

CHARACTERIZATION OF NON TRIM TRAJECTORIES OF AN AUTONOMOUS UNDERACTUATED AIRSHIP IN A LOW VELOCITY FLIGHT

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Abstract : The objective of this paper is to generate a desired flight path to be followed by an autonomous airship. The space is supposed without obstacles. As there is six degrees of freedom and only three inputs for the LSC airship in a low velocity flight, three equality constraints appear due to the under-actuation. When the roll ϕ and pitch θ angles as well as the longitudinal velocity u are imposed, the first constraint gives a differential equation on the yaw angle ψ , the second equality gives a differential equation on the lateral velocity v and the third one an equation on the vertical velocity w .

Index terms : Autonomous airship, trajectory generation, under-actuation.

I - INTRODUCTION

A basic problem that has to be solved by autonomous vehicles is the problem of trajectory generation. Trajectory generation means the generation and execution of a plan for moving from one location to another location in space to accomplish a desired task. The motion generation module generates a nominal state space trajectory and a nominal control input. Trajectory prediction consists in computing reference values to be given to the controller.

In the first part of the paper [BH01], the trajectories considered are trim trajectories. The general condition for trim requires that the rate of change of the magnitude of the velocity vector is identically zero, in the body fixed frame. The trim problem is generally formulated as a set of non-linear algebraic equations. The solution trajectories are helices with constant curvature and torsion. The most general trim condition resembles a spin mode. The spin axis is always directed vertically in the trim. In the second part, variable curvature helices are investigated. In [A04], a 3rd order expansion is used for transition maneuvering between 2 helices. [DMC03] investigate optimal trajectory planning for hot air balloons in linear wind fields. The objective function to be minimized is fuel consumption with respect to free end states.

This article is concerned with methods of computing a trajectory in 6 degrees of freedom space that describes the desired motion. The

contribution of this paper is the characterization of trajectories, considering the under-actuation.

This paper consists of 6 sections. Section 2 presents the kinematics while the following one introduces the dynamics. Section 4 introduces the relationship between trajectory generation and under-actuation. Simulation results are discussed in Section 5 and finally some concluding remarks are given in section 6.

II - KINEMATICS

Consider a rigid body moving in free space. Assume any inertial reference frame $\{F\}$ fixed in space and a frame $\{M\}$ fixed to the body at the center of gravity. At each instant, the configuration (position and orientation) of the rigid body can be described by a homogeneous transformation matrix corresponding to the displacement from frame $\{F\}$ to frame $\{M\}$.

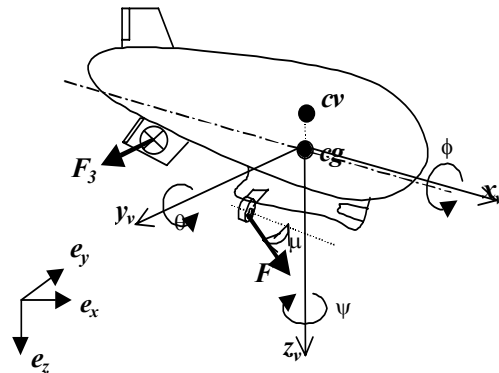


Figure 1

cg: center of gravity, cv: center of volume = cb: center of buoyancy.

The origin C of $\{M\}$ coincides with the center of gravity of the vehicle. Its axes $(X_c \ Y_c \ Z_c)$ are the principal axes of symmetry when available. They must form a right-handed orthogonal normed frame. The position of the vehicle C in $\{F\}$ can be described by $\eta_1 = (x \ y \ z)^T$ While the orientation is given by $\eta_2 = (\phi \ \theta \ \psi)^T$ with ϕ Roll, θ pitch and ψ Yaw angles. The orientation matrix R is given by:

$$R = \begin{pmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix}$$

where $c\theta = \cos(\theta)$ and $s\theta = \sin(\theta)$

This description is valid in the region $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

A singularity of this transformation exists for $\theta = \frac{\pi}{2} \pm k\pi; k \in Z$.

The kinematics of the airship can be expressed in the following way:

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} R & 0_{3*3} \\ 0_{3*3} & J(\eta_2) \end{pmatrix} \begin{pmatrix} V \\ \omega \end{pmatrix} \quad \text{eq 1}$$

$$\text{where } J(\eta_2) = \begin{pmatrix} 1 & s\phi \tan\theta & c\phi \tan\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{pmatrix}$$

$$V = (u \ v \ w)^T \quad \omega = (p \ q \ r)^T$$

ω corresponds to the angular velocity of the rigid body, while V is the linear velocity of the origin C of the frame $\{M\}$.

III - MECHANICAL SYSTEM

In this section, analytic expressions for the forces and moments of a system with added mass and inertia such as an airship are introduced [BH00, T00, F96]. An airship is a lighter than air vehicle using a lifting gas (helium in this particular case). We will make in the sequel some simplifying assumptions: the earth fixed frame is inertial, the gravitational field is constant, the airship is well inflated, the density of air is uniform. [ABL02] consider the case of an airship with small deformations analyzed via the Updated Lagrangian Method. An airship is propelled by thrust. Propellers are designed to exert thrust to drive the airship forward. The most popular propulsion system layout for pressurized non rigid airships is twin ducted propellers mounted either side of the envelope bottom. Another one exists in the tail for torque correction and attitude control. In aerostatics hovering (floating), its stability is mainly affected by its center of lift in relation to the center of gravity. The airship's center of gravity can be adjusted to obtain either stable, neutral or unstable conditions. Putting all weight on the top would create a highly unstable blimp with a tendency to roll over in a stable position. In aerodynamics flight, stability can be affected by fins and the general layout of the envelope. Control

inertia can be affected by weight distribution, dynamic (static) stability and control power (leverage) available.

The translational part being separated from the rotational part [BH00], the dynamic equations (Euler – Poincaré) are given by :

$$M\dot{V} = -\omega^*MV - b(.) + f(u) \quad \text{eq 2}$$

$$J\dot{\omega} = -\omega^*J\omega - V^*MV - \beta(.) + \tau(u)$$

where M and J are respectively the vehicle's mass and rotational tensors and τ , β and f , b represent respectively the control and non-conservative torques and forces in body axes.

For a system with added masses, the term V^*MV is non zero. We can propose

$$b(.) = R^T e_3 (mg - B) + \text{diag}(D_V).V$$

$$\beta(.) = (R^T e_3 * \overline{BG})B + \text{diag}(D_\omega).\omega \quad \text{eq 3}$$

m is the mass of the airship, the propellers and actuators. M includes both the airship's actual mass matrix as well as the virtual mass elements associated with the dynamics of buoyant vehicles. J includes both the airship's actual inertias as well as the virtual inertia elements associated with the dynamics of buoyant vehicles. As the airship displays a very large volume, its added masses and inertias become very significant [F96]. We will assume that the added mass coefficients are constant. They can be estimated from the inertia ratios and the airship weight and dimension parameters.

$\text{Diag}(D_V)$ is the 3*3 aerodynamics forces diagonal matrix. $\text{Diag}(D_\omega)$ is the 3*3 aerodynamics moments diagonal matrix.

$$e_3 = (0 \ 0 \ 1)^T \quad \text{a unit vector.}$$

$B e_3$: The 3*1 buoyancy force vector. $B = \rho \Delta g$

where Δ is the volume of the envelope, ρ is the difference between the density of the ambient atmosphere ρ_{air} and the density of the helium ρ_{helium} in the envelope, g is the constant gravity acceleration.

\overline{BG} represents the position of the center of buoyancy with respect to the body fixed frame. If the center of gravity is below the center of buoyancy, then $\overline{BG} = (0 \ 0 \ z_b)^T$

The aerodynamic force can be resolved into two component forces, one parallel and the other perpendicular to the direction of motion. Lift is the component of the aerodynamic force perpendicular to the direction of motion and drag is the component opposite to the direction of motion. As

the airship is a slow moving object in the air, we can assume a linear relationship between the speed and the drag.

$$\begin{aligned} \text{diag}(D_v) &= \text{diag}(-X_u \quad -Y_v \quad -Z_w) \\ \text{diag}(D_\Omega) &= \text{diag}(-L_p \quad -M_q \quad -N_r) \end{aligned}$$

The airship AS200 is an under-actuated system with two types of control in a low velocity flight : forces generated by thrusters and angular inputs controlling the direction of the thrusters (γ is the tilt angle of the propellers):

$$F_1 = (T_M \sin \gamma \quad 0 \quad T_M \cos \gamma)^T$$

$$F_2 = (0 \quad T_T \quad 0)^T$$

where T_M and T_T represent respectively the main

and tail thrusters. $\overline{P_1 G} = (0 \quad 0 \quad P_1^3)^T$ $\overline{P_2 G} = (P_2^1 \quad 0 \quad 0)^T$
If we consider the plane XZ as a plane of symmetry, the mass and inertia matrices can be written as :

$$M = \begin{pmatrix} m + X_u & 0 & X_z \\ 0 & m + Y_v & 0 \\ Z_x & 0 & m + Z_w \end{pmatrix}$$

$$J = \begin{pmatrix} I_x + L_p & 0 & -I_{xz} \\ 0 & I_y + M_q & 0 \\ -I_{xz} & 0 & I_z + N_r \end{pmatrix}$$

It is important to gain insight into the geometric structure of the equations since this knowledge can be useful in areas such as motion planning and control.

IV - TRAJECTORY GENERATION AND UNDERACTUATION

In this paragraph, the three equality constraints deriving from the under-actuation are sought. Considering the dynamics of the airship and its propulsion, the following under-actuation constraints can be formulated. From

$$f(u) = M \dot{V} + \omega * MV + b(\cdot) = \begin{pmatrix} T_M \sin \gamma \\ T_T \\ T_M \cos \gamma \end{pmatrix} \quad \text{eq 4}$$

$$\tau(u) = J \dot{\omega} + \omega * J\omega + V * MV + \beta(\cdot) = \begin{pmatrix} 0 \\ P_1^3 T_M \sin \gamma \\ P_2^1 T_T \end{pmatrix}$$

where the kinematics are :

$$p = \dot{\phi} - \dot{\psi} S\theta$$

$$q = \dot{\theta} C\phi + \dot{\psi} S\phi C\theta \quad \text{eq 5}$$

$$r = -\dot{\theta} S\phi + \dot{\psi} C\phi C\theta$$

we obtain the three equality constraints.

First equality constraint : The roll moment being zero, $\tau_1 = 0$ gives

$$\begin{aligned} & J_{11} \left(\ddot{\phi} - \ddot{\psi} S\theta - \dot{\psi} \dot{\theta} C\theta \right) + \\ & (J_{33} - J_{22}) \left(\dot{\theta} C\phi + \dot{\psi} S\phi C\theta \right) \left(-\dot{\theta} S\phi + \dot{\psi} C\phi C\theta \right) + \\ & D_p \left(\dot{\phi} - \dot{\psi} S\theta \right) + z_b BC\theta S\phi = 0 \end{aligned} \quad \text{eq 6}$$

Second equality constraint $P_1^3 f_1 = \tau_2$ gives

$$\begin{aligned} & J_{22} \left(\ddot{\theta} C\phi - \dot{\theta} \dot{\phi} S\phi + \ddot{\psi} S\phi C\theta + \dot{\psi} \dot{\phi} C\phi C\theta - \dot{\psi} \dot{\theta} S\phi S\theta \right) + \\ & (M_{11} - M_{33}) uv + (J_{11} - J_{33}) \left(\dot{\phi} - \dot{\psi} S\theta \right) \left(-\dot{\theta} S\phi + \dot{\psi} C\phi C\theta \right) + \\ & D_q \left(\dot{\theta} C\phi + \dot{\psi} S\phi C\theta \right) - z_b BS\theta + P_1^3 M_{11} \dot{u} + P_1^3 M_{22} \dot{w} \left(\dot{\theta} C\phi + \dot{\psi} S\phi C\theta \right) \\ & - P_1^3 M_{22} v \left(-\dot{\theta} S\phi + \dot{\psi} C\phi C\theta \right) + P_1^3 D_u u + P_1^3 (mg - B) S\theta = 0 \end{aligned} \quad \text{eq 7}$$

Third equality constraint : $P_2^1 f_2 = \tau_3$ gives

$$\begin{aligned} & J_{33} \left(-\ddot{\theta} S\phi - \dot{\theta} \dot{\phi} C\phi + \ddot{\psi} C\phi C\theta - \dot{\psi} \dot{\phi} S\phi C\theta - \dot{\psi} \dot{\theta} C\phi S\theta \right) \\ & + (M_{22} - M_{11}) uv + (J_{22} - J_{11}) \left(\dot{\phi} - \dot{\psi} S\theta \right) \left(\dot{\theta} C\phi + \dot{\psi} S\phi C\theta \right) \\ & + N_r \left(-\dot{\theta} S\phi + \dot{\psi} C\phi C\theta \right) + P_1^2 M_{22} \dot{v} - P_1^2 M_{22} w \left(\dot{\phi} - \dot{\psi} S\theta \right) + \\ & P_1^2 M_{11} u \left(-\dot{\theta} S\phi + \dot{\psi} C\phi C\theta \right) + P_1^2 D_v v - P_1^2 (mg - B) S\phi C\theta = 0 \end{aligned} \quad \text{eq 8}$$

The following approach is considered. The variations of the roll and pitch angles as well as the longitudinal velocity are imposed and the influence of the under-actuation on the variations of the yaw angle, the lateral and vertical velocities is studied. The first equality constraint (eq. 6) is equivalent to the resolution of an ordinary differential equation of the form

$$a(t) \ddot{\psi} + b(t) \left(\dot{\psi} \right)^2 + c(t) \dot{\psi} + d(t) = 0 \quad \text{eq 9}$$

where

$$\begin{aligned}
a(t) &= J_{11}(S\theta) \\
b(t) &= (J_{33} - J_{22})(C\phi S\phi C^2\theta) \\
c(t) &= -S\theta D_p - (J_{11} + J_{33} - J_{22})\dot{\theta} C\theta + 2(J_{33} - J_{22})C\theta C^2\phi \\
d &= -z_b BC\theta S\phi - J_{11}\ddot{\phi} + (J_{33} - J_{22})\dot{\theta}^2 S\phi C\phi - D_p\dot{\phi}
\end{aligned} \tag{eq 10}$$

If $\Xi(t) = \dot{\psi}(t)$ then the non autonomous generalized logistic equation must be solved :

$$a(t)\dot{\Xi}(t) + b(t)(\Xi(t))^2 + c(t)\Xi(t) + d(t) = 0 \tag{eq 11}$$

The third equality constraint (eq. 8) can be written as :

$$w(t) = \alpha_0 + \alpha_1 u + \alpha_2 v + \alpha_3 uv + \alpha_4 \dot{v} \tag{eq 12}$$

where

$$\begin{aligned}
\alpha_0 &= \frac{\left(-\dot{\psi}^2 S\theta C\theta S\phi - \dot{\psi}\dot{\theta} S\theta C\phi + \dot{\psi}\dot{\phi} C\theta S\phi + \dot{\theta}\dot{\phi} C\phi \right) (J_{22} - J_{11})}{-P_1^2 M_{22} (\dot{\psi} S\theta - \dot{\phi})} + \\
&+ \frac{-P_1^2 (mg - B) C\theta S\phi + D_p (\dot{\psi} C\theta C\phi - \dot{\theta} S\phi)}{-P_1^2 M_{22} (\dot{\psi} S\theta - \dot{\phi})} \\
&+ \frac{J_{33} (\dot{\psi} C\theta C\phi - \dot{\theta} S\phi - \dot{\psi}\dot{\theta} S\theta C\phi - \dot{\psi}\dot{\phi} C\theta S\phi - \dot{\theta}\dot{\phi} C\phi)}{-P_1^2 M_{22} (\dot{\psi} S\theta - \dot{\phi})}
\end{aligned}$$

$$\alpha_1 = -\frac{(\dot{\psi} C\theta C\phi - \dot{\theta} S\phi) M_{11}}{(\dot{\psi} S\theta - \dot{\phi}) M_{22}} \quad \alpha_2 = \frac{-D_v}{(\dot{\psi} S\theta - \dot{\phi}) M_{22}} \tag{eq 13}$$

$$\alpha_3 = \frac{M_{11} - M_{22}}{P_1^2 M_{22} (\dot{\psi} S\theta - \dot{\phi})} \quad \alpha_4 = \frac{-1}{(\dot{\psi} S\theta - \dot{\phi})}$$

The second equality constraint (eq. 7) gives :

$$\begin{aligned}
\beta_0 + \beta_1 u + \beta_2 u^2 + \beta_3 uv + \beta_4 u^2 v + \beta_5 u \dot{v} \\
+ \beta_6 v + \beta_7 \dot{v} + \beta_8 \dot{u} = 0
\end{aligned} \tag{eq 14}$$

where

$$\begin{aligned}
\beta'_0 &= J_{22} (\dot{\psi} C\theta S\phi + \ddot{\theta} C\phi + \dot{\psi}\dot{\phi} C\theta C\phi - \dot{\psi}\dot{\theta} S\theta S\phi - \dot{\theta}\dot{\phi} S\phi) \\
&+ (J_{11} - J_{33}) \left(-\dot{\theta}\dot{\phi} S\phi + \dot{\psi}\dot{\phi} C\phi C\theta + \dot{\psi}\dot{\theta} S\theta S\phi - \dot{\psi}^2 C\phi C\theta S\theta \right) \\
&+ D_q (\dot{\psi} C\theta S\phi + \dot{\theta} C\phi) - B z_b S\theta + P_1^3 (mg - B) S\theta
\end{aligned}$$

$$\begin{aligned}
\beta'_1 &= P_1^3 X_u \\
\beta'_2 &= -P_1^3 M_{22} (\dot{\psi} C\theta C\phi - \dot{\theta} S\phi) \\
\beta'_3 &= P_1^3 M_{22} (\dot{\psi} C\theta S\phi + \dot{\theta} C\phi) \\
\beta'_4 &= P_1^3 M_{11} \\
\beta'_5 &= (M_{11} - M_{22})
\end{aligned}$$

$$\begin{aligned}
\beta_0 &= \beta'_0 + \beta'_3 \alpha_0 & \beta_5 &= \beta'_5 \alpha_4 \\
\beta_1 &= \beta'_1 + \alpha_1 \beta'_3 + \alpha_0 \beta'_5 & \beta_6 &= \beta'_2 + \alpha_2 \beta'_3 \\
\beta_2 &= \beta'_2 \alpha_1 & \beta_7 &= \alpha_3 \beta'_3 \\
\beta_3 &= \beta'_3 \alpha_2 + \alpha_3 \beta'_5 & \beta_8 &= \beta'_4 \\
\beta_4 &= \beta'_4 \alpha_2
\end{aligned} \tag{eq 15}$$

A Roll and Pitch angles constant

The differential equation

$$a\ddot{\psi} + b\left(\dot{\psi}\right)^2 + c\dot{\psi} + d = 0 \tag{eq 16}$$

where

$$\begin{aligned}
a &= -J_{11}(S\theta) & b &= (J_{33} - J_{22})(S\phi C\theta)(C\phi C\theta) \\
c &= D_p(-S\theta) & d &= -z_b BC\theta S\phi
\end{aligned}$$

admits as general solution

$$\begin{aligned}
\psi(t) &= \frac{1}{2} \left(-t c - t \sqrt{c^2 - 4 d b} + \ln \left(-c^3 \sqrt{c^2 - 4 d b} + 2 d b \sqrt{c^2 - 4 d b} c + 2 d^2 b^2 \right. \right. \\
&\quad \left. \left. - 2 d b c^2 + 2 e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} d b \sqrt{c^2 - 4 d b} c + \frac{1}{2} \left(e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} \right)^2 \right. \right. \\
&\quad \left. \left. + \frac{1}{2} (c^2 - 4 d b) c^2 + 2 \left(e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} \right)^2 d^2 b^2 - \frac{1}{2} \left(e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} \right)^2 (c^2 - 4 d b) c^2 \right. \right. \\
&\quad \left. \left. + 4 e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} d^2 b^2 - 2 \left(e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} \right)^2 c^2 d b - 2 e^{\left(\frac{t \sqrt{c^2 - 4 d b}}{a} \right)} d b c^2 + \frac{1}{2} c^4 \right) / (\right. \\
&\quad \left. \left. c^4 - 6 d b c^2 - c^3 \sqrt{c^2 - 4 d b} + 4 d b \sqrt{c^2 - 4 d b} c + 8 d^2 b^2 \right) \right) / b
\end{aligned} \tag{eq 17}$$

By using the method of separation of variables and integration by partial fractions, in the constant coefficient case, logistic equation can be solved and the behavior of all solutions is analyzed.

$$\begin{aligned}
\text{For } \phi = 0; \psi(t) &= \psi_0 e^{-L_p t / I_x} \\
\text{For } \theta = 0; \psi(t) &= t \sqrt{\frac{B z_b}{C \phi (I_z - I_y)}} + \psi_0
\end{aligned}$$

For the particular case, where $\dot{\psi}$ is constant, classical trim trajectories are encountered. When $\dot{\psi}$ is not constant, Figure 2 shows the solution $\psi(t)$ of the differential equation (eq 12) while Figure 3 shows its derivative $\dot{\psi}(t)$.

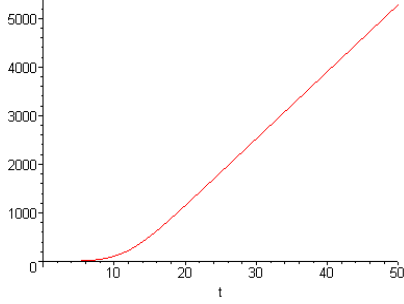


Figure 2

Even though there is a nonlinear variation of ψ in the beginning of the simulation, the yaw angle has a linear variation after a certain time .

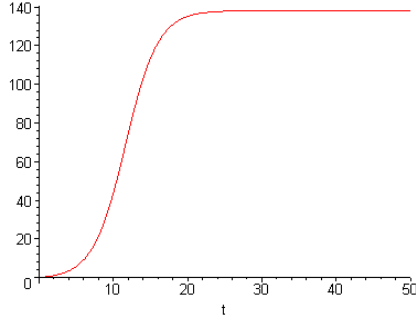


Figure 3

A transitional behavior can be recognized before the yaw velocity attains a permanent (constant) value.

B Roll and Pitch angles linear functions of time

In this paragraph, the roll and pitch angles are assumed to have linear variations:

$$\theta = \dot{\theta}_0 t + \theta_0 \quad \phi = \dot{\phi}_0 t + \phi_0$$

When the coefficients of the non autonomous logistic equation are no longer constant, no explicit solutions can be found in general and the equilibrium point may become unstable [G93]. For a study to be complete, the existence of stable periodic or stable bounded solutions is an essential part of qualitative theory of this differential equation, in order to determine non trivial solutions and study their behavior [GP03, G93, JWZ02, N00]. Nkashama[N00] proved that the logistic equation with positive non autonomous bounded coefficients has exactly one bounded solutions that is positive and does not tend to the zero solution.

Solving the first equality constraint (eq. 6), the roll moment being null, $\forall t$, implies

$$L_p \dot{\phi}_0 = 0 \Rightarrow \dot{\phi}_0 = 0.$$

Rearranging the first equality constraint with this requirement gives : $\dot{\theta}_0 C \phi_0 S \theta_0 = 0$, three cases are possible

$$\dot{\theta}_0 = 0 \quad or \quad \phi_0 = 0 \quad or \quad \phi_0 = \frac{\pi}{2}$$

If the roll angle is zero, the following differential equations must be solved :

$$-\ddot{\psi}(I_x S \theta) + \dot{\psi} \left(-L_p S \theta + \dot{\theta}_0 C \theta (I_z - I_y - I_x) \right) = 0 \quad \text{eq 18}$$

$$\begin{aligned} or \quad \ddot{\psi} + \dot{\psi} \left(a + b \dot{\theta}_0 C \theta / S \theta \right) &= 0 \\ a = L_p / (I_x) \quad b = -(I_z - I_y - I_x) / (I_x) &\quad \text{eq 19} \end{aligned}$$

the following derivative $\dot{\psi}(t)$ is obtained

$$\dot{\psi}(t) = - \frac{\dot{\theta}_0 S \theta_0^b S \theta^{-b} e^{\frac{a\theta}{\dot{\theta}_0}}}{-\cosh\left(\frac{a\theta_0}{\dot{\theta}_0}\right) + \sinh\left(\frac{a\theta_0}{\dot{\theta}_0}\right)} \quad \text{eq 20}$$

The case $\phi_0 = \frac{\pi}{2}$ gives the following differential equations

$$\begin{aligned} \ddot{\psi} + \dot{\psi} \left(a_1 + a_2 \dot{\theta}_0 C \theta / S \theta \right) + a_3 C \theta / S \theta &= 0 \\ a_1 = L_p / (I_x) \quad a_2 = (I_z - I_y + I_x) / (I_x) \quad a_3 = Bz_b / I_x &\quad \text{eq 21} \end{aligned}$$

The third equality constraint (eq. 8) gives

$$w = \delta_1 + \delta_2 uv + \delta_3 u + \delta_4 v + \delta_5 \dot{u} \quad \text{eq 22}$$

$$\delta_1 = \dot{\theta}_0 \frac{I_y + I_z - I_x}{P_2^1 M_y} - \frac{N_r C \theta}{P_2^1 S \theta M_y} - \frac{I_z \ddot{\psi} C \theta}{P_2^1 \dot{\psi} S \theta M_y}$$

$$\delta_2 = - \frac{M_y - M_x}{P_2^1 \dot{\psi} S \theta M_y}$$

$$\delta_3 = - \frac{C \theta M_y}{S \theta M_y} \quad \delta_4 = - \frac{Y_v}{\dot{\psi} S \theta M_y}$$

$$\delta_5 = - \frac{1}{\dot{\psi} S \theta}$$

Once the yaw angle is calculated, the linear and angular velocities are determined as well as the 3D path, using the kinematics.

V - SIMULATION RESULTS

The lighter than air platform is the AS200 by Airspeed Airships. It is a remotely piloted airship designed for remote sensing. It is a non rigid 6m long, 1.4m diameter and $8.6 m^3$ volume airship equipped with two vectorable engines on the sides of the gondola and 4 control surfaces at the stern. The four stabilisers are externally braced on the full and rudder movement is provided by direct linkage to the servos. Envelope pressure is maintained by air fed from the propellers into the two ballonets located inside the central portion of the hull. These ballonets are self regulating and can be fed from either engine. The engines are standard model aircraft type units. The propellers can be rotated through 120 degrees. During flight the ruddervators (Rudder and elevator) are used for all movements in pitch and yaw.

In this paragraph, three cases are presented for a normalized simulation time = 1.

$$1 - \phi_0 = \pi/12 \quad \theta_0 = \pi/6 \quad \dot{\phi}_0 = 0 \quad \text{fig. 4}$$

$$\dot{\theta}_0 = 0.1 \quad u = 1 \quad \dot{u} = 0$$

$$2 - \phi_0 = \pi/12 \quad \theta_0 = \pi/6 \quad \dot{\phi}_0 = 0.1 \quad \text{fig. 5}$$

$$\dot{\theta}_0 = 0 \quad u = 1 \quad \dot{u} = 0$$

$$3 - \phi_0 = \pi/12 \quad \theta_0 = \pi/6 \quad \dot{\phi}_0 = 0.1 \quad \text{fig. 6}$$

$$\dot{\theta}_0 = 0.1 \quad u = 1 \quad \dot{u} = 0.1$$

For each case, four subplots are presented : the first one presents the trajectory in space, the second one the variation of the yaw angle ψ , the linear velocities v and w and finally the angular velocities p , q , r .

This 3D trajectory (figure 4) represents a part of a helix with constant curvature and torsion. The yaw angle has a linear variation while the angular and linear velocities are constant.

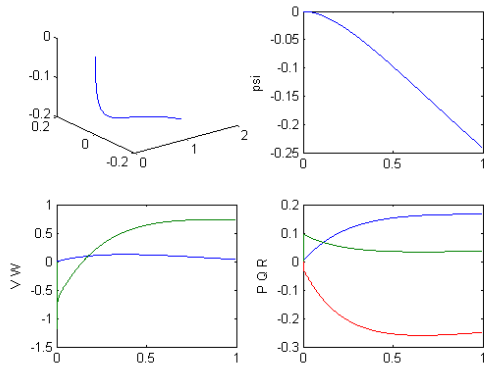


Figure 4

After a transient phenomenon, the yaw angle has a linear variation and the path tends to a classical helix with constant curvature and torsion.

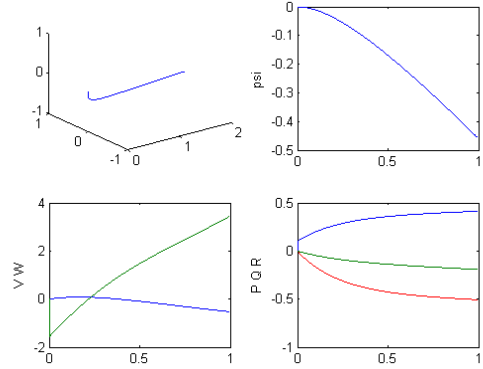


Figure 5

In this simulation, the angular and linear velocities have a slight nonlinear variation. After a certain time, they tend to have permanent values.

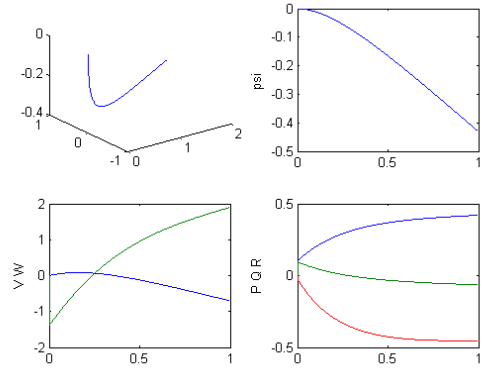


Figure 6

When the derivative of the longitudinal velocity is non zero, the nonlinear phenomenon is amplified.

VI - CONCLUSIONS

This paper addresses the problem of characterizing continuous paths on the group of rigid body motions in 3D, taking into account the under-actuation. Three differential algebraic equations must be solved as there is six degrees of freedom and three inputs. The constraints on the yaw angle is in the form of a generalized logistic equation while the others are differential algebraic equations in v and w , when the variations of the longitudinal velocity u , and the pitch and roll angles θ, ϕ are imposed. The role of the trajectory generator is to generate a feasible time trajectory for the UAV. Once the path has been calculated in the Earth fixed frame, motion must be investigated and reference

trajectories determined taking into account actuators constraints. This is the subject of our actual research. This method can be suitable for precise flight path tracking tasks, such as in landing approach. As a further application of the trajectory determination, the prediction of a cone of feasible future positions can be determined to evaluate the influence of the different kinematic parameters on the future flight path.

This methodology can be applied to other types of UAV, taking into account their characteristics. For fixed wing aircraft or helicopter, the added mass and inertia are neglected.

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