. The solution to this problem is to require zero velocity at each trajectory extremum [2].

3.1 Point to Point Motion Description

In order to satisfy the smoothness assumption we assume that the motion is represented as a fifth degree polynomial interpolation of time between two points [9] :

$$q(t) = q_i + D\,r(t/t_f)$$
$$\text{with } D = q_f - q_i \qquad (9)$$
$$r(t/t_f) = 10(t/t_f)^3 - 15(t/t_f)^4 + 6(t/t_f)^5$$

Using time derivation on equation (9) gives the joint speeds and accelerations, with $x=t/t_f$:

$$\dot{q} = \frac{D}{t_f}\frac{dr}{dx}\Big|_{x=t/t_f} \qquad \ddot{q} = \frac{D}{t_f^2}\frac{d^2r}{dx^2}\Big|_{x=t/t_f} \qquad (10)$$

3.2 Via Points Motion Description

Now the manipulator has to pass through m+1 via points. The motion is supposed to have a continuous acceleration. Start and end points are chosen to have joint speeds and accelerations equal to zero. For those reasons we represent the motion with a fifth degree polynomial between each crossing point rather than cubic splines that offer less possibilities :

$$q_{j,k}(t) = \sum_{i=0}^{5} a_{i,j,k}\left(\frac{t}{t_{f,k}}\right)^i \qquad \begin{cases} 1 \leq k \leq m \\ 1 \leq j \leq n \end{cases} \qquad (11)$$

Assuming continuity of joint positions, speeds and accelerations for each polynomial, we obtain the following equations :

$$q_{j,k}=q_{j,k}(0) \quad \dot{q}_{j,k}=\dot{q}_{j,k}(0) \quad \ddot{q}_{j,k}=\ddot{q}_{j,k}(0) \tag{12}$$
$$q_{j,k+1}=q_{j,k}(t_{f,k}) \quad \dot{q}_{j,k+1}=\dot{q}_{j,k}(t_{f,k}) \quad \ddot{q}_{j,k+1}=\ddot{q}_{j,k}(t_{f,k})$$

We finally obtain, for each polynomial, the coefficients as a function of the various positions, speeds and accelerations. But, in order to have expressions which are not explicit functions of the final time $t_{f,k}$, we operate the following change of parameters :

$$\dot{q}_k=\hat{\dot{q}}_k / t_{f,k} \quad \ddot{q}_k=\hat{\ddot{q}}_k / t_{f,k}^2 \quad \Phi_k=\frac{t_{f,k+1}}{t_{f,k}} \tag{13}$$

Then the previous equations (12) lead to :

$$a_{0,k}=q_k \qquad a_{1,k}=\hat{\dot{q}}_k \qquad a_{2,k}=\frac{1}{2}\hat{\ddot{q}}_k \tag{14}$$

$$a_{3,k}=10(q_{k+1}-q_k)-\left(6\hat{\dot{q}}_k+4\frac{\hat{\dot{q}}_{k+1}}{\Phi_k}\right)+\frac{1}{2}\left(\frac{\hat{\ddot{q}}_{k+1}}{\Phi_k^2}-3\hat{\ddot{q}}_k\right)$$

$$a_{4,k}=-15(q_{k+1}-q_k)+\left(8\hat{\dot{q}}_k+7\frac{\hat{\dot{q}}_{k+1}}{\Phi_k}\right)-\frac{1}{2}\left(2\frac{\hat{\ddot{q}}_{k+1}}{\Phi_k^2}-3\hat{\ddot{q}}_k\right)$$

$$a_{5,k}=6(q_{k+1}-q_k)-\left(3\hat{\dot{q}}_k-3\frac{\hat{\dot{q}}_{k+1}}{\Phi_k}\right)+\frac{1}{2}\left(\frac{\hat{\ddot{q}}_{k+1}}{\Phi_k^2}-\hat{\ddot{q}}_k\right)$$

In addition for each polynomial, the duration $t_{f,k}$ could be expressed with the value of the first one $t_{f,1}$:

$$t_{f,k+1}=\left[\prod_{j=1}^{k}\Phi_j\right]t_{f,1} \tag{15}$$

The general problem of minimum time motion may be formulated as follows :

$$\text{Min}\ \{t_{f,1}\}\ \text{subject to} \tag{16}$$
$$t_{f,1}\in\left\{\begin{array}{l} t_{f,1}\in\mathbf{R}^+ / |\dot{q}_j|\le qp_{\max,j}, |I_j|\le I_{\max,j}, |U_j|\le U_{\max,j}, \\ \left|\frac{dI_j}{dt}\right|\le dI_{\max,j}, \sqrt{\frac{1}{t_f}\int_0^{t_f} I_j^2(t)dt}\le Ieff_{\max,j}\end{array}\right\}$$

Note that in point to point motion $t_{f,1}=t_f$. In via point motion the variables to be optimized are the previous parameters introduced in (13) :

$$X=\left[\hat{\dot{q}}_{j,k},\hat{\ddot{q}}_{j,k},\Phi_k\right] \quad \begin{cases}1\le j\le n\\ 2\le k\le m-1\end{cases} \tag{17}$$

4 Resolution Method

4.1 Proposed Minimum Time Approach

The optimization theory gives the solution of problem (16). It is located in the vertex of the admissible set. The resolution will be organized as follows. First, this problem will be solved for each constraint (3)-(7), then the greatest value of all the proposed times will be taken as the predicted arrival time t_f. Let us assume the robot moves using the maximum motor capabilities.

First we consider joint speeds limitations (7). The minimal time is obtained when, using (10) :

$$qp_{\max}=\frac{D}{t_f}\frac{dr}{dx} \quad \text{or} \quad t_{fj/qp}(x)=\left|\frac{D_j}{qp_{\max,j}}\frac{dr}{dx}\right| \tag{18}$$

t describes the interval $[0,t_f]$ or equivalently x belongs to [0,1]. Thus, if we use x as a variable, we may write Γ as a separate function of t_f and x. From (1) and (10), we obtain the following expression :

$$\begin{aligned}
&\Gamma=\frac{1}{t_f^2}\tilde{A}(x)+\frac{1}{t_f}\tilde{B}(x)+G(x)\\
&\tilde{A}(x)=A(q(x))D\frac{d^2r}{dx^2}+D^T B(q(x))D\left(\frac{dr}{dx}\right)^2\\
&\tilde{B}(x)=F_v D\frac{dr}{dx}
\end{aligned} \tag{19}$$

We also write the following equations which will be used in the sequel having time derivative of Γ :

$$\frac{d\Gamma}{dt}=\frac{1}{t_f^3}\tilde{C}(x)+\frac{1}{t_f^2}\tilde{E}(x)+\frac{1}{t_f}\tilde{F}(x) \tag{20}$$
$$\tilde{C}(x)=A(q(x))D\frac{d^3r}{dx^3}+\frac{dA}{dq}(q(x))D^2\frac{d^2r}{dx^2}\frac{dr}{dx}+$$
$$2D^T B(q(x))D\frac{dr}{dx}\frac{d^2r}{dx^2}+D^T\frac{dB}{dq}(q(x))D^2\left(\frac{dr}{dx}\right)^3$$
$$\tilde{E}(x)=F_v D\frac{d^2r}{dx^2} \qquad \tilde{F}(x)=\frac{dG}{dq}(q(x))D\frac{dr}{dx}$$

Using eqns (2) and (19), current bounds (3) lead to the following second degree equation in t_f :

$$(\pm KI_{max}-G(x))t_f^2-\tilde{B}(x)t_f-\tilde{A}(x)=0 \tag{21}$$

For every joint (j=1,n), the solution $t_{fj/I}(x)$ of (21) can be approximated, neglecting the gravity and viscous friction, by :

$$t_{fj/I}(x) \approx \sqrt{\tilde{A}(x)/(\pm K I_{max})} \qquad (22)$$

Then, we can suppose that there always exists a real positive root for $t_{fj/I}(x)$.

The candidate times t_f for the constraints on current derivative (5) and voltage (4) are also obtained when one of these values reaches its bounds. We then have two third degree equations in t_f to solve :

$$\pm dI_{max} t_f^3 - K^{-1}\tilde{F}(x) t_f^2 - K^{-1}\tilde{E}(x) t_f - K^{-1}\tilde{C}(x) = 0 \qquad (23)$$

$$\begin{aligned} &\pm U_{max} t_f^3 - \hat{A}(x) t_f^2 - \hat{B}(x) t_f - \hat{C}(x) = 0 \\ &\text{with} \quad \hat{A}(x) = KD\frac{dr}{dx} + LK^{-1}\tilde{F}(x) + RK^{-1}\tilde{E}(x) \\ &\qquad \hat{B}(x) = LK^{-1}\tilde{E}(x) + RK^{-1}\tilde{A}(x) \\ &\qquad \hat{C}(x) = LK^{-1}\tilde{C}(x) \end{aligned} \qquad (24)$$

A third degree polynomial equation can be solved analytically. It may have 1 or 3 real roots from which we choose the smallest positive value (or zero if it does not exist). Such equations can be solved using the formal calculus software MAPLE. The respective solutions are called $t_{fj/dI}(x)$ and $t_{fj/U}(x)$.

The constraint concerning the current root mean square value (6) leads to a fourth degree equation in t_f :

$$\begin{aligned} &Ieff_{max}^2 t_f^4 - t_f^4\int_0^1 \vec{E}(x)dx - t_f^3\int_0^1 \vec{D}(x)dx - \\ &t_f^2\int_0^1 \vec{C}(x)dx - t_f\int_0^1 \vec{B}(x)dx - \int_0^1 \vec{A}(x)dx = 0 \end{aligned} \qquad (25)$$

where :

$$\vec{A}(x) = \left(K^{-1}\tilde{A}(x)\right)^2 \qquad \vec{D}(x) = 2K^{-1}\tilde{B}(x)K^{-1}G(x)$$

$$\vec{B}(x) = 2K^{-1}\tilde{B}(x)K^{-1}\tilde{A}(x) \qquad \vec{E}(x) = \left(K^{-1}G(x)\right)^2$$

$$\vec{C}(x) = \left(\left(K^{-1}\tilde{B}(x)\right)^2 + 2K^{-1}G(x)K^{-1}\tilde{A}(x)\right)$$

Such an equation can be solved numerically, giving all the four solutions, using the software MAPLE (the smallest real positive solution is called $t_{fj/Ieff}$).

4.2 Numerical Implementation

All the differents matrices $\tilde{A}$, ..., $\tilde{F}$, $\hat{A}$, ..., $\hat{C}$, and $\vec{A}$, ..., $\vec{E}$ are obtained analytically. Then, in point to point motions, the numerical implementation consists in choosing the start and end points of the path. We then calculate the solutions of (18), (21), (23) and (24) for every x belonging to [0,1] (i.e. with a sufficient discretisation). Besides, the numerical calculus of the coefficient of (25) allowed MAPLE to give all the complex solutions. The minimal time is the following maximum value :

$$t_f = \max_{1\le j\le n}\left(\left\{t_{fj/qp}(x), t_{fj/I}(x), t_{fj/dI}(x), t_{fj/U}(x) / x \in [0,1]\right\}, t_{fj/Ieff}\right) \qquad (26)$$

In via point motions, the problem implies an optimization of the variables X (17). Using those parameters allows to calculate for each polynomial, as in point to point motion, the roots of (18), (21), (23) and (24). But using (15), showing the relation of time $t_{f,k}$ with X, we prefer to obtain the value $t_{f,1}^{(k)}$ of $t_{f,1}$ for the k^{th} polynomial. To ensure the constraints on the whole trajectory, the result is then :

$$t_{f,1} = \max_{1\le k\le m}\left(t_{f,1}^{(k)}, \max_{1\le j\le n}\left(t_{fj/Ieff}\right) / \prod_{i=1}^{m-1}\Phi_i\right) \qquad (27)$$

$$t_{f,1}^{(k)} = \max_{1\le j\le n}\left(t_{fj/qp}(x), t_{fj/I}(x), t_{fj/dI}(x), t_{fj/U}(x) / x \in [0,1]\right)$$

The result is a function of the optimization variables X. As in many optimization problems one of the hardest part of the study lies in the initial estimate [8]. We provide initial values to X, solving a minor optimization problem under kinematic constraints, not involving models (1) and (2). The initial estimate of this problem is easily obtained using classical results [5], for a point to point motion on each polynomial of the curve :

$$t_f = \max_{1\le j\le n}\left(\frac{15|D_j|}{8K_{vj}}, \sqrt{\frac{10|D_j|}{\sqrt{3}K_{aj}}}\right) \qquad (28)$$

Because of the large number of parameters allowed by 5^{th} degree polynomial (speeds and accelerations) we notice that for non restricting axis, the value of $\dot{q}_{j,k}$ and $\ddot{q}_{j,k}$ do not change. Then we optimize each axis separately, find the restricting axis jc and generate new values of $\dot{q}_{j,k}$ and $\ddot{q}_{j,k}$ ($j \ne jc$) :

$$j = jc \Rightarrow \begin{cases} t_{f,k} \\ \dot{q}_{jc,k} \\ \ddot{q}_{jc,k} \end{cases} \quad j \ne jc \Rightarrow \begin{cases} \alpha = \dfrac{1}{2}\left(\dfrac{t_{f,k}^{j\ne jc}}{t_{f,k}} + \dfrac{t_{f,k+1}^{j\ne jc}}{t_{f,k+1}}\right) \\ \alpha\, \dot{q}_{j,k} \qquad \alpha^2\, \ddot{q}_{j,k} \end{cases} \qquad (29)$$

For non restricting axis, the new speeds and accelerations are obtained using a medium value based on the times of adjacent polynomials. This choice has been used because it leads to smooth trajectories with no overshoots or loops.

kinematic constraints are satisfied, and an initial estimate of X is obtained with the values of $t_{f,k}$, $\dot{q}_{j,k}$ and $\ddot{q}_{j,k}$ (13).

We are now able to compute an unconstrained optimization problem, solving polynomial equations, using the initial estimate. Our formulation ensure that all the constraints are satisfied.

5 Numerical Examples

5.1 Robot Characteristics

We performed numerical simulations with a two degrees-of-freedom SCARA like robot (simulating our lab robot) that arm lengths are 0.5 m and 0.3 m.

The different values of the actuators limitations are :

I_{max} = [11.53,7.29] A U_{max} = [40.0,26.3] V
dI_{max} = [10^4,10^4] A/s $Ieff_{max}$ = [12.0,10.0] A
qp_{max} = [7.0,21.0] rad/s

Besides, the maximum admissible joint speeds and accelerations, for our robot, are :

K_v = [7.0,10.0] rad/s K_a = [3.84,49.6] rad/s^2

We did not consider the constraint on current root mean square value (6) here to simplify the problem.

5.2 Simulation Results

In the beginning, we present two examples of a point to point motion for constraints (3)-(7), for a fifth degree polynomial interpolation between two points (9) :

Example		cartesian position	joint position
1	start point	[0.5,0.3]m	[0.0,1.57]rad
	end point	[-0.2,0.4]m	[1.399,2.056]rad
2	start point	[0.5,0.3]m	[0.0,1.57]rad
	end point	[0.78,0.08]m	[-.051,0.412]rad

table 1

For the first example, when we use the classical results of (28), we obtain the minimum time t_{fk1} = 1.45 s. With our new formulation (26), involving actuator constraints, the value is t_{fa1} = 1.36 s. Then, we see that we have a gain on the final time about 7%, which is non negligible when the task is highly repetitive.

Figures 1 and 2 are relative to the first example, and present the current of the first axis (for which the bound is reached), when $t_f = t_{fk1}$ and when $t_f = t_{fa1}$. The differences between those two results justify our approach.

In the second example, we see that (8) are not acceptable approximations. The time obtained with (28), t_{fk2} = 0.37 s, is lower than the one for (26), t_{fa2} = 0.39 s, but the current of the first joint exceeds its bound (figure 3). In fact, the values K_v and K_a do not correspond to the worst admissible case (especially when Coriolis and Centrifugal forces are important).

Besides, because of the result of this second example, for which (28) leads to a non admissible motion, we calculate the real maximum values of the accelerations K_{ar} reached by the robot during the motion. The new value K_{ar} = [2.5,45.0] rad/s^2 is adopted for the third example.

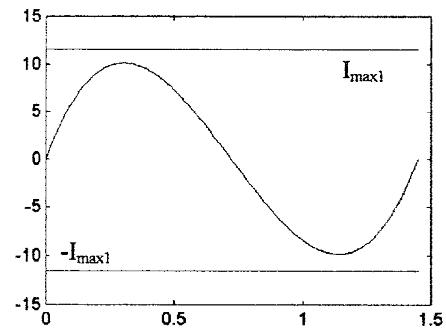

figure 1

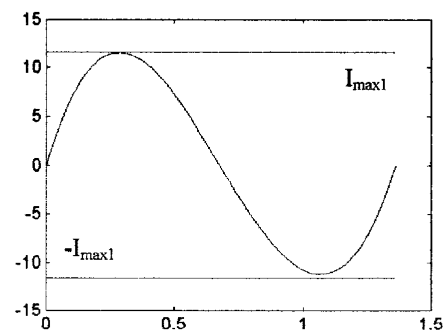

figure 2

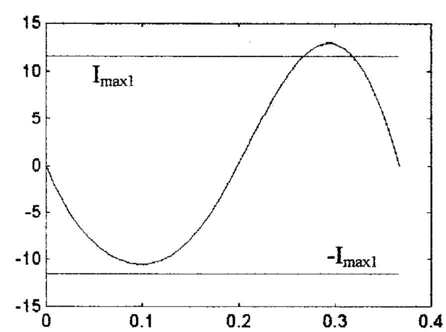

figure 3

A third example is proposed for a via point motion (11), with 6 crossing points : [0.3,0.1] m, [0.4,0.1] m, [0.6,0.2] m, [0.6,0.3] m, [0.4,0.4] m and [0.2,0.5] m.

Figure 4 represents first axis current with respect to kinematical constraints with the method described in (29). A greater value than K_{ar}, will conduct to a non admissible motion. Figure 5 shows current with respect to actuators constraints calculated with (27).

Using a fifth degree polynomial, the obtained trajectory is smooth and does not present overshoots or loops at via-points.

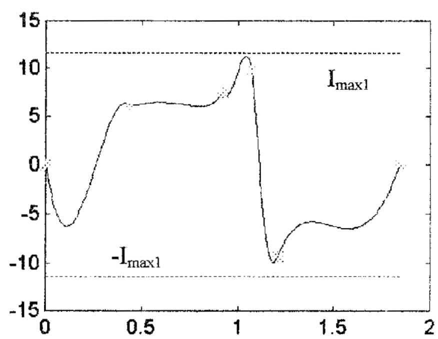

figure 4

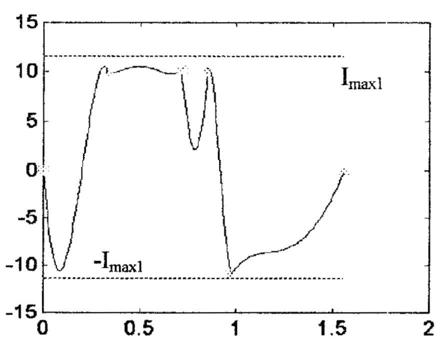

figure 5

Final times are, respectively for figures 4 and 5, of 1.84 s and 1.50 s, representing a 20% improvement.

Even if the computation time for our formulation is longer and depends on the discretisation adopted, it leads to good results. A special fixed movement has often to be repeated thousand of times. In such cases, generation of smooth trajectories which can be performed in minimum time becomes interesting even at the price of longer off line computation times (10 mn instead of 10 s). On line computation times, involving few parameters (9) or (11)-(14), remain short.

Those examples clearly show that traditional kinematical constraints are not satisfying. All the constraints (3)-(7) are important to ensure an admissible motion.

6 Conclusions

In specifying a trajectory, the physical limits of the system must be considered. It is common to model these limits as constant maximum values for acceleration and velocity. The trajectory goes from the initial to the final position with initial and final velocities zero, subject to limits on speed and acceleration. These assumptions are often unrealistic. These considerations mean that even for joint level trajectories, any assumptions about fixed acceleration limits must be based on the worst case. This results in motions that are usually slower than necessary or else the actuators may be unable to follow the requested trajectory. A more realistic assumption means that the limits on the amount of voltage and current a motor may generate are given limits.

The proposed motion generation algorithm uses the solution of polynomial equations in t_f to find the predicted arrival time. Besides, the polynomial interpolation with only few parameters, allows to generate easily the path on line. In the future, this approach will be applied to a real manipulator.

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