TRAJECTORY TRACKING OF A DIRIGIBLE IN A HIGH CONSTANT ALTITUDE FLIGHT.

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Abstract **:** After being neglected during many decades, airships are experiencing a new interest, specially the stratospheric lighter than air vehicle. This paper presents a nonlinear motion control method of a dirigible in a high constant altitude flight, for path tracking. We use the property that the system is input/output linearizable. In path generation, we use the cubic spiral while the polynomial approach is preferred in motion generation. Then, some simulation results are presented.

Keywords: Airship, path planning, trajectory tracking, controllability, input-output linearization.

1. INTRODUCTION

Development of stratospheric Lighter Than Air (LTA) platforms is attracting great attention in many countries, for novel informational systems applications (Onda, 1999). Some other possible missions are presented in (Campos and Souza Colho, 1999; Elfes, Siquera Bueno and Bergerman, 1999).
An unmanned LTA platform can be maneuvered on a guided path or held geostationary in the stratosphere (20km). It is an easily modifiable, sub-orbital platform. Missions span many fields and include scientific and commercial applications (Onda, 1999) :
- high-resolution, real time remote sensing,
- environmental monitoring,
- telecommunications relays.

Compared to satellites, the airship platform in the lower stratosphere has the advantages of being closer to the ground for better resolution images and requires less power for radio wave relay. Since a stratospheric LTA platform has to be light and large in its displacement volume, a non rigid structured hull would be most adequate. Winds in the stratosphere are weak at the altitude around 20 km above the ground, where the atmospheric pressure is about 40 hPa. The average temperature at this altitude is –50°C. Air density at this altitude is about 1/20 of that at sea-level and the LTA envelope needs to be large enough to yield necessary buoyancy. This kind of airship is a super-light weight membrane structure. They are assumed super-pressurized, buoyant helium is expected to remain above atmospheric pressure inside the envelope, independent of variations in the environment of the airship. Neither the volume nor the mass changes during a flight, assuming that no ballast is dropped.
One of the basic problem is the control of this stratospheric airship. Some analysis were made in (Cook, Lipscombe and Goineau, 2000; Paiva, Bueno, gomes, Ramos and Bergerman, 1999), dependent on assumptions made from linear models for each studied airship (moving in troposphere). The linear models were obtained from non linear simulation models by linearising about a number of chosen trim speeds representative of a typical speed envelope. The decoupled linear models comprised the longitudinal and lateral motions of the neutrally buoyant airship, for speeds from the hover (0-0.1m/s) to 30m/s. In this paper, we propose a non linear approach : Input / Output linearization. Each input must go through four integrators to go to the output. Then we present some simulation results for path tracking, and finally some concluding remarks.

2. MODELLING

2.1 Dynamic model

In (Bestaoui and Hamel, 2000) we have presented a dynamic model taking into account the six degrees of freedom, using the Newton-Euler approach, to determine the position and orientation of the aerial vehicle. Buoyancy has also some effects presented in (Turner, 1973). A high constant altitude flight occurs in a horizontal plane. To simulate the horizontal trajectory of dirigibles, we assume that the difference between the horizontal velocity components of the wind and the horizontal velocity components of the airship is identically zero. Dirigible is a platform where the correct control input to be issued is at the level of forces and torques. They are propelled by thrust, and are under-actuated systems. An under-actuated mechanical system is one that does not have all of its degrees of freedom independently actuated. We model the dirigible as a neutrally buoyant rigid body in an ideal fluid. When we consider the simplified case where the dirigible just move in 2D plane, then the system dynamics in an inertial frame are (Sastry, 1999; Zhang and Ostrowski, 1999) :

$$\begin{aligned} m_x \ddot{x} &= (F_1 + F_2)\cos\theta - k_x \dot{x} \\ m_y \ddot{y} &= (F_1 + F_2)\sin\theta - k_y \dot{y} \\ J_z \ddot{\theta} &= (F_1 - F_2)\left(k_t l_x + l_y\right) - k_\tau \dot{\theta} \end{aligned} \quad \textbf{(1)}$$

where l_x and l_y are acting lengths and k_t is a rotor dependant constant. F_1 and F_2 are actuating forces of the motors.

Neglecting the air resistance and all kind of damping and friction, and assuming than the added mass coefficients are identical in the x and y directions, we can write :

$$\begin{aligned} \ddot{x} &= u_1 \cos\theta \\ \ddot{y} &= u_1 \sin\theta \\ \ddot{\theta} &= u_2 \end{aligned} \quad \textbf{(2)}$$

where

$$\begin{aligned} u_1 &= (F_1 + F_2)\big/ m_x \\ u_2 &= \left((k_t l_x + l_y) \big/ J_z \right) (F_1 - F_2) \end{aligned} \quad \textbf{(3)}$$

This an underactuated system, non-holonomic in the acceleration. The non integrable condition arising in terms of acceleration, is called the second order non holonomic condition.

*2.2 Properties of the dynamic model***.**

The aim of this paragraph is to study some properties of the model (2), such as controllability.

<u>State-space formulation</u>

If the state-space variable is respectively defined as

$$X = \begin{pmatrix} x & y & \theta & \dot{x} & \dot{y} & \dot{\theta} \end{pmatrix}^T \quad \textbf{(4)}$$

then the state space model can be expressed as:

$$\dot{X} = f_0(X) + f_1(X)u_1 + f_2(X)u_2 = f_0(X) + g(X)U \quad \textbf{(5)}$$

$$Y = h(X) = \begin{pmatrix} x \\ y \end{pmatrix} \quad \textbf{(6)}$$

Where :

$$f_0(x) = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \cos(x_3) & 0 \\ \sin(x_3) & 0 \\ 0 & 1 \end{pmatrix}$$

we must choose a number of outputs to be controlled equal to number of its inputs to make the system be square, for this aim we select x, y as outputs of interest.

These equations represent an affine system with drift. The vector fields f_0, f_1, f_2 are smooth typically real analytic on $\Re^6$. The control $U = (u_1 \quad u_2)^T$ is a measurable function taking values in a compact subset U of $\Re^2$ containing zero in its interior.

Driftless non-holonomic control systems have been studied by (Brockett, 1983; Enos, 1993; Sastry, 1999) Several important results have been derived based on the structure of Lie algebra generated by the control vector fields. The discussion of non-holonomic systems with drift has been concentrated on the dynamic extension of drift free systems.

Controllability.

Controllability indicates the existence of a path that connects an initial configuration to the desired final configuration, given a non holonomic sytem. There are many possible approaches to finding conditions for local controllability leading to different results and requiring different hypotheses. For analytic affine systems, the entire information about local properties of the system such as local controllability is contained in the values of the iterated Lie brackets of the vector fields f_0, f_1, f_2 . Moreover, these values are easily computable. Therefore it is a natural approach to look for conditions for local controllability in terms of the elements of the Lie Algebra generated by the vector fields f_0, f_1, f_2.

In the sequel, the control characteristic indices σ_i, equal to the least order of the time derivative of the output Y which is directly affected by some input, are introduced.

$$Y_i^{(\sigma_i)} = L_{f_0}^{\sigma_i} h_i + \sum_{j=1}^{m} L_{f_j} h_i u_j \quad 1 \le i \le m = 2 \qquad \textbf{(7)}$$

Proposition 1: A system represented by the equation (4) is locally controllable and the non-holonomy order is r =4, while the growth vector is (2,4,6).(The relative growth vector is (2,2,2)).

The Control characteristic indices associated with system (4) are given as follows:

$$\sigma_1 = 2$$
$$\sigma_2 = 2$$

The proof of Proposition 1 can be found in the Appendix .

As $\sigma = \sum \sigma_i = 4 = n,$ there exists no zero dynamics. In this case, the controllability indices are equal to the control characteristic indices. The system is of minimum phase.

3. TRAJECTORY GENERATION.

For trajectory generation, we must find answers to two questions. The first one is : Given a controllable non holonomic system, how does one construct a path that connects an initial configuration to the desired final configuration ? The second question being : How can we generate the motion on this given path ? For both questions, we have used knowledge acquired in terrestrial mobile robots and manipulators.

3.1. Path generation

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An understanding of the characteristic of trajectories of constant altitude, stratospheric platforms is important because safety planning is influenced by trajectories. An autonomous dirigible has the obvious advantage of freedom in motion. If smooth paths are used, path tracking tasks become easier and faster navigation is possible. We use in the sequel a well known path in mobile robotics : the spiral cubic with continuous and differentiable curvature. The curvature is defined as :

$$K(s) = b_2 s^2 + b_1 s + b_0 \quad \textbf{(8)}$$

where s represents the curvilinear abscissa and b_2, b_1, b_0 are polynomial coefficients depending on the boundary conditions.

The second part of this section presents a method for generating smooth motion for vehicles on a given path.

3.2. Motion generation

Designing reference trajectories is essentially an optimization problem. In specifying a trajectory, the physical limits of the system must be considered.

The minimum time trajectory generation can be solved in a number of ways, following the usual approach, i.e. taking as feasible limits purely kinematics constraints on vehicle velocity and acceleration.

We use the classical fifth polynomial interpolation for motion generation :

$$\begin{aligned} & s(\tau) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + a_4\tau^4 + a_5\tau^5 \\ & a_0 = a_1 = a_2 = 0 \\ & a_3 = \frac{10L}{t_f^3} \qquad a_4 = \frac{-15L}{t_f^4} \qquad a_5 = \frac{6L}{t_f^5} \end{aligned} \qquad \textbf{(9)}$$

where L represents the length of the path and t_f the total final time.

With kinematics considerations such as the maximal velocity of the airship v_{max} or its maximal acceleration a_{max}, we have :

$$t_f = \max\left(\frac{15}{8} \frac{L}{v_{max}} ; \sqrt{\frac{10}{\sqrt{3}} \frac{L}{a_{max}}} \right) \quad \textbf{(10)}$$

This approach should be generalized to constraints on the actuators and the propulsors.

4. CONTROL DESIGN

The next step is to control the motion of the vehicle onto the path. For kinematics models, the stabilization problem has essentially been solved with two types of control laws:

- time-varying piecewise continuous control.
- Time-varying continuous control.

An analogous study must be made for dynamics models.

If we are given a desired state trajectory, we would like to construct a controller which stabilizes the system to this trajectory. The system given by (1) is not input – state linearizable. However, this system having a well defined relative degree can be Input/Output linearizable. The key of this method is to transform the non linear system into a linear one by applying a state feedback and state transformation.

Proposition 2 : If the control U is chosen such that

$$\mathrm{U} = \mathrm{E}(\mathrm{X})^{-1}\left(-\mathrm{a}(\mathrm{X}) + \mathrm{V}\right) \qquad \textbf{(11)}$$

Then the system (2) can be equivalently written as

$$\ddot{\mathrm{y}} = \mathrm{V} \qquad \textbf{(12)}$$

Where V is the new input.

If the decoupling matrix E(X) is non singular then the system is locally decouplable and Input Output linearizable by state feedback

$$\begin{aligned} \mathrm{E}(\mathrm{X}) &= \frac{\partial \mathrm{h}}{\partial \mathrm{X}}\frac{\partial \mathrm{f}_0}{\partial \mathrm{X}}\mathrm{f}_0 \\ \mathrm{a}(\mathrm{X}) &= \frac{\partial \mathrm{h}}{\partial \mathrm{X}}\frac{\partial \mathrm{f}_0}{\partial \mathrm{X}}\mathrm{g} \end{aligned} \qquad \textbf{(13)}$$

The output to be controlled is the output of a chain of cascaded integrators fed by a nonlinear but invertible forcing term.

In the case of our airship, $r_1 = 2$, $r_2 = 2$ and

$$E(x) = \begin{pmatrix} \cos(x_3) & 0 \\ \sin(x_3) & 0 \end{pmatrix} \qquad \textbf{(14)}$$

where E is a singular matrix. To skip this problem , we can continue differentiating the outputs four times to appear all inputs but in this case the determinant of the decoupling matrix depend on the force u_1, this causes a serious problem when $u_1 = 0$. We can avoid this drawback by choosing the point of interest different from the gravitational center of airship.

Let :

$$\begin{aligned} x_p &= x + l\cos(x_3) \\ y_p &= y + l\sin(x_3) \end{aligned} \qquad \textbf{(15)}$$

be the coordinate of this point X_p.

In this case we have:

$$f(x) = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ x_6^2 l\cos(x_3) \\ x_6^2 l\sin(x_3) \\ 0 \end{pmatrix}, \; h(x) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \textbf{(16)}$$

applying the same calculus we can obtain: $r_1 = 2$ and $r_2 = 2$ with

$$a(x) = \begin{pmatrix} -x_6^2 l\cos(x_3) \\ -x_6^2 l\sin(x_3) \end{pmatrix}$$

$$E(x) = \begin{pmatrix} \cos(x_3) & -l\sin(x_3) \\ \sin(x_3) & l\cos(x_3) \end{pmatrix} \textbf{(17)}$$

and $\det(E(x)) = l$

The linearized model system does not contain an unobservable zero dynamics. Thus, using a stable tracking law, we can make the point X_p tracking the reference trajectory.

5. SIMULATION RESULTS

In this section, we illustrate the results of the simulation in two parts, the first part is devoted to present the reference trajectory generation using the spiral cubic technique. we give the general case, non parallel non symmetric initial and final configurations. We have chosen :

$$\begin{aligned} X_i &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T \\ X_f &= \begin{pmatrix} 100 & 100 & 0 & -45 & 0 & 0 \end{pmatrix}^T \end{aligned}$$

The distances (first three coordinates) are expressed in meters and the angles (last three coordinates : Euler representation) in degrees.

For the first part, path planning, to solve the problem of searching the optimal intermediate posture, a genetic algorithm is implemented.

In the second part we apply our control design based on the input-output linearization to allow the system tracking of reference trajectory. The airship is an under actuated system, the number of its inputs is less than the number of its outputs, we must select a number of outputs to be controlled equal to those of its inputs. We used the MATLAB command "ode23" and 0.005s as a step of simulation to simulate the control system.

Figure 1 presents the geometrical trajectories for an airship moving at constant altitude, while figures 2 and 3 show respectively the errors in the x and y directions, differences between the reference and measured trajectories. Figure 4 shows the airship gemetrical postures. We can see that the point X_p track the reference trajectory, but the airship turns around this point, we cannot control the orientation. This situation should be avoided in the future control algorithms.

6. DISCUSSIONS AND FUTURE WORK

This paper has presented an example of a dynamic model suitable for path planning and control studies, under the assumptions of ideal conditions. The non-holonomic state space model is formulated where the control inputs are those produced by the system's actuators. Some properties of the affine system are studied.

Our next objective is to determine the class of local motions that can be exhibited by the airship. We are currently studying motion planning based on Lie vectors. Motion in these directions are possible by appropriate choice of both controls U.

All of the above analyses are based on the assumption that the airship is neutrally buoyant. However, airships are commonly operated heavy and develop a small fraction of their total lift from 'body' incidence. This general case requires a 3D analysis of the trajectories and the design of an ad-hoc control method.

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<u>Appendix A</u> : Proof of Proposition 1.

For this particular class of affine non-holonomic system, to check the controllability, we have to consider the following Lie brackets.

$$f_3 = [f_0, f_1]$$
$$f_4 = [f_0, f_2]$$

Where $ad_f^i g = [f, ad_f^{i-1} g]$

with :

$$rank\begin{pmatrix} f_1 & f_2 \end{pmatrix} = 2$$
$$rank\begin{pmatrix} f_1 & f_2 & f_3 & f_4 \end{pmatrix} = 4$$
$$rank\begin{pmatrix} f_1 & f_2 & f_3 & f_4 & f_5 & f_7 \end{pmatrix} = 6$$

Thus the non-holonomy order is 4 while the growth vector is (2, 4, 0, 6) and the relative growth vector is (2,2,0,2).

It has been checked the Lie algebra spans the entire space (R^6). We can thus deduce that the system is locally controllable.

$$L_{g_{js}} L_f^k h_i(x) = 0; 1 \le j \le m; 0 \le k \le \rho_i - 2, \forall x \in U_0$$
$$L_{g_j} L_f^{\rho_i - 1} h_i(x) \ne 0;\ for.some.j; 1 \le j \le m; \forall x \in U_0$$

where $L_f h = \frac{\partial h}{\partial x} f \qquad L_f^i h = L_f\left(L_f^{i-1} h\right)$

$$L_f^0 h = h$$

The above definition of control characteristic indices is given about the origin: it may be given around any point $\bar{x} \in \mathbf{R}^n$ such that $rankG\left(\bar{x}\right) = 2$ and

$$rank\left\{dh_1(\bar{x}),\ldots,dh_m\left(\bar{x}\right)\right\} = 2$$

$$\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \Phi_1(X)\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} + \Phi_0(X)$$

with $\Phi_1 = \begin{pmatrix} a_1 & a_1 \\ -a_2 & a_2 \end{pmatrix}$ and $\Phi_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Thus we can conclude that the relative degree is: $\begin{matrix}\sigma_1 = 2 \\ \sigma_2 = 2\end{matrix}$ for the vehicle.

<u>Appendix B</u> : Proof of Proposition 2.

Input-output linearization of the system is obtained by differentiating the outputs y_i until the inputs appear. So, the system become :

$$y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{2} L_{g_i} L_f^{r_i - 1} h_i u_j$$

with $L_{g_j} L_f^{r_i-1} h_i \neq 0$ for at least one j. rewrite those equations in the compact form

$$\begin{pmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \end{pmatrix} = \begin{pmatrix} L_f^{r_1} h_1 \\ L_f^{r_2} h_2 \end{pmatrix} + \begin{pmatrix} L_{g_1} L_f^{r_1-1} h_1 & L_{g_2} L_f^{r_1-1} h_1 \\ L_{g_1} L_f^{r_2-1} h_2 & L_{g_2} L_f^{r_2-1} h_2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

or

$$\begin{pmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \end{pmatrix} = a(x) + E(x)U$$

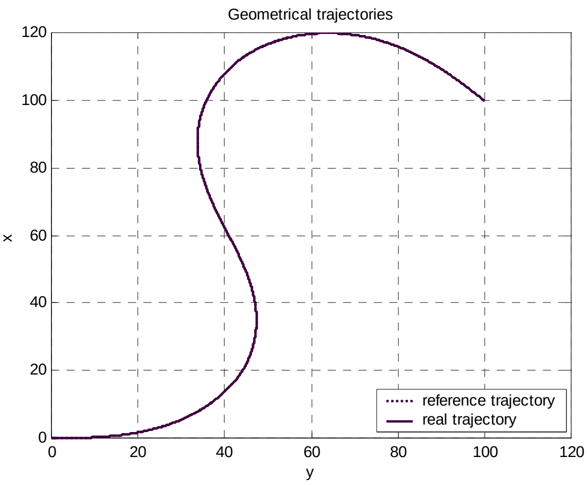
and the control that canceling the no-linearity can be done by the equation :

$U = E^{-1}(x)\begin{pmatrix} v_1 - L_f^{r_1} h_1 \\ v_2 - L_f^{r_2} h_2 \end{pmatrix}$ where E(x) is called the decoupling matrix must be non-singular, v_i are the auxiliary inputs can be chosen as follows :

$v = yd_i^{r_i} - k_1^i e_i^{r_i - 1} - \cdots - k_{r_i}^i e_i$, where $e_i = yd_i - y_i$. The coefficients k_j^i must be chosen such that the polynoms:

$$e_i^{r_i} + k_1^i e_i^{r_i-1} + \cdots + k_{r_i}^i e_i = 0$$

is Herwitz to guarantee the asymptotic convergence of the errors to zero.



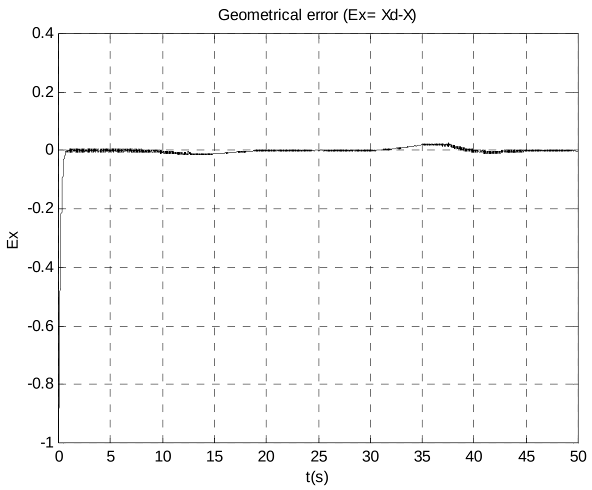


Fig 2

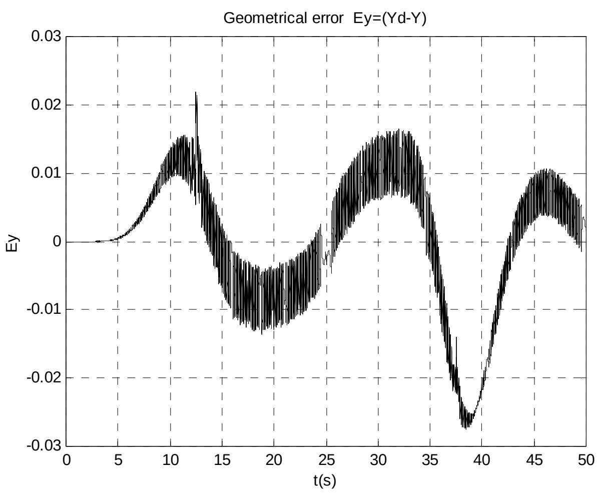


Fig 3

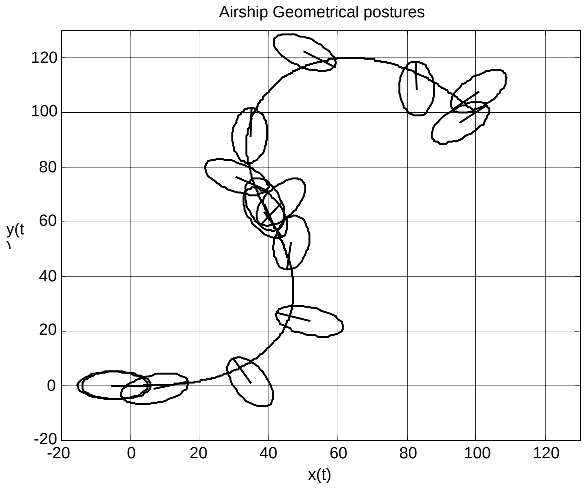

Airship Geometrical postures
y(t)
x(t)
120
100
80
60
40
20
0
-20
-20
0
20
40
60
80
100
120

Fig 4