DYNAMIC MODELLING OF SMALL AUTONOMOUS BLIMPS

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Abstract : We are interested in blimps. A blimp is a small airship that has no metal framework and collapses when deflated. In this paper, dynamic modeling of autonomous blimps is presented, using the Newton-Euler approach. This study discusses the motion in 6 degrees of freedom since 6 independent coordinates are necessary to determine the position and orientation of this vehicle.

Key-words : Autonomous Airship, Modeling, Under-actuated systems.

1. INTRODUCTION

Unmanned aerial vehicles are a new focus of research, because of their important application potential. They can be divided into three different types: reduced scale fixed wing vehicles (airplanes), rotary wing aircraft (helicopter) or lighter than air (airships). Lighter than air vehicles suit a wide range of applications, ranging from advertising, aerial photography and survey work to surveillance and monitoring tasks. They are safe, cost-effective, durable, environmentally benign and simple to operate. Airships offer the advantage of quiet hover with noise levels much lower than helicopters. Unmanned remotely-operated airships have already proved themselves as camera and TV platforms and for specialized scientific tasks. An actual trend is toward autonomous airships.

A lighter than air craft is any vehicle that flies because it is lighter than air. This includes balloons and airships, also known as dirigibles. What makes a vehicle lighter than air is the fact that it uses a lifting gas (i.e. helium or hot air) in order to be lighter than the surrounding air. The principle of Archimedes applies in the air as well as under water. The difference between airships and balloons is that: balloons simply follow the direction of the winds. In contrast, airships are powered and have some means of controlling their direction. Non rigid airships or blimps are the most common form nowadays. They are basically large gas balloons. Their shape is maintained by their internal overpressure. The only solid parts are the gondola, the set of propeller (a pair of propeller mounted at the gondola and a tail rotor with horizontal axis of rotation) and the tail fins. The envelope holds the helium that makes the blimp lighter than air. In addition to the lift provided by helium, airships derive aerodynamic lift from the shape of the envelope as it moves through the air. The most common form of a dirigible is an ellipsoid. It is a highly aerodynamically profile with good resistance to aerostatics pressures.

A mathematical description of a dirigible flight dynamics needs to embody the important aerodynamic, structural and other internal dynamic effects (engine, actuation) that combine to influence the response of the blimp to the controls and external atmospheric disturbances. The blimp is a member of the family of under-actuated systems because it has fewer inputs than degrees of freedom.

In some studies such as [1, 2, 6, 9], motion is referenced to a system of orthogonal body axes fixed in the airship, with the origin at the center of volume assumed to coincide with the gross center

of buoyancy. In this paper, the origin of the body fixed frame is the center of gravity G.
The paper is organized as follows. Kinematics and dynamics are the subject of the second section while the dynamic model is adapted for control purposes in section three. Finally, some conclusions and perspectives are presented in the last section.

2. AIRSHIP DYNAMIC MODELING

In this section, analytic expressions for the forces and moments on the dirigible are derived. The forces and moments are referred to a system of body-fixed axes, centered at the blimp center of gravity. There are in general two approaches in deriving equations of motion. One is to apply Newton's law and Euler's law which can give some physical insight through the derivation. The other one is more complicated, it provides the linkage between the classical framework and the Lagrangian or Hamiltonian framework. In this paper, applying Newton's laws of motion relating the applied forces and moments to the resulting translational and rotational accelerations assembles the equations of motion for the 6 degrees of freedom. We will make in the sequel some simplifying assumptions : the vehicle is rigid and the earth fixed reference frame is inertial, the gravitational field is constant, the airship is supposed to be a rigid body, meaning that it is well inflated, the density of air is supposed to be uniform, and the influence of gust is considered as a continuous disturbance, ignoring its stochastic character. The buoyancy system lifetime will be limited by a number of components and factors. Included is the corrosion of unprotected airship skin, degradation of the airship skin due to thermal cycling and temperature exposure and buoyant gas leakage. High temperature will increase permeability of the airship skin and increase leakage. Introducing these factors into the dynamic model would result in very complicated partial differential equations.

2.1. Kinematics.

Two reference frames are considered in the derivation of the kinematics and dynamical equations of motion. These are the Earth fixed frame R_f and the body fixed frame R_m. The position and orientation of the vehicle should be described relative to the inertial reference frame while the linear and angular velocities of the vehicle should be expressed in the body-fixed coordinate system [4, 7].
The origin C of R_m coincides with the center of gravity of the vehicle. Its axes $\begin{pmatrix} X_c & Y_c & Z_c \end{pmatrix}$ are the principal axes of symmetry when available. They must form a right handed orthogonal normed frame.

The position of the vehicle C in R_f can be described by : $\eta_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ **eq 1**

While the orientation is given by

$$\eta_2 = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \qquad \textbf{eq 2}$$

with ϕ Roll, θ pitch and ψ Yaw angles.

Let $R \in SO(3)$ denote the orthogonal rotation matrix that specifies the orientation of the airship frame relative to the inertial reference frame in inertial reference frame coordinates. SO(3) is the special orthogonal group of order 3 which is represented by the set of all 3*3 orthogonal rotation matrices that characteristics are :

$$R^T R = I_{3*3} \; and \; \det(R) = 1 \qquad \textbf{eq 3}$$

I_{3*3} represents the 3*3 identity matrix.
The orientation matrix R is given by:

$$R = \begin{pmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix} \qquad \textbf{eq 4}$$

Where $c\theta = \cos(\theta)$ and $s\theta = \sin(\theta)$
This description is valid in the region $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. A singularity of this transformation exists for: $\theta = \frac{\pi}{2} \pm k\pi; k \in Z$.

If we use the manipulators formulation, at each instant, the configuration (position and orientation) of the airship can be described by an homogeneous transformation matrix corresponding to the displacement from frame R_f to frame R_m. The set of all such matrices is called SE(3) [10], the special Euclidean group of rigid-body transformations in three dimensions.

$$SE(3) = \left\{ A \middle| \; A = \begin{bmatrix} R & \eta_1 \\ 0 & 1 \end{bmatrix}, R \in SO(3) \subset \Re^{3*3}, \; \eta_1 \in \Re^3; R^T R = I_{3*3}; \det(R) = 1 \right\}$$ **eq 5**

SE(3) is a Lie group. $\Re^3$ represents the set of 3*1 real vectors and $\Re^{3*3}$ the set of 3*3 real matrices.

Let's introduce $V = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$ as the linear velocity (or velocity of translation of the origin C expressed in R_m and $\Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ as the angular velocity expressed in R_m.

The kinematics of the airship can be expressed in the following way :

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} R & 0_{3*3} \\ 0_{3*3} & J\ (\eta_2) \end{pmatrix} \begin{pmatrix} V \\ \Omega \end{pmatrix}$$ **eq 6**

Where

$$J\ (\eta_2) = \begin{pmatrix} 1 & s\phi.\tan\theta & c\phi.\tan\theta \\ 0 & c\phi. & -s\phi \\ 0 & s\phi / c\theta & c\phi / c\theta \end{pmatrix}$$ **eq 7**

This formulation is used for underwater vehicles as well.

If we use the metric formulation, the tangent space of SE(3), denoted by se(3) is given by:

$$se(3) = \left\{ \begin{bmatrix} sk(\Omega) & V \\ 0 & 0 \end{bmatrix}; sk(\Omega) \in \Re^{3*3}; V \in \Re^3 \right\}$$ **eq 8**

where sk(Ω) represents the skew-matrix :

$$sk(\Omega) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}$$ **eq 9**

This matrix has the property that for an arbitrary vector $U \in \Re^3$

$$sk(\Omega)U = \Omega \times U$$ **eq 10**

$\times$: represents the cross vector product in $\Re^3$.

This tangent space se(3) has the structure of a Lie algebra.

2.2. Dynamics.

The dynamics model is defined as the set of equations relying the situation of the vehicle in its position, velocity and acceleration to the control vector. The translational part is separated from the rotational part. As the blimp displays a very large volume, its virtual mass and inertia properties become significant.

The dynamic model of the autonomous dirigible is expressed in the body fixed frame as:

$$\dot{X} = -\Omega \times X + V$$

$$M\dot{V} = -\Omega \times MV - diag(D_V).V + R^T.e_3.(mg - B) + F_1 + F_2$$

$$\dot{R} = R.Sk(\Omega)$$

$$I\dot{\Omega} = -\Omega \times I\Omega - diag(D_\Omega).\Omega + \left(R^T.e_3 \times \overline{BG} \right).B - F_1 \times \overline{P_1G} - F_2 \times \overline{P_2G}$$ **eq 11**

where

$$X = R^T \eta_1$$ **eq 12**

represents the position in the body fixed frame.

m : is the mass of the airship, the propellers and actuators.

M : is the 3*3 mass matrix and includes both the airship's actual mass as well as the virtual mass elements associated with the dynamics of buoyant vehicles.

I : is the 3*3 inertia matrix and includes both the airship's actual inertias as well as the virtual inertia elements associated with the dynamics of buoyant vehicles, with respect to G .

Diag(D_V) : The 3*3 aerodynamics forces diagonal matrix.

Diag(D_Ω) : The 3*3 aerodynamics moments diagonal matrix.

F_1 *and* F_2 : Vector of propulsion forces

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$ unit vector.

Be_3 : The 3*1 buoyancy force vector. $B = \rho \Delta g$ where Δ is the volume of the envelope, ρ is the difference between the density of the ambient atmosphere ρ_{air} and the density of the helium ρ_{helium} in the envelope, g is the constant gravity acceleration.

$\overline{P_i G}$ represents the position of the i[th] propeller.

The terms $\Omega \times MV$ and $\Omega \times M\Omega$ show the centrifugal and Coriolis components.

The radiation induced forces and moments can be identified as the sum of three components [3] :

- Added mass due to the inertia of the surrounding fluid,
- Radiation induced potential damping due the energy carried away by the wind,
- Restoring forces due to Archimedes (weight and buoyancy).

Added mass should be understood as pressure – induced forces and moments due to a forced harmonic motion of the body which are proportional to the acceleration of the body. In order to allow the vehicle to pass through the air, the fluid must move aside and then close behind the vehicle [8]. As a consequence, the fluid passage possesses kinetic that it would lack if the vehicle was not in motion.

The mass of the dirigible is assumed to be concentrated in the center of gravity

$$M = \begin{pmatrix} m + X_x & X_y & X_z \\ Y_x & m + Y_y & Y_z \\ Z_x & Z_y & m + Z_z \end{pmatrix} \quad \textbf{eq 13}$$

X_x, Y_y, Z_z are the virtual mass terms of X, Y, Z axes respectively.

$$I = \begin{pmatrix} I_x + K_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y + M_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z + N_z \end{pmatrix} \quad \textbf{eq 14}$$

K_x, M_y, N_z are the virtual inertia terms of X, Y, Z about GX, GY, GZ axes respectively.

The mass and inertia matrices are positive definite.

We will assume that the added mass coefficients are constant. They can be estimated from the inertia ratios and the airship weight and dimension parameters.

The aerodynamic force can be resolved into two component forces, one parallel and the other perpendicular to the direction of motion [5]. Lift is the component of the aerodynamic force perpendicular to the direction of motion and drag is the component opposite to the direction of motion.

$$diag(D_V) = diag\left(-X_u - X_{uu}|u| \quad -Y_v - Y_{vv}|v| \quad -Z_w - Z_{ww}|w|\right) \quad \textbf{eq 15}$$

$$diag(D_\Omega) = diag\left(-L_p - L_{pp}|p| \quad -M_q - M_{qq}|q| \quad -N_r - N_{rr}|r|\right) \quad \textbf{eq 16}$$

For a slow moving object in the air, we can assume a linear relationship between the speed and the drag.

The gravitational force vector is given by the difference between the airship weight and the buoyancy acting upwards on it:

$$R^T e_3 (mg - B) = \begin{pmatrix} (mg - B)s\theta \\ -(mg - B)c\theta.s\phi \\ -(mg - B)c\theta.c\phi \end{pmatrix} \quad \textbf{eq 17}$$

The gravitational and buoyant moments are given by:

$$\left(R^T.e_3 \times \overline{BG}\right).B = B\begin{pmatrix} z_b c\theta s\phi - y_b c\theta c\phi \\ x_b c\theta c\phi + z_b s\theta \\ -y_b s\theta - x_b c\theta s\phi \end{pmatrix} \quad \textbf{eq 18}$$

where $\overline{BG} = (x_b \quad y_b \quad z_b)$ represents the position of the center of buoyancy with respect to the body fixed frame.

2.3. Propulsion

Actuators provide the means for maneuvering the airship along its course. A blimp is propelled by thrust. Propellers are designed to exert thrust to drive the airship forward. The most popular propulsion system layout for pressurized non rigid airships is twin ducted propellers mounted either side of the envelope bottom. Another one exists in the tail for torque correction and attitude control. A propeller consists of a certain number of blades rotating about an axis. Six blades per propeller are considered to be the minimum required to produce smooth and continuous thrust without excessive turbulence and inter-blade flow interference. Required power to keep a position against winds increases in proportion to the wind velocity cubed.

In aerostatics hovering (floating), its stability is mainly affected by its center of lift in relation to the center of gravity. The blimp's center of gravity can be adjusted to obtain either stable, neutral or unstable conditions. Putting all weight on the top would create a highly unstable blimp with a tendency to roll over in a stable position.

In aerodynamics flight, stability can be affected by fins and the general layout of the envelope.

Control inertia can be affected by weight distribution, dynamic (static) stability and control power (leverage) available.

The best way to obtain a well behaved and easy to control blimp in real time is :

- to reduce control inertia by putting all weight as centered as possible (main concentration close to the center of gravity).
- to increase control power, propellers should be far away from the center of gravity for maximum leverage.
- to maximize stability around 'unwanted' degrees of freedom (mainly the roll angle and velocity), the propellers should be installed such that the 'unwanted ' degrees are not controlled.

If the compensating thrust system is found to be impratical, or only partially effective, the airship must move laterally.

A blimp is an under-actuated system with two types of control : forces generated by thrusters and angular inputs controlling the direction of the thrusters (γ is the tilt angle of the propellers):

$$F_1 = \begin{pmatrix} T_M \sin\gamma \\ 0 \\ T_M \cos\gamma \end{pmatrix} \qquad F_2 = \begin{pmatrix} 0 \\ T_T \\ 0 \end{pmatrix} \qquad \textbf{eq 19}$$

where T_M and T_T represent respectively the main and tail thrusters.

Thus in building the non linear six degrees of freedom mathematical model, the additional following assumptions are made :

$$\overline{P_1G} = \begin{pmatrix} 0 \\ 0 \\ P_1^3 \end{pmatrix} \qquad \overline{P_2G} = \begin{pmatrix} -P_2^1 \\ 0 \\ 0 \end{pmatrix} \qquad \textbf{eq 20}$$

If we consider the plane XZ as a plane of symmetry, the mass and inertia matrices can be written as :

$$M = \begin{pmatrix} m + X_x & 0 & X_z \\ 0 & m + Y_y & 0 \\ Z_x & 0 & m + Z_z \end{pmatrix} \qquad \textbf{eq 21}$$

$$I = \begin{pmatrix} I_x + K_x & 0 & -I_{xz} \\ 0 & I_y + M_y & 0 \\ -I_{xz} & 0 & I_z + N_z \end{pmatrix} \qquad \textbf{eq 22}$$

If the center of gravity sits below the center of buoyancy, then

$$\overline{BG} = \begin{pmatrix} 0 & 0 & z_b \end{pmatrix}^T \qquad \textbf{eq 23}$$

The approach and landing, ground maneuvering and masting phases of flight demand the highest degree of control precision. However, due to the airship's susceptibility to gusts and thermals, its inherent aerodynamic instability and the evaporation of aerodynamic control at low airspeeds, the control system must accomplish these tasks at a time when the control over the airship is greatly reduced. The problems associated with gust sensitivity and aerodynamic instability are fundamental to ellipsoidal airships. One of the current principal challenge is the development of quiet, high efficiency propulsion systems. Any airship designed to achieve near-autonomous mast-docking must have significant mass dedicated to low speed handling. In order to estimate the performance of any airship, it is necessary to estimate the cruise drag coefficient and propulsive efficiency. In general, the thruster force and moment vector will be a complicated function depending on the vehicle linear and angular velocity and the control variables. However, under some assumptions, a linear form can be proposed. In addition, most thruster systems are driven by small DC motors designed for aerial operating conditions. Dynamics of the DC motor should also be included in the dynamics.

It is important to gain insight into the geometric structure of the equations since this knowledge can be useful in areas such as motion planning and control. Important issues for mission planning are recovery and safety.

3. ADAPTING THE DYNAMIC MODEL FOR CONTROL DESIGN

In this section, the above dynamic model (eq 11) is considered and a new equivalent model is derived, which is more convenient to work with for the purposes of control design. Equations (11) provide a full dynamic model of the blimp under consideration. However, from a control perspective, the equations are complex and difficult to work with. The system dynamics become much simpler to understand if the relationship between principal forces and torques is solved . In the dynamic equations of the system (eq 11), consider solely those forces and torques corresponding directly to the forces applied by the thrusters.

In the translation dynamics, the generated forces are

$$F = T_M(\sin\gamma).e_1 + T_T.e_2 + T_M(\cos\gamma).e_3$$

eq 24

where e_1, e_2, e_3 are unit orthogonal vectors (a base).

In the rotation dynamics, the torque contributions from the thrusters are :

$$\Gamma = T_M.P_1^3(\sin\gamma).e_2 - T_T P_2^1.e_3$$

eq 25

To simplify notation, let's set $\Gamma = \begin{pmatrix} 0 \\ \tau_2 \\ \tau_3 \end{pmatrix}$

Substituting from (eq 24), and recalling that P_1^3 and P_2^1 are not zero, yields

$$F = \begin{pmatrix} \tau_2 / P_1^3 \\ -\tau_3 / P_2^1 \\ f_3 \end{pmatrix}$$ **eq 26**

where the third component of translational forces

$$f_3 = T_M(\cos\gamma)$$ **eq 27**

Thus, the translation force can be rewritten in a form consisting of three terms. The third component corresponds intuitively to the force necessary to sustain the dirigible in stationary (or neutral) flight. Control input for longitudinal and lateral translation dynamics must be achieved by re-orienting the dirigible, using torques.

Blimps are members of the family of non-holonomic systems, characterized by acceleration level equality constraints. The longitudinal, vertical and pitch equations are related through a constant parameter, and the lateral equation is related to the yaw equation.

4. CONCLUSIONS

In this paper, we have discussed dynamic modeling of small autonomous blimps. Blimps are a highly interesting study object due to their stability properties. Llike the helicopter, they possess different properties depending on their flight modus.

Here, motion is referenced to a system of orthogonal body axes fixed in the airship, with the origin assumed to coincide with the center of gravity. The equations of motion are derived from the Newton-Euler approach. In classical aerodynamics study, the rigid body frame axes are taken as the wind axes. Although the model defined below does not correspond to any specific airship, it exhibits the general qualitative characteristics.

The airship is supposed to form a rigid body so that aero-elastic effects can be ignored. We did not discuss the case of a partially inflated blimp.

We have chosen a classical propulsion mode. Other kind of propulsion could be used such as a system of differential propellers with different tilt angles. Another formulation of the equations of motion using quaternions is actually under study. This formulation is useful because it exhibits no kinematics singularity.

After establishing the equations of motion of blimps, some questions arise :

- What are their controllability and stabilizability properties?
- How can trimming trajectories be generated for different flight operating modes?
- How can closed loop control systems be solved?

Finding answers to these questions is a part of our actual work.

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